adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Algebra and functions

Mark scheme

23 questions

Exam questions and marking instructions remain credited to their board.

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Question	Scheme	Marks	AOs
1 (a)	$(4,-4)$ **or e.g.** $x=4, y=-4$ o.e.	B1	1.1b
		(1)	
(b)	$(-4,6)$ **or e.g.** $x=-4, y=6$ o.e.	B1	1.1b
		(1)	
(c)	$(6, ...)$ **or** $(..., 5)$ **or** $x=6$ **or** $y=5$ o.e.	M1	1.1b
	$(6, 5)$ **or** $x=6$ **and** $y=5$	A1	1.1b
		(2)	
			(4 marks)

Notes:

General guidelines for all parts:

Remember to check answers written against the questions.

If there is any contradiction, mark the answers given in the body of the scripts.

If there is no labelling, mark the responses in the order given.

The coordinates need to be values not just a calculation e.g. **not** $6-2$ for 4

Points can be written as a coordinate pair or separately as $x=..., y=...$

Do not allow coordinates written the wrong way round but isw if necessary

e.g. $x=4, y=-4 \rightarrow (-4, 4)$ scores B1 and isw

Condone missing brackets (one or both) e.g. $x=4, y=-4$ **or** $(4, -4$ **or** $4, -4$ for $(4,-4)$

Condone a missing comma e.g. $(4\ -4)$ for $(4,-4)$

Condone use of a semi-colon e.g. $(4\ ;-4)$ for $(4,-4)$

Condone vector notation e.g. $\begin{pmatrix}4\\-4\end{pmatrix}$ for $(4,-4)$ and condone $\left(\frac{4}{-4}\right)$

(a)

B1: $(4,-4)$ o.e. see above

(b)

B1: $(-4,6)$ o.e. see above

(c)

M1: One correct coordinate. See above.

A1: Both coordinates correct. See above.

Note that M0A1 is not a possible mark profile.

Note that in part (c), some candidates show their thinking by transforming the point piecewise e.g. $(6,-4) \rightarrow (6,4) \rightarrow (6,8) \rightarrow (6,5)$

In such cases, mark their final **pair** of coordinates

Question	Scheme	Marks	AOs
6 (a)	Attempts to use $h = A - B\,t^{1.5}$ to form one correct equation with either $17.6 = A - B \times 4^{1.5}$ or $11.9 = A - B \times 9^{1.5}$	M1	3.1b
	Correct equations $\begin{matrix} A-8B=17.6 \\ A-27B=11.9 \end{matrix}$	A1	1.1b
	Solves simultaneously to find values for A and B	dM1	1.1b
	$h = 20 - 0.3\,t^{1.5}$	A1	3.3
		(4)	
(b)	20 metres	B1ft	3.2a
		(1)	
(c)	$0 = 20 - 0.3\,T^{1.5} \left(\Rightarrow T^{1.5} = \frac{200}{3} \right) \Rightarrow T = ...$	M1	3.4
	$\left(T =\right) 16.4$	A1	1.1b
		(2)	
			(7 marks)

Notes:

(a)

M1: Forms one correct equation either $17.6 = A - 8B$ or $11.9 = A - 27B$
May be unsimplified e.g. $17.6 = A - B \times 4^{1.5}$ or $11.9 = A - B \times 9^{1.5}$

A1: $17.6 = A - 8B$ and $11.9 = A - 27B$ which may be unsimplified

dM1: Solves simultaneously to find values for *A* and *B*. It is dependent on the previous method mark so at least one equation must be correct. Do not be concerned with the process as calculators may be used. Score if values for *A* and *B* are reached from a pair of simultaneous equations.

A1: $h = 20 - 0.3\,t^{1.5}$ o.e. e.g. $t = \left(\frac{10}{3}\left(20 - h\right) \right)^{\frac{2}{3}}$ isw once a correct equation is found.

Requires the complete equation including $h = \ldots$
Just stating the values for *A* and *B* is A0 but allow A1 to be scored if the correct equation is seen in (b) or (c)

(b)

B1ft: 20 metres but ft on their *A* metres (provided $A > 0$). Condone if they have a value for *A* which is given to a greater degree of accuracy than 3sf but they round this to 3sf or better in (b).
Requires units as well (allow m for metres)

(c) **Do not be concerned with the use of *t* or *T*. If awrt 16.4 is seen in (a) it must be seen or used in (c) to score.**

M1: Sets $"20"-"0.3"T^{1.5}=0$ and proceeds to a value or expression for T e.g. $\left(\frac{"20"}{"0.3"}\right)^{\frac{2}{3}}$

Do not be concerned by the use of any inequalities instead of "="
You do not need to be concerned by the mechanics of the rearrangement, they just need to achieve a value or expression.

If no equation is seen then may be implied by a correct value for T (to the nearest integer e.g. awrt "16") or a correct expression for T for their A and B. You may need to check $\left(\frac{"20"}{"0.3"}\right)^{\frac{2}{3}}$ on your calculator. They may also achieve this by trial and improvement.

A1: $(T=)$ awrt 16.4 including $\left(\frac{200}{3}\right)^{\frac{2}{3}}$ or exact equivalent e.g. $\sqrt[\frac{3}{2}]{\frac{200}{3}}$ ignore any units for time. If an exact value is given condone the radical symbol not fully covering the fraction provided it is not clearly e.g. $\frac{\sqrt[\frac{3}{2}]{200}}{3}$

Condone $(0\leq)\ T\leq$ awrt 16.4 o.e. but not $(0\leq)\ T<$ awrt 16.4

Do not accept any greater than (or equal) inequalities e.g. $T\geq$ awrt 16.4

isw if there is a written response to (c) once a correct value or valid expression for T is seen.

Question	Scheme	Marks	AOs
1	$2x\left(x^2-12x+20\right)$ or $x\left(2x^2-24x+40\right)$	B1	1.1b
	$x^2-12x+20=(x-2)(x-10)$ or e.g. $2x^2-24x+40=(2x-4)(x-10)$	M1	1.1b
	$2x(x-2)(x-10)$	A1	1.1b
		(3)	

(3 marks)

Notes

B1: Takes a factor of $2x$ or x out of the given cubic **correctly**.
Dividing by 2 and then achieving $x\left(x^2-12x+20\right)$ does not score this mark unless recovered.

M1: Attempts to factorise their **quadratic**. Invisible brackets may be implied by later work.
Score for $(x\pm b)(x\pm d)$ where $|bd|=20$ coming from $x^2\pm12x\pm20$
or for $(ax\pm b)(cx\pm d)$ where $|ac|=2$ and $|bd|=40$ coming from $2x^2\pm24x\pm40$
May be scored if they have divided by x but not from e.g. $2x^3-24x^2+40x\rightarrow(2x-4)(x-10)$ without clear indication that they have divided by or taken out a factor of x or $2x$.
There may be incorrect intermediate steps such as $(2x-20)(2x-4)$ which do not score the mark on their own but may be ignored if they return to e.g. $(2x-20)(x-2)$.

A1: $2x(x-2)(x-10)$ or e.g. $2(x-10)(x-2)x$. Do not accept e.g. $x(2x-4)(x-10)$. Ignore = 0
ISW after a fully correct factorisation e.g. $2x(x-2)(x-10)$ that becomes $x(x-2)(x-10)$ and ignore any attempt to find roots (before or after factorisation). Allow $(2x)(x-2)(x-10)$

Alternative:

B1: Takes a factor of $(x-2)$ or $(x-10)$ out **correctly** i.e. $(x-2)\left(2x^2-20x\right)$ or $(x-10)\left(2x^2-4x\right)$. Must be seen as a product of factors and not just in a division attempt but may be implied by later work.

M1: Attempts to factorise their **quadratic**. Invisible brackets may be implied by later work.
Score for $ax(bx\pm c)$ where $|ab|=$"2" and $|ac|=$ their "20" or "4" coming from Ax^2+Bx

A1: As main scheme.

Note: Solutions that solve the cubic = 0 to achieve x = 0, 2 and 10 and then arrive at e.g. $x(x-2)(x-10)$ score no marks without a prior line of working such as $x\left(x^2-12x+20\right)$ (would score M1) or $x\left(2x^2-24x+40\right)$ (would score B1).

Some examples:

- $2x^3-24x^2+40x\rightarrow x(x-2)(x-10)$ scores B0M0A0
- $2x^3-24x^2+40x\rightarrow(2x-4)(x-10)$ scores B0M0A0
- $x^2-12x+20\rightarrow(x+2)(x-10)$ scores B0M1A0
- $x^3-12x^2+20x\rightarrow x(x-2)(x-10)$ scores B0M1A0
- $x\left(2x^2-24x+40\right)\rightarrow x(x-2)(x-10)$ scores B1M0A0
- $x\left(2x^2-24x+40\right)\rightarrow\ \{x(2x+2)(2x+40)\rightarrow\}\ x(2x+2)(x+20)$ scores B1M1A0
- $2x\left(x^2-12x+20\right)\rightarrow 2x(x-1)(x+20)$ scores B1M1A0
- $2x(x-2)(x-10)$ on its own scores B1M1A1

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
7	Any one of $A=3,\ B=-2,\ C=4,\ D=-5$ correct from correct work	B1	2.2a
	Finds the value of B, C or D using a valid method e.g. $3x^3-8x^2-6x-11=(Ax+B)(x+1)(x-3)+C(x-3)+D(x+1)$ **and** substitutes $x=-1\Rightarrow\ldots=\ldots C\Rightarrow C=\ldots$ **or** substitutes $x=3\Rightarrow\ldots=\ldots D\Rightarrow D=\ldots$	M1	1.1b
	Valid method for finding all four constants. e.g. Deduces $A = 3$, substitutes $x = 3$ leading to $D = \ldots$ **and** substitutes $x = -1$ leading to $C = \ldots$ **and** substitutes e.g. $x = 0$ leading to $B = \ldots$	M1 (A1 on EPEN)	1.1b
	$A=3,\ B=-2,\ C=4$ and $D=-5$	A1	2.1
		(4)	
			(4 marks)

Notes

Note: Condone different uses of '*A*', '*B*' etc for the M marks provided the intention is clear but values must be correctly attributed to each letter or implied (if embedded in a final expression) for the A mark.

B1: Any one of $A=3,\ B=-2,\ C=4,\ D=-5$ correct (usually $A = 3$ is deduced) from correct work. May be implied by $3x$ seen. Condone $A = 3x$ for this mark.

M1: This mark is for finding the value of *B*, *C* or *D*, using a valid method, which may be in one of the following ways:

- Attempts algebraic division of $\left(3x^3-8x^2-6x-11\right)$ by their expanded $(x+1)(x-3)=\left("x^2-2x-3"\right)$ **or** separately by both $(x+1)$ **and** $(x-3)$ as far as $\mathbf{3x\pm 2}$
  Ignore any remainder for this mark. For reference, the correct remainder is $-x-17$

- Algebraic division might be seen as $3x\left(x^2-2x-3\right)\pm 2\left(x^2-2x-3\right)+"9x-4x-6-6x-11"$ o.e. and scores the B1 for the $3x$ in the correct place and M1 for both the $3x$ and the ± 2 in the correct places.

  In the next two bullets, they should achieve $3x^3-8x^2-6x-11=(Ax+B)(x+1)(x-3)+C(x-3)+D(x+1)$ o.e. but condone small slips e.g. sign errors or a slip on a constant. The bracketed expressions should be in the correct places and Ax and B should both be multiplied by $(x+1)(x-3)$.
  Note that $3x^3-8x^2-6x-11=C(x-3)+D(x+1)$ without sight of the identity above (condoning the previously mentioned slips) is an incorrect method and scores M0.

- Attempts to multiply up by $(x+1)(x-3)$ and substitutes one of $x=-1$ **or** $x=3$ leading to $C=...$ **or** $D=...$ (you do not need to check the accuracy of their substitution)

- **or** attempts to multiply up by $(x+1)(x-3)$, expands and compares coefficients leading to a value for *B*, *C* or *D*. Do not be concerned about how they expand the brackets.

- Attempts at the cover up method may be seen. If unsure then send to review.

M1: For finding values of all four constants via a valid method.

- If attempting algebraic division, they must have a quotient of $3x \pm k$ for constant k ($3x \pm 2$ is **not** required for this mark) **and** they must go on to deal with their remainder using a correct method, i.e., $"-x-17" = C(x-3)+D(x+1)$ followed by either comparing coefficients or substitution of appropriate values. You do not need to check the accuracy of their substitution.

- For those attempting repeated algebraic division, the values of C and D must come from a correct method and so cannot be equal to e.g. the remainders from successive division by $(x+1)$ and $(x-3)$ or vice versa. If, e.g., they divide by $(x+1)$ (to get remainder R_1) and then $(x-3)$ (to get remainder R_2) they should get $3x \pm k + \frac{R_1}{(x+1)(x-3)} + \frac{R_2}{(x-3)}$. They can then either combine the two fraction terms before using partial fractions or they can use partial fractions on $\frac{R_1}{(x+1)(x-3)}$ before combining their two terms in $\frac{1}{(x-3)}$.

- If using $3x^3-8x^2-6x-11 = (Ax+B)(x+1)(x-3)+C(x-3)+D(x+1)$ o.e. they may use a combination of substitution and comparing coefficients, but the method must be valid. Do not be concerned about how they expand the brackets, and you do not need to check the accuracy of their substitution.

- If using substitution only they are likely to use $x = -1$, 3 and 0 but the final value for x could be anything (and in fact they could use three/four different values for x). You do not need to check the accuracy of their substitution.

- If attempting to compare coefficients, do not be concerned about how they expand the brackets.

A1: Requires:

- all of $A=3$, $B=-2$, $C=4$ and $D=-5$ which may be stated separately in their work ($A = 3x$ that is not recovered would score final A0).
- **or** $3x-2+\frac{4}{x+1}-\frac{5}{x-3}$ seen. Condone $3x+-2+\frac{4}{x+1}+\frac{-5}{x-3}$

There is no requirement to write out the form given in the question once correct values for A, B, C and D have been seen.

ISW after the four correct values (e.g. if they swap C and D in the fractions so $A=3,\ B=-2,\ C=4,\ D=-5$ followed by $3x-2-\frac{5}{x+1}+\frac{4}{x-3}$ scores full marks).

Repeated Algebraic Division

An example for repeated algebraic division. Note that the repeated division could be carried out in the other order – i.e. dividing by $(x - 3)$ first followed by $(x + 1)$.

$$x+1 \overline{)\ 3x^3 - 8x^2 - 6x - 11}\quad 3x^2 - 11x + 5$$

$3x^3 + 3x^2$

$-11x^2 - 6x$

$-11x^2 - 11x$

$5x - 11$

$5x + 5$

-16

Followed by

$$x-3 \overline{)\ 3x^2 - 11x + 5}\quad 3x - 2$$

$3x^2 - 9x$

$-2x + 5$

$-2x + 6$

-1

B1 Scored for either '3' **or** '–2' embedded in the quotient.

1st M1 Scored for achieving $3x \pm 2$

$$\frac{3x^3 - 8x^2 - 6x - 11}{(x+1)(x-3)} = 3x - 2 + \frac{-16}{(x+1)(x-3)} + \frac{-1}{(x-3)}$$

Correctly deals with both remainders from the repeated division.

$$\frac{3x^3 - 8x^2 - 6x - 11}{(x+1)(x-3)} = 3x - 2 + \frac{-16}{(x+1)(x-3)} + \frac{-(x+1)}{(x+1)(x-3)}$$

Sets up an identity in x and C and D using partial fractions.

$$\frac{3x^3 - 8x^2 - 6x - 11}{(x+1)(x-3)} = 3x - 2 + \frac{-x - 17}{(x+1)(x-3)}$$

$$-x - 17 = C(x-3) + D(x+1)$$

$$x = 3 \Rightarrow D = -5 \text{ and } x = -1 \Rightarrow C = 4$$

2nd **M1** Finds the values of all four constants via a correct method (in this case by substitution of appropriate values).

$A = 3$, $B = -2$, $C = 4$. $D = -5$

A1 $A = 3$, $B = -2$. $C = 4$ and $D = -5$

Alternatively, splits the first fraction using partial fractions and sets this equal to their remainder divided by $(x + 1)(x - 3)$ and multiplies up correctly.

$$\frac{3x^3 - 8x^2 - 6x - 11}{(x+1)(x-3)} = 3x - 2 + \frac{F}{(x+1)} + \frac{G}{(x-3)} + \frac{-1}{(x-3)}$$

$$-16 = G(x+1) + F(x-3)$$

$$x = -1 \Rightarrow F = 4,\ x = 3 \Rightarrow G = -4$$

2nd **M1** Finds the values of all four constants via a valid method.

$$\frac{3x^3 - 8x^2 - 6x - 11}{(x+1)(x-3)} = 3x - 2 + \frac{4}{(x+1)} + \frac{-4}{(x-3)} + \frac{-1}{(x-3)}$$

$$\frac{3x^3 - 8x^2 - 6x - 11}{(x+1)(x-3)} = 3x - 2 + \frac{4}{(x+1)} - \frac{5}{(x-3)}$$

A1 $A = 3$, $B = -2$, $C = 4$ and $D = -5$ seen embedded in a final expression.

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
11(a)	$\frac{300-1500}{50-30}=(-60)$	M1	3.1b
	$y-1500="-60"(x-30)$	dM1	1.1b
	$y=3300-60x$	A1	3.3
		(3)	
(b)	Either $x("3300-60x")$ or $-10("3300-60x")$ seen	M1	1.1b
	$(P=)\ \frac{x("3300-60x")-10("3300-60x")-8000}{1000}$	dM1	3.3
	$P=3.3x-0.06x^2-33+0.6x-8$ $P=-0.06x^2+3.9x-41*$	A1*	2.1
		(3)	
(c)	$\frac{-3.9\pm\sqrt{3.9^2-4(-0.06)(-41)}}{2\times-0.06}$	M1	3.1b
	$13.18<x<51.82$ (or $13.19\leq x\leq 51.81$)	A1	3.2a
		(2)	
(d)	£32.50	B1ft	3.4
		(1)	
(e)	$(P=)\ -0.06(35)^2+3.9(35)-41=...$	M1	3.4
	£22 000 which is close to £21 750 so the model is suitable.	A1	3.5a
		(2)	
			(11 marks)

Notes

(a)

M1: Attempts the gradient of the straight line either way round.

Look for $\frac{300-1500}{50-30}=(-60)$ o.e. or $\frac{50-30}{300-1500}=\left(-\frac{1}{60}\right)$ o.e.

Condone 1 sign/copying slip only if a correct formula is seen or implied e.g. $\frac{y_2-y_1}{x_2-x_1}$

Alternatively, sets up simultaneous equations $1500=30a+b$ and $300=50a+b$ (or $30=1500a+b$ and $50=300a+b$) and attempts to solve, leading to a value for a and/or b.

If using simultaneous equations, condone slips.

dM1: Attempts the equation relating x and y with either coordinate (50, 300) or (30, 1500) used correctly. Look for $y-1500="-60"(x-30)$ or $x-30="-\frac{1}{60}"(y-1500)$

or using (50, 300). Note that e.g. $y-30="-60"(x-1500)$ scores dM0 as the values have been used incorrectly. If using $y=mx+c$ they must proceed to $c=\ldots$

If using simultaneous equations, they must proceed to values for a **and** b

(Note for reference x in terms of y is $x=-\frac{1}{60}y+55$)

If they use e.g. $\frac{y-300}{1500-300}=\frac{x-50}{30-50}$ both M's can be scored together.

Condone 1 sign/copying slip only if a correct formula is seen or implied.

A1: cao Either $y=3300-60x$ or $y=-60x+3300$ or equivalent with y in terms of x

The correct equation with no working scores full marks.

(b)

M1: Attempts either $\pm x\left("3300-60x"\right)$ or $\pm 10\left("3300-60x"\right)$ with or without dividing by 1000.

May see both in one step, i.e., $\pm\left(x\pm 10\right)\left("3300-60x"\right)$

Alternatively, allow $\left(P=\right)\ \pm xy\pm 10y\pm 8000$ with or without dividing by 1000.

Allow their model for y to be any expression in x (i.e. it does not need to be linear) for M1.

dM1: Full attempt to find P in terms of x only. There is no need to see a LHS for this mark.

$\left(P=\right)\ \dfrac{x\left("3300-60x"\right)-10\left("3300-60x"\right)-8000}{1000}$ or $\left(P=\right)\ \dfrac{\left(x-10\right)\left("3300-60x"\right)-8000}{1000}$

Condone the consistent absence of division by 1000 e.g.

$\left(P=\right)\ x\left("3300-60x"\right)-10\left("3300-60x"\right)-8000$ or $\left(P=\right)\ \left(x-10\right)\left("3300-60x"\right)-8000$

Condone invisible brackets if recovered. Dependent on the previous method mark.

Their model for y must now be a linear expression in x for dM1.

A1*: Achieves the given answer through rigorous argument with all elements pulled together clearly. Requires P = (not just "profit = ") at some point which may be on a previous line.

Any brackets seen must have been expanded in an intermediate line before the given answer.

Allow otherwise correct work leading to $P=-60x^2+3900x-41000$ followed by $P=-0.06x^2+3.9x-41$ (without justifying the division by 1000).

Allow recovery if they write e.g. 800 but it is recovered before the given answer.

Note: Attempts by verification are unlikely to score any marks. If unsure, send to review.

e.g. $x=10\rightarrow P=-0.06\left(10\right)^2+3.9\left(10\right)-41=-8$ without an accompanying argument involving the (x, y) values (50, 300) and (30, 1500) scores M0dM0A0*.

(c)

M1: Attempts to find either end point of the interval which may come from a calculator.

The exact values are $\dfrac{195\pm 5\sqrt{537}}{6}$ but may just be seen in the quadratic formula, e.g., $\dfrac{-3.9\pm\sqrt{3.9^2-4\left(-0.06\right)\left(-41\right)}}{2\times -0.06}$ or using completing the square – see general marking principles.

Accept awrt 13.2 or awrt 51.8 for this mark.

A1: Either $13.18<x<51.82$ or $13.19\leq x\leq 51.81$ No units are required but 2d.p. are required.

Correct answer scores both marks. Allow e.g. $13.18<x\leq 51.81$ or $13.19\leq x<51.82$

Allow e.g. "$x>13.18$ and $x<51.82$" or "$x>13.18\cap x<51.82$" but not use of exact values and not "$x>13.18$, $x<51.82$" or "$x>13.18\cup x<51.82$" or "$x>13.18$ or $x<51.82$"

Must be x or e.g. "selling price" for the range, not y or P.

(d)

B1ft: £32.50. Units and 2dp required. Not £32.50p. Ignore attempts to find maximum profit.

Allow FT on the mean of their end points from (c) (their end points must be > 0, their answer requires £ and 2dp).

May come directly from the given answer to (b) using $x=-\dfrac{b}{2a}=-\dfrac{3.9}{2\times -0.06}=$ £32.50

or from $\dfrac{\mathrm{d}P}{\mathrm{d}x}=-0.12x+3.9=0$ or directly from a calculator. No working required.

Note: Following use of 13.2 and/or 51.8 (not e.g. 51.80) with either type of inequality in part (c), £32.5 will score B1ft so that the missing decimal place(s) is only penalised once.

(c)

M1: Attempts to substitute 35 (and not any other value) into the given equation for P to find a value for P.
May be implied by 22 or 22000. Condone slips.
Alternatively, substitutes $P = 21.75$ into the model, and solves via any acceptable method, including by calculator, to find a value for x. If using 21750 it must be substituted into their $(P =) \ -60x^2 + 3900x - 41000$. Note $x = 35.73$ or 29.27

A1: Requires:

- A correct value. Usually (£)22 000 but accept, e.g., 22 thousand (pound). If using 22 they must compare with 21.75. The alternative requires x = awrt 35.7 to be compared to 35.
- A correct comparison. Some examples below:
  - "Close to (£)21 750" o.e. e.g. "the model predicts 22000 and they make close to this amount"
  - "Agree to 2sf". Do not condone "it rounds to 22 000" without mention of the degree of accuracy.
  - 1.1% error (accept approx. 1%) Do not be concerned about the mechanics of any percentage error calculation seen.
  - "The difference is small"
  - Condone e.g. $22 \approx 21.75$ o.e.
  - "**Only** £250 off". However, just stating "£250 off" without suggesting this is a (relatively) small difference is not sufficient for this component. Similarly, "£350 off is a small difference" is incorrect (requires correct £250).
- A conclusion. e.g. Concludes it (the model) is suitable / reliable / good / fairly accurate / accurate.

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
12(a)	$4\lvert 1-3\rvert -5=...$ or $4\lvert -1-3\rvert -5=...$	M1	1.1b
	3 and 11	A1	1.1b
		(2)	
(b)	$2(-5)+17=...$	M1	1.1b
	$\text{gf}(x)\geq 7$	A1	2.2a
		(2)	
(c)	$k\ldots -4$	B1	2.2a
	Minimum at $(3,-5)$ so $k\ldots -\frac{5}{3}$	M1	3.1a
	$k<"-\frac{5}{3}"$	A1ft	1.1b
		A1	2.5
		(4)	
			(8 marks)

Notes

(a)

M1: Attempts $4\lvert 1-3\rvert -5=...$ or $4\lvert -1-3\rvert -5=...$ proceeding to a value.
Either correct value (3 or 11) that is clearly their answer can imply the M mark.
Alternatively, attempts any of $-4(-1-3)-5$ or $-4(1-3)-5$ or $-4(-1)+7$ or $-4(1)+7$
i.e. without the modulus signs using the 'left' branch of the function.

A1: Both 3 and 11 found and no other values that are clearly meant to be their answers (i.e. ignore reference to $a = 1$ or $a = -1$). Accept "3, 11" or "3 or 11" or "3 and 11" or {3, 11}
but not (3, 11). Condone $y = 3$, 11 but not $x = 3$, 11.

(b)

M1: Attempts $2(-5)+17$ which may be implied by sight of 7 used in their range, including in an incorrect range e.g. f $(x) > 7$
Alternatively, attempts $\text{gf}(x)=2(4\lvert x-3\rvert -5)+17$ $(=8\lvert x-3\rvert +"7")$ **and** replaces $\lvert x-3\rvert$ with 0 or x with 3 or simply deduces the minimum value of their "7" in this case.
Must be an attempt at gf and not fg.

A1: Deduces the range $\text{gf}(x)\geq 7$ using correct notation.
Condone $y \geq 7$ but **not** e.g. $x \geq 7$ or $\text{g}(x)\geq 7$ or $\text{f}(x)\geq 7$ or $\text{fg}(x)\geq 7$
Other acceptable notation includes: $\text{gf}(x)\in[7,\infty)$
Do **not** accept $\text{gf}(x)\in[7,\infty]$ or $\text{gf}(x)\in(7,\infty)$

(c)

B1: Deduces $k\ldots -4$ allow any equality or inequality here. Condone $k\ldots \pm 4$ (but not just $k \ldots 4$) and condone e.g. "gradient … – 4"
May be seen coming from e.g. $kx=-4x+7\rightarrow (k+4)x=7$ or $x=\frac{7}{k+4}$ which is acceptable but see the note below. Should not be x or y for this mark but may be m.
Note: Finding the 'discriminant' of a linear equation e.g. $(k+4)x-7=0$ in order to obtain $k \ldots -4$ is invalid and cannot earn either the B1 or the final A1 mark unless there is an alternative valid reason given.

M1: Attempts to use the vertex to find the (upper) limit for k. Look for $\frac{\pm 5}{\pm 3}$ but allow this mark to be scored for $\frac{\pm B}{\pm A}$ if there is clear evidence that they think the vertex is at (A, B)

Allow use of *m*, *k*, *y* or another variable for this mark.

Alt 1 via squaring and the discriminant:

$$kx = 4|x-3|-5 \Rightarrow kx+5 = 4|x-3|$$
$$\Rightarrow k^2x^2+10kx+25 = 16\left(x^2-6x+9\right)$$
$$\Rightarrow \left(k^2-16\right)x^2+\left(10k+96\right)x-119 = 0$$
$$\Rightarrow \left(10k+96\right)^2-4\left(k^2-16\right)\left(-119\right) = 0$$
$$\Rightarrow 576k^2+1920k+1600 = 0 \Rightarrow k = -\frac{5}{3}$$

Scores M1 for setting $kx = 4|x-3|-5$, isolating $|x-3|$ (or $4|x-3|$), squaring both sides, using $b^2-4ac\ldots 0$ where … is any equality or inequality, and solving the resulting 3TQ using the usual rules and may be by calculator, leading to a value for *k*.
Condone slips in expanding the brackets.

Alt 2 via solving simultaneous equations:

$$\text{e.g. } kx = 4(x-3)-5 \Rightarrow x = \frac{-17}{k-4}$$
$$kx = 4(3-x)-5 \Rightarrow \left(k+4\right)x = 7$$
$$\Rightarrow \left(k+4\right)\left(\frac{-17}{k-4}\right) = 7 \Rightarrow k = -\frac{5}{3}$$

Scores M1 for setting $kx = 4(x-3)-5$ and $kx = 4(3-x)-5$, eliminating *x*, and solving for *k*

A1ft: Uses their value of *k*, found using one of the above methods, as the upper end of the range for *k*, i.e., $k < "-\frac{5}{3}"$ which may be seen as part of their range. Condone $k \leq "-\frac{5}{3}"$ for this mark.
Score once seen as an upper limit and do not withhold if they then incorrectly combine as e.g. $-\frac{5}{3} < k < -4$
Condone the use of *m* for this mark, but not *x* or *y*.

A1: $-4 \leq k < -\frac{5}{3}$ o.e. and no other solutions seen. Their range must use *k* and not e.g. *x*, *y* or *m*.
Other acceptable notation includes: $k \in \left[-4, -\frac{5}{3}\right)$, "$k < -\frac{5}{3}$ and $k \geq -4$", $k < -\frac{5}{3} \cap k \geq -4$
Do not accept e.g. "$k < -\frac{5}{3}$, $k \geq -4$" or "$k < -\frac{5}{3}$ or $k \geq -4$" or "$k < -\frac{5}{3} \cup k \geq -4$"
Allow $-1.\dot{6}$ but **not** –1.6 or –1.6…for $-\frac{5}{3}$

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
1	$g(3)=3(3)^3-20(3)^2+3(k+17)+k=0$	M1	3.1a
	$4k-48=0\Rightarrow k=...$	M1	1.1b
	$\{k=\}12$	A1	1.1b
		(3)	

(3 marks)

Notes

Note: Ignore any use of $f(x)$ in place of $g(x)$ throughout.

M1: Attempts $g(3)=0$ to set up a linear equation in *k*. The = 0 may implied by their value of *k*.

Expect to see 3 substituted for *x* at least twice but condone minor slips copying the function.
May be scored for e.g. 81 – 180 + 3(*k* + 17) + *k* = 0
Missing brackets may be recovered.

Attempting $g(-3)=0$ scores M0 but note that the second M1 is available.

If algebraic division is attempted, they need to achieve a linear remainder in *k* only and set =0 Condone slips in their calculations.

As a minimum, expect to see $3x^2+\lambda x,\ \lambda\neq 0$ as their quotient **leading to a linear remainder in *k* only set = 0** (the = 0 may be implied by their value for *k*).

For reference, the correct division is

$$\begin{array}{r} 3x^2-11x+k-16 \\ x-3\overline{)\,3x^3-20x^2+(k+17)x+k} \\ \underline{3x^3-9x^2} \\ -11x^2+(k+17)x+k \\ \underline{-11x^2+33x} \\ (k-16)x+k \\ \underline{(k-16)x-3k+48} \\ 4k-48=0 \end{array}$$

You may also see variations on the table below.

Here, the M1 is scored when the sum of both coefficients of *x* are equated to (*k* + 17)

	$3x^2$	$-11x$	$-\frac{k}{3}$
x	$3x^3$	$-11x^2$	$-\frac{k}{3}x$
-3	$-9x^2$	$33x$	k

$33-\frac{k}{3}=k+17$ scores M1

M1: Scored for attempting to solve a linear equation in *k* having attempted g (±3) = 0

Do not be concerned about the process, e.g. $-81+180-3(k+17)+k=0\rightarrow k=...$ scores M1.

Via division they must have a linear remainder in *k* set = 0

The = 0 may be implied by their value for *k* in all approaches.

A1: Obtains $\{k=\}12$ only. Do not accept e.g. $\frac{48}{4}$ Allow slips in working to be recovered.

Condone e.g. $x=12$ provided it has come from a linear equation in *k*.

Note that e.g. $3(3)^3-20(3)^2+3(k+17)+k\{=0\}\rightarrow k=12$ and

$81-180+3k+51+k\{=0\}\rightarrow k=12$ are sufficient to imply M1M1A1.

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
5(a)	$\mathrm{f}(x)=\dfrac{2x-3}{x^2+4}\Rightarrow \mathrm{f}'(x)=\dfrac{2\left(x^2+4\right)-2x(2x-3)}{\left(x^2+4\right)^2}$ **or** $\mathrm{f}(x)=(2x-3)\left(x^2+4\right)^{-1}\Rightarrow \mathrm{f}'(x)=2\left(x^2+4\right)^{-1}-2x(2x-3)\left(x^2+4\right)^{-2}$	M1 A1	1.1b 1.1b
	$\mathrm{f}'(x)=\dfrac{-2x^2+6x+8}{\left(x^2+4\right)^2}$	A1	1.1b
		(3)	
(b)	$-2x^2+6x+8=0\Rightarrow -2(x+1)(x-4)=0\Rightarrow x=-1,\ 4$	B1ft (M1 on EPEN)	1.1b
	Chooses correct region for their numerator and their critical values $x<-1$ or $x>4$	M1 A1	1.1b 2.2a
		(3)	
			(6 marks)

Notes

(a)

M1: Attempts the quotient rule to obtain an expression of the form $\dfrac{P\left(x^2+4\right)-Qx(2x-3)}{\left(x^2+4\right)^2}$ $P,Q>0$

condoning bracketing errors/omissions or minor slips (e.g. $2x+3$ or $x+4$).

Condone, e.g. $\left\{\mathrm{f}'(x)=\right\}\dfrac{2\left(x^2+4\right)-2x(2x-3)}{\left(x^2+4\right)}$ provided an incorrect formula is not quoted.

May also see the product rule applied to $(2x-3)\left(x^2+4\right)^{-1}$ to obtain an expression of the form $\left\{\mathrm{f}'(x)=\right\}P\left(x^2+4\right)^{-1}-Qx(2x-3)\left(x^2+4\right)^{-2}$ $P,Q>0$ condoning bracketing errors/omissions or minor slips (e.g. $2x+3$ or $x+4$)

A1: Fully correct differentiation in any form with correct bracketing which may be implied by subsequent working.

A1: $\mathrm{f}'(x)=\dfrac{-2x^2+6x+8}{\left(x^2+4\right)^2}$ or simplified equivalent, e.g. numerator terms in a different order.

Allow recovery from "invisible" brackets earlier and apply isw once a correct answer is seen. Note that the complete form of the answer is not given so allow candidates to go from e.g.

$\mathrm{f}'(x)=\dfrac{2\left(x^2+4\right)-2x(2x-3)}{\left(x^2+4\right)^2}\rightarrow\dfrac{-2x^2+6x+8}{\left(x^2+4\right)^2}$ for the final mark. The denominator $\left(x^2+4\right)^2$ may go "missing" on an intermediate line provided it is present in their initial derivative **and** recovered in the final answer. Allow recovery from incorrect expansion of the denominator.

The $\mathrm{f}'(x)=$ must appear at some point but allow e.g. $"\dfrac{\mathrm{d}y}{\mathrm{d}x}="$

Note that just e.g. $\mathrm{f}'(x)=\dfrac{-2\left(x^2-3x-4\right)}{\left(x^2+4\right)^2}$ without sight of a correct derivative in the correct form scores A0.

(b) **Note:** it is possible to score B0M1A1 in this question due to the demand to "use algebra".
Note: if their numerator from (a) is not a 3 term quadratic then no marks can be scored in (b).

B1ft: Uses algebra to solve their $ax^2+bx+c...0$ with $a,b,c\neq 0$ where … is any equality or inequality, finding the correct, real critical values for their 3TQ.
The … 0 may be implied by their method.
They must show their working for this mark, so expect to see factorisation, substitution into the correct quadratic formula or completing the square.
Correct values for their quadratic do **not** imply this mark.
Approaches via factorisation must have completely correct factorisation, e.g.

$-2x^2+6x+8\{=0\}\Rightarrow -2(x+1)(x-4)\{=0\}\Rightarrow x=-1,\ 4$ scores B1ft

$-2x^2+6x+8\{=0\}\Rightarrow (2x+2)(4-x)\{=0\}\Rightarrow x=-1,\ 4$ scores B1ft

$-2x^2+6x+8\{=0\}\Rightarrow x^2-3x-4\{=0\}\Rightarrow (x+1)(x-4)\{=0\}\Rightarrow x=-1,\ 4$ scores B1ft

$-2x^2+6x+8\{=0\}\Rightarrow (2x+2)(x-4)\{=0\}\Rightarrow x=-1,\ 4$ scores B0ft

$-2x^2+6x+8\{=0\}\Rightarrow (x+1)(x-4)\{=0\}\Rightarrow x=-1,\ 4$ scores B0ft

M1: Selects the "correct" region for their critical values and their *a* from part (a). Must be *x* not f(*x*)
CVs may have been found using a calculator and may be implied if they are correct for their 3TQ. CVs may be incorrect due to errors in their calculations (but not errors in their method).

- For $a<0$ and roots $\alpha<\beta$ they need e.g. $x<\alpha$, $x>\beta$ (or e.g. $x\ ,,\ \alpha$ or $x\ \ldots\beta$)
- For $a>0$ and roots $\alpha<\beta$ they need e.g. $\alpha<x<\beta$ (or e.g. $x\ldots\alpha,\ x\ ,,\ \beta$)

Do not be overly concerned about their use of = , > , < in reference to their $-2x^2+6x+8...0$ for this mark or for the A1.
Indicating the region on a sketch is not sufficient. Allow , / or / and / $\cup$ / $\cap$ for the M1.
If they have complex roots (or they use the discriminant to find there are no real roots) then they can score this mark for concluding:

- if *a* < 0, "all values for *x* (have f decreasing)" or "f is always decreasing" or $x\in\mathrm{R}$
- if *a* > 0, "no values for *x* (have f decreasing)" or "f is never decreasing"

A1: Correct solution $x<-1$ or $x>4$ (allow $x\ ,,\ -1$ or $x\ \ldots 4$) coming from the correct numerator.
Do not isw if they go on to select e.g. $x>4$ or combine incorrectly to $4<x<-1$
Allow full marks to be scored in (b) from an incorrect denominator (but it must be positive for all *x*), e.g. from $\mathrm{f}'(x)=\dfrac{-2x^2+6x+8}{(x+4)^2}$ or $\mathrm{f}'(x)=\dfrac{-2x^2+6x+8}{4x^2}$ or $\mathrm{f}'(x)=\dfrac{-2x^2+6x+8}{x^2+16}$

Examples: Just "$x<-1$ or $x>4$" stated scores B0M1A1

$-2x^2+6x+8\{=0\}\Rightarrow -2(x+1)(x-4)=0\Rightarrow x<-1, x>4$ scores B1ftM1A1

$-2x^2+6x+8\{=0\}\Rightarrow x^2-3x-4\{=0\}\Rightarrow (x+1)(x-4)=0\Rightarrow x,,\ -1,\ x..4$ scores B1ftM1A1

$-2x^2+6x+8\{=0\}\Rightarrow (2x+2)(x-4)\Rightarrow x<-1, x>4$ scores B0ftM1A1

$-2x^2+6x+8<0\Rightarrow x^2-3x-4<0\Rightarrow (x+1)(x-4)<0\Rightarrow x<-1, x>4$ scores B1ftM1A1 (as this has correct factorisation shown, the region follows from *a* < 0 (M1) and we condone reference to $x^2-3x-4<0$ as part of their working to find critical values (A1).)

Acceptable notation: allow a "," , "or" , "and" or "$\cup$" to link the two regions, which may also be in set notation. e.g. $x<-1$ or $x>4$; $x\ ,,\ -1$, $x\ \ldots 4$; $x<-1$ and $x>4$
$x\ ,,\ -1\cup x\ \ldots 4$; $\{x:x<-1\cup x>4\}$; $\{x\in\mathbb{R}:x\ ,,\ -1\}\cup\{x\in\mathbb{R}:x\ \ldots 4\}$;
$x\in(-\infty,-1)\cup(4,\infty)$; $(-\infty,-1]\cup[4,\infty)$ etc.

Do not accept $4<x<-1$ or use of the $\cap$ symbol e.g. $(-\infty,-1]\cap[4,\infty)$ for the final mark, but they may be condoned for the M1. Note also that $[-\infty,-1]\cup[4,\infty]$ scores A0.

Question	Scheme	Marks	AOs

3(i)	$(5,-2)$ **or e.g.** $x=5, y=-2$ o.e.	B1	1.1b
		(1)	
(ii)	$(1.5,-2)$ **or e.g.** $x=1.5, y=-2$ o.e.	B1	1.1b
		(1)	
(iii)	$(-3, \ldots)$ **or** $(\ldots, -1)$ **or** $x=-3$ **or** $y=-1$ o.e.	B1	1.1b
	$(-3,-1)$ **or** $x=-3$ **and** $y=-1$ o.e.	B1	1.1b
		(2)	
			(4 marks)

Notes

General guidelines for all parts:

Remember to check answers written against the questions.

If there is any contradiction, mark the answers given in the body of the script.

If there is no labelling, mark the responses in the order given.

The coordinates need to be values not just a calculation e.g. **not** $-2\times3+5$ for -1

Points can be written as a coordinate pair or separately as $x = \ldots, y = \ldots$

Do **not** allow coordinates written the wrong way round but isw if necessary

e.g. $x=5, y=-2 \rightarrow (-2, 5)$ scores B1 and isw

Condone missing brackets (one or both) e.g. $5, -2$ **or** $(5, -2$ **or** $5, -2)$ for $(5, -2)$

Condone a missing comma e.g. $(5\ \ -2)$ for $(5, -2)$

Condone use of a semi-colon e.g. $(5\ ;-2)$ for $(5, -2)$

Condone vector notation e.g. $\begin{pmatrix}5\\-2\end{pmatrix}$ for $(5, -2)$ and condone $\left(\frac{5}{-2}\right)$

(i)

B1: $(5,-2)$ o.e. see above

(ii)

B1: $(1.5,-2)$ o.e. see above

(iii)

B1: One correct coordinate. See above.

B1: Both coordinates correct. See above.

Note that B0B1 is not a possible mark profile.

Note that in part (iii), some candidates show their thinking by transforming the point piecewise e.g. $(3, -2) \rightarrow (-3, -2) \rightarrow (-3, -6) \rightarrow (-3, -1)$

In such cases, mark their **final** pair of coordinates.

Question	Scheme	Marks	AOs

2(a)	$\left(\mathrm{f}(a)=\right)\ 4a^3+5a^2-10a+4a=0\Rightarrow a(....)=0$	M1	3.1a
	$a(4a^2+5a-6)=0$ *	A1*	1.1b
		(2)	
(b)(i)	$a=\dfrac{3}{4}$	B1	2.2a
(ii)	$4x^3+5x^2-10x+4\times"\dfrac{3}{4}"=3\Rightarrow 4x^3+5x^2-10x\ (=0)$	M1	1.1b
	$x=0$	B1	1.1b
	$x=\dfrac{-5\pm\sqrt{185}}{8}$	A1	1.1b
		(4)	
			(6 marks)

Notes

(a)

M1: Attempts $\mathrm{f}(a)=0$ leading to an equation in a only and attempts to take a factor of a out. Condone slips. The = 0 may appear on a later line which is fine, but must be seen at some point in their solution in part (a).

A1*: Achieves the given answer with no errors including brackets.
Minimum acceptable is $4a^3+5a^2-10a+4a=0\Rightarrow a(4a^2+5a-6)=0$
If the = 0 is absent at the start of their solution, it must appear before achieving the given answer.
Do not allow attempts to find the value of a and substitute that into $\mathrm{f}(x)$

More difficult alternative methods may be seen:

Alt 1: You may see attempts via division / inspection

$$\begin{array}{r|llll} & 4x^2 & +(5+4a)x & & +(-10+5a+4a^2) \\ x-a & 4x^3 & +5x^2 & -10x & +4a \end{array}$$

$$\begin{array}{ll} 4x^3 & -4ax^2 \\ & (5+4a)x^2 \quad -10x \\ & (5+4a)x^2 \quad -a(5+4a)x \\ & \qquad (-10+5a+4a^2)x \quad +4a \\ & \qquad (-10+5a+4a^2)x-a(-10+5a+4a^2) \\ & \qquad\qquad -6a+5a^2+4a^3 \end{array}$$

Then sets remainder $-6a+5a^2+4a^3=0$

M1: For dividing the cubic by $(x-a)$ leading to a quadratic quotient in x and a cubic remainder in a which is then set = 0 and attempts to take a factor of a out.

A1*: Completely correct with $a(4a^2+5a-6)=0$

Alt 2: You may also see a grid or an attempt at factorisation via inspection

	$4x^2$	$+(5+4a)x$	$+(-10+5a+4a^2)$
x	$4x^3$	$+(5+4a)x^2$	$(-10+5a+4a^2)x$
$-a$	$-4ax^2$	$-a(5+4a)x$	$-a(-10+5a+4a^2)$

OR $4x^3+5x^2-10x+4a \equiv (x-a)(4x^2+px-4)$ which should be followed by equating the x terms and x^2 terms to form two equations which can be solved simultaneously.
$-10=-ap-4$ and $5=-4a+p \Rightarrow p=5+4a$
$\Rightarrow -10=-a(5+4a)-4 \Rightarrow 4a^2+5a-6=0 \Rightarrow a(4a^2+5a-6)=0$

The above are examples. There may be other correct attempts so look at what is done.

M1: For an attempt to set up two simultaneous equations by equating coefficients for x and equating coefficients for x^2. Condone slips.

A1*: $4a^2+5a-6=0 \Rightarrow a(4a^2+5a-6)=0$ Completely correct with no errors.

(b) Mark (i) and (ii) together

(i)

B1: Deduces that $a=\frac{3}{4}$ only. May be implied by their resultant cubic. If they do (b)(ii) multiple times using other roots for which $a \neq \frac{3}{4}$, then the solutions arising from using the other roots $a \neq \frac{3}{4}$ must be rejected

(ii)

M1: Attempts to substitute their $a=\frac{3}{4}$ (which must be positive) into $f(x)$, sets their $f(x)=3$ and collects terms on one side (= 0 may be implied). Condone arithmetical and sign slips. Condone if they repeat this step using their other root(s).

B1: $(x=)\ 0$

A1: $(x=)\ \frac{-5\pm\sqrt{185}}{8}$ (**and these values only**) or exact equivalent (ignore 0 for this mark). Withhold this mark if the fraction line was clearly not intended to be under both terms. This mark cannot be scored if they proceed directly to the roots from $4x^3+5x^2-10x$ without taking a factor of or dividing by x first to **see the quadratic factor**. Isw once the correct answers are seen if they proceed to provide rounded answers after.

e.g. $4x^3+5x^2-10x+4\times"\frac{3}{4}"=3 \Rightarrow 0, \frac{-5\pm\sqrt{185}}{8}$ is M0B1A0

e.g. $4x^3+5x^2-10x=0 \Rightarrow 0, \frac{-5\pm\sqrt{185}}{8}$ is M1B1A0

e.g. $4x^3+5x^2-10x=0 \Rightarrow 4x^2+5x-10=0 \Rightarrow \frac{-5\pm\sqrt{185}}{8}$ is M1B0A1

e.g. $4x^3+5x^2-10x=0 \Rightarrow x(4x^2+5x-10)=0 \Rightarrow 0, \frac{-5\pm\sqrt{185}}{8}$ is M1B1A1

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
2	Sets $f(-2)=0 \Rightarrow (-2-4)\left((-2)^2-3\times-2+k\right)-42=0$	M1	3.1a
	$-6(k+10)=42 \Rightarrow k=...$	M1	1.1b
	$k=-17$	A1	1.1b
		(3)	
			(3 marks)
Notes:			

M1: Attempts $f(-2)=0$ leading to an equation in *k*. So $(-2-4)\left((-2)^2-3\times-2+k\right)-42=0$ is fine

Condone slips but expect to see a first bracket of $(-2-4)$.

"-42" must not be omitted but could appear as +42 with a sign slip.

There may have been attempts to expand $f(x)=(x-4)(x^2-3x+k)-42$ before attempting to set $f(-2)=0$. This is acceptable and condone slips/errors in the expansion, but the 42 must be present. FYI the expanded (and simplified) $f(x)=x^3-7x^2+(12+k)x-4k-42$

M1: Solves a **linear** equation in *k* as a result of setting $f(\pm 2)=0$.

The ± 42 must be there at some point when the substitution is made.

Allow minimal evidence here. A linear equation leading to a solution is fine.

If f (*x*) is expanded then it is dependent upon being a cubic which contains a *kx* term and a '42'

A1: $k=-17$ correct answer following correct work but allow recovery from invisible brackets

.....

Answers of $k=-17$ may appear with very little or no working, perhaps via trial and improvement. If so, then marks can only be allocated if evidence is shown.

E.g. $k=-17 \Rightarrow f(x)=(x-4)(x^2-3x-17)-42$

$f(-2)=(-6)\times(-7)-42=0$. Hence $(x+2)$ is a factor.

.....

More difficult alternative methods may be seen

.....

Alt I : You may see attempts via division / inspection

$$x+2\overline{)x^3-7x^2+(12+k)x-4k-42}\qquad x^2-9x+(k+30)$$

Then sets remainder $-6k-102=0\Rightarrow k=-17$

$$\underline{\quad\quad\quad\quad}$$
$$-6k-102$$

M1: For dividing their cubic by (x+2) which has both an x and a constant coefficient in k, leading to a quadratic quotient and a linear remainder in k which is then set = 0

M1: Solves a equation resulting from setting a linear remainder in k equal to 0 . It is dependent on the first M via this route

A1: Completely correct with $k=-17$

.....

Alt II: You may also see a grid or an attempt at factorisation via inspection

	x^2	$-9x$	$-2k-21$
x	x^3	$-9x^2$	$(-2k-21)x$
$+2$	$2x^2$	$-18x$	$-4k-42$

OR $x^3-7x^2+(12+k)x-4k-42\equiv(x+2)\left(x^2-9x-2k-21\right)$

which should be followed by equating the x terms to form an equation in k

$$12+k=-18-2k-21\Rightarrow 3k=-51\Rightarrow k=-17$$

OR $x^3-7x^2+(12+k)x-4k-42\equiv(x+2)\left(x^2-9x+k+30\right)$

which should be followed by equating the constant terms to form an equation in k

$$-4k-42=2\left(k+30\right)\Rightarrow 6k=-102\Rightarrow k=-17$$

The above are examples. There may be other correct attempts so look at what is done.

M1: For an attempt at factorising E.g. $x^3-7x^2+(12+k)x-4k-42\equiv(x+2)\left(x^2+bx+c\right)$ and attempting to set up three equations in b, c and k. E.g $2+b=-7, 2b+c=12+k, 2c=-4k-42$

The expanded f(x) must be a cubic which contains both a kx term and a '42'

M1: Solves the equations set up from an allowable equation to find k. It is dependent via this route.

A1: Completely correct with $k=-17$

.....

Question	Scheme	Marks	AOs
5 (a)	Attempts to use $h^2 = at + b$ with either $t = 2, h = 2.6$ or $t = 10, h = 5.1$	M1	3.1b
	Correct equations $2a + b = 6.76$, $10a + b = 26.01$	A1	1.1b
	Solves simultaneously to find values for a and b	dM1	1.1b
	$h^2 = 2.41t + 1.95$ cao	A1	3.3
		(4)	
(b)	Substitutes $t = 20$ into their $h^2 = 2.41t + 1.95$ and finds h or h^2 Or substitutes $h = 7$ into their $h^2 = 2.41t + 1.95$ and finds t	M1	3.4
	Compares the model with the true values and concludes "good model" with a minimal reason E.g. I Finds $h = 7.08\,(\text{m})$ and states that it is a good model as 7.08 (m) is close to 7 (m) E.g II Finds t =19.5 years and states that the model is accurate as 19.5 (years) $\approx$ 20 (years)	A1	3.5a
		(2)	
			(6 marks)
Notes:			

(a)

M1: For translating the problem into mathematics. Attempts to use the given equation o.e. with either of the pieces of information to form one correct equation.

Award for unsimplified equations as well, such as $2.6^2 = 2a + b$ or $2.6 = \sqrt{2a + b}$

A1: Two correct (and different) equations which may be unsimplified

dM1: Solves simultaneously to find values for *a* and *b*. It is dependent upon the previous M

Don't be too concerned with the process here as calculators may be used.

Score if values of *a* and *b* are reached from a pair of simultaneous equations

A1: Establishes **the full equation of the model** with values of *a* and *b* given to **exactly** 3sf. Award if seen in either (a) or (b). It is not scored for the values of *a* and *b*.

Allow either $h^2 = 2.41t + 1.95$ or $h = \sqrt{2.41t + 1.95}$

If they go on to square root each term from $h^2 = 2.41t + 1.95$ then it is A0. E.g. $h = 1.55t + 1.40$

.....

Special case for candidates who mistakenly use $h = at + b$

For $2.6 = 2a + b, \quad 5.1 = 10a + b \quad \Rightarrow h = 0.3125t + 1.975$ or $h = 0.313t + 1.98$

can score M1 correct equations with attempt to solve and A1 for either correct answer shown above.

These are the only marks available to them for a maximum mark of 1100 00

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(b)

M1: A full and valid attempt to

either substitute $t = 20$ into their $h^2 = 2.41t + 1.95$ o.e. and find a value for *h* or h^2

or substitute $h = 7$ into their $h^2 = 2.41t + 1.95$ o.e. and find a value for *t*

(to enable the candidate to compare real life data with that of the model.)

The equation of the model must be of the correct form, either $h^2 = at + b$ or $h = \sqrt{at + b}$

Do not be too concerned with the mechanics of the solution but the square or $\sqrt{}$ must have been used appropriately to enable the comparison to be made.

In cases with no working you will need to check the calculation

A1: Compares their *h*=7.08m to 7m o.e using h^2 or their *t* =19.5 years to 20 years and makes valid conclusion with reason.

For this mark you require

- a statement that it is a "good" or "accurate' ' model or similar wording
- a reason such as "the values are close", "the values are similar" or "the predicted values are within 5% of the true values."
- a model with equation $h^2 = a\,t + b$ o.e. where $a =$ awrt 2.4 and $b \in \left[1.9, 2.0\right]$
- correct calculations

Condone a statement like ‘ the model is pretty accurate as it predicted 7.08m and the actual value is 7m’

Do not allow incorrect statements such as the model is incorrect as it does not give 7 metres.

Do not allow just "the model gives an underestimate of the true value."

Do not allow ‘bad’ or ‘poor’ model

Question	Scheme	Marks	AOs

10(a)	Attempts to solve $\frac{3}{2}=\frac{8x+5}{2x+3}\Rightarrow x=...$ Or substitutes $x=\frac{3}{2}$ into $\frac{5-3x}{2x-8}$	M1	3.1a
	$\left(f^{-1}\left(\frac{3}{2}\right)=\right)-\frac{1}{10}$	A1	1.1b
		(2)	
(b)	$\left(\frac{8x+5}{2x+3}=\right)4\pm\frac{...}{2x+3}$	M1	1.1b
	$\left(\frac{8x+5}{2x+3}=\right)4-\frac{7}{2x+3}$	A1	2.1
		(2)	
(c)	$0\leqslant g^{-1}(x)\leqslant 4$	B1	2.2a
		(1)	
(d)	Attempts either boundary $f(0)=\frac{8\times0+5}{2\times0+3}$ **or** $f(4)=\frac{8\times4+5}{2\times4+3}$	M1	3.1a
	Attempts both boundaries $f(0)=\frac{8\times0+5}{2\times0+3}$ **and** $f(4)=\frac{8\times4+5}{2\times4+3}$	dM1	1.1b
	Range $\frac{5}{3}\leqslant fg^{-1}(x)\leqslant\frac{37}{11}$	A1	2.1
		(3)	
	Alternative by attempting $fg^{-1}(x)$		
	$g^{-1}(x)=\sqrt{16-x}\Rightarrow fg^{-1}(x)=\frac{8\sqrt{16-x}+5}{2\sqrt{16-x}+3}$ $fg^{-1}(0)=\frac{8\times4+5}{2\times4+3}$ **or** $fg^{-1}(16)=\frac{8\times0+5}{2\times0+3}$	M1	3.1a
	$fg^{-1}(0)=\frac{8\times4+5}{2\times4+3}$ **and** $fg^{-1}(16)=\frac{8\times0+5}{2\times0+3}$	dM1	1.1b
	Range $\frac{5}{3}\leqslant fg^{-1}(x)\leqslant\frac{37}{11}$	A1	2.1
		(3)	
			(8 marks)

Notes:

(a)

M1: Attempts to solve $\frac{3}{2}=\frac{8x+5}{2x+3}\Rightarrow x=...$ You can condone poor algebra as long as they reach a value for *x*.

Alternatively attempt to substitute $x=\frac{3}{2}$ into $f^{-1}(x)=\frac{\pm5\pm3x}{\pm2x\pm8}$ or equivalent (may be in terms of *y*). Note that attempts to find e.g. $f'(x)$ or $\frac{1}{f(x)}$ which may be implied by values such as $\frac{6}{17}$, $\frac{17}{6}$, $\frac{7}{18}$, $\frac{18}{7}$ score M0

A1: Achieves $\left(f^{-1}\left(\frac{3}{2}\right)=\right)-\frac{1}{10}$. Do not be concerned what they call it, just look for the value e.g. $x=-\frac{1}{10}$ or just $-\frac{1}{10}$ is fine. Correct answer with no (or minimal) working scores both marks.

M1: Attempts to divide $8x+5$ by $2x+3$

Look for $4\pm\frac{\ldots}{2x+3}$ where … is a constant or $8x+5=A(2x+3)+B$ with A or B correct (which may be in a fraction) or in a long division attempt and obtains a quotient of 4 or attempts to express the numerator in terms of the denominator e.g. $\frac{8x+5}{2x+3}=\frac{4(2x+3)+\ldots}{2x+3}$

A1: A full and complete method showing $\frac{8x+5}{2x+3}=4-\frac{7}{2x+3}$ or $\frac{8x+5}{2x+3}=4+\frac{-7}{2x+3}$

Also allow for correct values e.g. $A = 4$, $B = -7$

Do not isw here e.g. if they obtain $A = 4$, $B = -7$ and then write $-7+\frac{4}{2x+3}$ score A0

(c)

B1: Deduces $0\leqslant g^{-1}(x)\leqslant 4$ o.e.

E.g. $0\leqslant y\leqslant 4$, $0\leqslant$ range $\leqslant 4$, $g^{-1}(x)\leqslant 4$ and $g^{-1}(x)\geqslant 0$, $0\leqslant g^{-1}\leqslant 4$, $[0, 4]$

but not e.g. $0\leqslant x\leqslant 4$, $0\leqslant g(x)\leqslant 4$, $(0, 4)$

(d)

M1: Attempts either boundary. Look for either $f(0)=\frac{8\times 0+5}{2\times 0+3}$ **or** $f(4)=\frac{8\times 4+5}{2\times 4+3}$

or uses (b) e.g. $f(0)=4-\frac{7}{2\times 0+3}$ **or** $f(4)=4-\frac{7}{2\times 4+3}$

dM1: Attempts both boundaries. Look for $f(0)=\frac{8\times 0+5}{2\times 0+3}$ **and** $f(4)=\frac{8\times 4+5}{2\times 4+3}$

or uses (b) e.g. $f(0)=4-\frac{7}{2\times 0+3}$ **and** $f(4)=4-\frac{7}{2\times 4+3}$

A1: Correct answer written in the correct form.

E.g. $\frac{5}{3}\leqslant fg^{-1}(x)\leqslant\frac{37}{11}$, $\frac{5}{3}\leqslant$ range $\leqslant\frac{37}{11}$, $\frac{5}{3}\leqslant y\leqslant\frac{37}{11}$, $fg^{-1}(x)\leqslant\frac{37}{11}$ and $fg^{-1}(x)\geqslant\frac{5}{3}$

$\frac{5}{3}\leqslant fg^{-1}\leqslant\frac{37}{11}$, $fg^{-1}(x)\leqslant\frac{37}{11}\cap fg^{-1}(x)\geqslant\frac{5}{3}$, $\left[\frac{5}{3},\ \frac{37}{11}\right]$ but not e.g. $\frac{5}{3}\leqslant x\leqslant\frac{37}{11}$

PTO for an alternative to (d)

Look for either $\text{fg}^{-1}(0)=\dfrac{8\times \text{g}^{-1}(0)+5}{2\times \text{g}^{-1}(0)+3}$ **or** $\text{fg}^{-1}(16)=\dfrac{8\times \text{g}^{-1}(16)+5}{2\times \text{g}^{-1}(16)+3}$

Or uses (b) e.g. $\text{fg}^{-1}(0)=4-\dfrac{7}{2\times \text{g}^{-1}(0)+3}$ **or** $\text{fg}^{-1}(16)=4-\dfrac{7}{2\times \text{g}^{-1}(16)+3}$

The attempt at $\text{fg}^{-1}(x)$ requires an attempt to substitute $\sqrt{16-x}$ (condone $\pm\sqrt{16-x}$) into f

dM1: Attempts both boundaries. Look for $\text{fg}^{-1}(0)=\dfrac{8\times \text{g}^{-1}(0)+5}{2\times \text{g}^{-1}(0)+3}$ **and** $\text{fg}^{-1}(16)=\dfrac{8\times \text{g}^{-1}(16)+5}{2\times \text{g}^{-1}(16)+3}$

Or uses (b) e.g. $\text{fg}^{-1}(0)=4-\dfrac{7}{2\times \text{g}^{-1}(0)+3}$ **and** $\text{fg}^{-1}(16)=4-\dfrac{7}{2\times \text{g}^{-1}(16)+3}$

The attempt at $\text{fg}^{-1}(x)$ requires an attempt to substitute $\sqrt{16-x}$ (condone $\pm\sqrt{16-x}$) into f

A1: Correct answer written in the correct form with exact values.

E.g. $\dfrac{5}{3}\leqslant \text{fg}^{-1}(x)\leqslant\dfrac{37}{11}$, $\dfrac{5}{3}\leqslant \text{range}\leqslant\dfrac{37}{11}$, $\dfrac{5}{3}\leqslant y\leqslant\dfrac{37}{11}$, $\text{fg}^{-1}(x)\leqslant\dfrac{37}{11}$ and $\text{fg}^{-1}(x)\geqslant\dfrac{5}{3}$

$\dfrac{5}{3}\leqslant \text{fg}^{-1}\leqslant\dfrac{37}{11}$, $\text{fg}^{-1}(x)\leqslant\dfrac{37}{11}\cap\ \text{fg}^{-1}(x)\geqslant\dfrac{5}{3}$, $\left[\dfrac{5}{3},\ \dfrac{37}{11}\right]$ but not e.g. $\dfrac{5}{3}\leqslant x\leqslant\dfrac{37}{11}$

Note that the $\dfrac{37}{11}$ is sometimes obtained fortuitously from incorrect working so check working carefully.

Question	Scheme	Marks	AOs
1	$f(1)=a(1)^3+10(1)^2-3a(1)-4=0$	M1	3.1a
	$6-2a=0\Rightarrow a=...$	M1	1.1b
	$a=3$	A1	1.1b
		(3)	
			(3 marks)
Notes			

Main method seen:

M1: Attempts f (1) = 0 to set up an equation in a It is implied by $a+10-3a-4=0$

Condone a slip but attempting f (-1) = 0 is M0

M1: Solves a linear equation in a .

Using the main method it is dependent upon having set f (± 1) = 0

It is implied by a solution of $\pm a\pm 10\pm 3a\pm 4=0$.

Don't be concerned about the mechanics of the solution.

A1: a = 3 (following correct work)

.....

Answers without working scores 0 marks. The method must be made clear. Candidates cannot guess. However if a candidate states for example, when $a=3$, $f(x)=3x^3+10x^2-9x-4$ and shows that $(x-1)$ is a factor of this $f(x)$ by an allowable method, they should be awarded M1 M1 A1

E.g. 1: $3x^3+10x^2-9x-4=(x-1)\left(3x^2+13x+4\right)$ Hence $a=3$

E.g. 2: $f(x)=3x^3+10x^2-9x-4$, $f(1)=3+10-9-4=0$ Hence $a=3$

The solutions via this method must end with the value for a to score the A1

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Other methods are available. They are more difficult to determine what the candidate is doing.
Please send to review if you are uncertain
It is important that a correct method is attempted so look at how the two M's are scored

Amongst others are:

Alt (1) by inspection which may be seen in a table/g

	ax^2	$(10+a)x$	4
x	ax^3	$(10+a)x^2$	$4x$
-1	$-ax^2$	$-(10+a)x$	-4

$ax^3+10x^2-3ax-4=(x-1)\left(ax^2+(10+a)x+4\right)$ and sets terms in x equal

$-3a=-(10+a)+4 \Rightarrow 2a=6 \Rightarrow a=3$

M1: This method is implied by a **correct** equation, usually $-3a=-(10+a)+4$

M1: Attempts to find the quadratic factor which must be of the form $ax^2+\mathrm{g}(a)x\pm 4$ and then forms and solves a linear equation formed by linking the coefficients or terms in x

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Alt (2) By division:

$$x-1\overline{)\ ax^3+10x^2-3ax-4\ }\ \overline{ax^2+(\pm 10\pm a)x+(10-2a)}$$

$$\underline{ax^3-ax^2}$$

$$(10+a)x^2-3ax$$

$$\underline{(10+a)x^2-(10+a)x}$$

$$(-2a+10)x$$

M1: This method is implied by a **correct** equation, usually $-10+2a=-4$

M1: Attempts to divide with quotient of $ax^2+(\pm 10\pm a)x+\mathrm{h}(a)$ and then forms and solves a linear equation in a formed by setting the remainder = 0.

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
4 (a)	Either attempts $\frac{3x-7}{x-2}=7\Rightarrow x=...$ Or attempts $f^{-1}(x)$ and substitutes in $x=7$	M1	3.1a
	$\frac{7}{4}$ oe	A1	1.1b
		(2)	
(b)	Attempts $ff(x)=\frac{3\times\left(\frac{3x-7}{x-2}\right)-7}{\left(\frac{3x-7}{x-2}\right)-2}=\frac{3\times(3x-7)-7(x-2)}{3x-7-2(x-2)}$	M1, dM1	1.1b 1.1b
	$=\frac{2x-7}{x-3}$	A1	2.1
		(3)	
			(5 marks)
Notes:			

(a)

M1: For either attempting to solve $\frac{3x-7}{x-2}=7$. Look for an attempt to multiply by the $(x-2)$ leading to a value for *x*.

Or score for substituting in $x=7$ in $f^{-1}(x)$. FYI $f^{-1}(x)=\frac{2x-7}{x-3}$

The method for finding $f^{-1}(x)$ should be sound, but you can condone slips.

A1: $\frac{7}{4}$

(b)

M1: For an attempt at fully substituting $\frac{3x-7}{x-2}$ into $f(x)$. Condone slips but the expression must have a correct form. E.g. $\frac{3\times\left(\frac{*-*}{*-*}\right)-a}{\left(\frac{*-*}{*-*}\right)-b}$ where *a* and *b* are positive constants.

dM1: Attempts to multiply **all** terms on the numerator and denominator by $(x-2)$ to create a fraction $\frac{P(x)}{Q(x)}$ where both $P(x)$ and $Q(x)$ are linear expressions. Condone $\frac{P(x)}{Q(x)}\times\frac{x-2}{x-2}$

A1: Reaches $\frac{2x-7}{x-3}$ via careful and accurate work. Implied by $a=2, b=-7$ following correct work.

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Methods involving $\frac{3x-7}{x-2}\equiv a+\frac{b}{x-2}$ may be seen. The scheme can be applied in a similar way

FYI $\frac{3x-7}{x-2}\equiv 3-\frac{1}{x-2}$

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
7	Attempts equation of line Eg Substitutes $(-2,13)$ into $y=mx+25$ and finds m	M1	1.1b
	Equation of l is $y=6x+25$	A1	1.1b
	Attempts equation of C Eg Attempts to use the intercept $(0,25)$ within the equation $y=a(x\pm 2)^2+13$, in order to find a	M1	3.1a
	Equation of C is $y=3(x+2)^2+13$ or $y=3x^2+12x+25$	A1	1.1b
	Region R is defined by $3(x+2)^2+13<y<6x+25$ o.e.	B1ft	2.5
		(5)	
			(5 marks)

Notes:

The first two marks are awarded for finding the equation of the line

M1: Uses the information in an attempt to find an equation for the line *l*.

E.g. Attempt using two points: Finds $m=\pm\frac{25-13}{2}$ and uses of one of the points in their $y=mx+c$ or equivalent to find *c*. Alternatively uses the intercept as shown in main scheme.

A1: $y=6x+25$ seen or implied. This alone scores the first two marks. Do not accept $l=6x+25$

It must be in the form *y* = ... but the correct equation can be implied from an inequality. E.g. $....<y<6x+25$

.

The next two marks are awarded for finding the equation of the curve

M1: A complete method to find the constant *a* in $y=a(x\pm 2)^2+13$ or the constants *a*, *b* in $y=ax^2+bx+25$.

An alternative to the main scheme is deducing equation is of the form $y=ax^2+bx+25$ and setting and solving a pair of simultaneous equations in *a* and *b* using the point (−2, 13) the gradient being 0 at *x* = −2. Condone slips. Implied by $C=3x^2+12x+25$ or $3x^2+12x+25$

FYI the correct equations are $13=4a-2b+25\,(2a-b=-6)$ and $-4a+b=0$

A1: $y=3(x+2)^2+13$ or equivalent such as $y=3x^2+12x+25$, $f(x)=3(x+2)^2+13$.

Do not accept $C=3x^2+12x+25$ or just $3x^2+12x+25$ for the A1 but may be implied from an inequality or from an attempt at the area, E.g. $\int 3x^2+12x+25\,dx$

B1ft: Fully defines the region *R*. Follow through on their equations for *l* and *C*.

Allow strict or non -strict inequalities as long as they are used consistently.

E.g. Allow for example $"3(x+2)^2+13<y<6x+25"$ $"3(x+2)^2+13\leqslant y\leqslant 6x+25"$

Allow the inequalities to be given separately, e.g. $y<6x+25,\ y>3(x+2)^2+13$. Set notation may be used so $\{(x,y):y>3(x+2)^2+13\}\cap\{(x,y):y<6x+25\}$ is fine but condone with or without any of $(x,y)\leftrightarrow y\leftrightarrow x$

Incorrect examples include " $y<6x+25$ **or** $y>3(x+2)^2+13$", $\{(x,y):y>3(x+2)^2+13\}\cup\{(x,y):y<6x+25\}$

Values of *x* could be included but they must be correct. So $3(x+2)^2+13<y<6x+25,\ x<0$ is fine

If there are multiple solutions mark the final one.

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
1	Attempts $f(-3)=3\times(-3)^3+2a\times(-3)^2-4\times-3+5a=0$	M1	3.1a
	Solves linear equation $23a=69\Rightarrow a=...$	M1	1.1b
	$a=3$ cso	A1	1.1b
		(3)	
			(3 marks)

M1: Chooses a suitable method to set up a correct equation in a which may be unsimplified.
This is mainly applying $f(-3)=0$ leading to a correct equation in a.
Missing brackets may be recovered.
Other methods may be seen but they are more demanding
If division is attempted must produce **a correct equation** in a similar way to the $f(-3)=0$ method

$$\begin{array}{r} 3x^2+(2a-9)x+23-6a \\ x+3\overline{)\,3x^3+2ax^2-4x+5a} \\ \underline{3x^3+9x^2} \\ (2a-9)x^2-4x \\ \underline{(2a-9)x^2+(6a-27)x} \\ (23-6a)x+5a \\ (23-6a)x+69-18a \end{array}$$

So accept $5a=69-18a$ or equivalent, where it implies that the remainder will be 0
You may also see variations on the table below. In this method the terms in x are equated to -4

	$3x^2$	$(2a-9)x$	$\frac{5a}{3}$
x	$3x^3$	$(2a-9)x^2$	$\frac{5a}{3}x$
3	$9x^2$	$(6a-27)x$	$5a$

$$6a-27+\frac{5a}{3}=-4$$

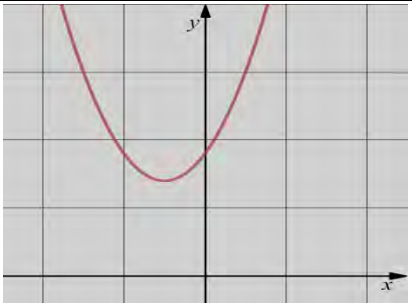
M1: This is scored for an attempt at solving a linear equation in a.
For the main scheme it is dependent upon having attempted $f(\pm3)=0$. Allow for a linear equation in a leading to $a=...$. Don't be too concerned with the mechanics of this.

Via division accept $x+3\overline{)\,3x^3+2ax^2-4x+5a}$ followed by a remainder in a set $=0\Rightarrow a=...$

or two terms in a are equated so that the remainder = 0
FYI the correct remainder via division is $23a+12-81$ oe

A1: $a=3$ cso

An answer of 3 with no incorrect working can be awarded 3 marks

5 (a)	$2x^2+4x+9=2(x\pm k)^2\pm....$ $\quad a=2$	B1	1.1b
	Full method $2x^2+4x+9=2(x+1)^2\pm...$ $\quad a=2\ \&\ b=1$	M1	1.1b
	$2x^2+4x+9=2(x+1)^2+7$	A1	1.1b
		(3)	
(b)	 U shaped curve any position but not through (0,0)	B1	1.2
	y - intercept at $(0,9)$	B1	1.1b
	Minimum at $(-1,7)$	B1ft	2.2a
		(3)	
(c)	(i) Deduces translation with one correct aspect.	M1	3.1a
	Translate $\begin{pmatrix}2\\-4\end{pmatrix}$	A1	2.2a
	(ii) $h(x)=\dfrac{21}{"2(x+1)^2+7"}\Rightarrow$ (maximum) value $\dfrac{21}{"7"}(=3)$	M1	3.1a
	$0<h(x)\leqslant 3$	A1ft	1.1b
		(4)	
			(10 marks)

(a)

B1: Achieves $2x^2+4x+9=2(x\pm k)^2\pm....$ or states that $a=2$

M1: Deals correctly with first two terms of $2x^2+4x+9$.

Scored for $2x^2+4x+9=2(x+1)^2\pm...$ or stating that $a=2$ and $b=1$

A1: $2x^2+4x+9=2(x+1)^2+7$

Note that this may be done in a variety of ways including equating $2x^2+4x+9$ with the expanded form of $a(x+b)^2+c\equiv ax^2+2abx+ab^2+c$

B1: For a U-shaped curve in any position not passing through $(0,0)$. Be tolerant of slips of the pen but do not allow if the curve bends back on itself

B1: A curve with a y - intercept on the +ve y axis of 9. The curve cannot just stop at $(0,9)$

Allow the intercept to be marked 9, $(0,9)$ but not $(9,0)$

B1ft: For a minimum at $(-1,7)$ in quadrant 2. This may be implied by -1 and 7 marked on the axes in the correct places. The curve must be a U shape and not a cubic say.

Follow through on a minimum at $(-b,c)$, marked in the correct quadrant, for their $a(x+b)^2+c$

(c)(i)

M1: Deduces translation with one correct aspect or states $\begin{pmatrix}2\\-4\end{pmatrix}$ with no reference to 'translate'.

Allow instead of the word translate, shift or move. $g(x)=f(x-2)-4$ can score M1

For example, possible methods of arriving at this deduction are:

- $f(x)\to g(x)$ is $2x^2+4x+9\to 2(x-2)^2+4(x-2)+5$ So $g(x)=f(x-2)-4$
- $g(x)=2(x-1)^2+3$ New curve has its minimum at $(1,3)$ so $(-1,7)\to(1,3)$
- Using a graphical calculator to sketch $y=g(x)$ and compares to the sketch of $y=f(x)$

In almost all cases you will not allow if the candidate gives two **different types of** transformations. Eg, stretch and .....

A1: Requires both 'translate' and '$\begin{pmatrix}2\\-4\end{pmatrix}$' Allow 'shift' or move' instead of translate.

So condone " Move shift 2 (units) to the right and move 4 (units) down

However, for M1 A1, it is possible to reflect in $x=0$ and translate $\begin{pmatrix}0\\-4\end{pmatrix}$, so please consider all responses.

SC: If the candidate writes translate $\begin{pmatrix}-2\\4\end{pmatrix}$ or " move 2 (units) to the left and 4 (units) up" score M1 A0

(c)(ii)

M1: Correct attempt at finding the maximum value (although it may not be stated as a maximum)

- Uses part (a) to write $h(x)=\dfrac{21}{"2(x+1)^2+7"}$ and attempts to find $\dfrac{21}{\text{their "7"}}$
- Attempts to differentiate, sets $4x+4=0\to x=-1$ and substitutes into $h(x)=\dfrac{21}{2x^2+4x+9}$
- Uses a graphical calculator to sketch $y=h(x)$ and establishes the 'maximum' value $(...,3)$

A1ft: $0<h(x)\leqslant 3$ Allow for $0<h\leqslant 3$ $(0,3]$ and $0<y\leqslant 3$ but not $0<x\leqslant 3$

Follow through on their $a(x+b)^2+c$ so award for $0<h(x)\leqslant\dfrac{21}{c}$

Question	Scheme	Marks	AOs

1	$2^x \times 4^y = \dfrac{1}{2\sqrt{2}} \quad \left\{ = \dfrac{\sqrt{2}}{4} \right\}$		
Special Case	If 0 marks are scored on application of the mark scheme then allow Special Case B1 M0 A0 (total of 1 mark) for any of • $2^x \times 4^y \rightarrow 2^{x+2y}$ • $2^x \times 4^y \rightarrow 4^{\frac{1}{2}x+y}$ • $\dfrac{1}{2^x 2\sqrt{2}} \rightarrow 2^{-x-\frac{3}{2}}$ • $\log 2^x + \log 4^y \rightarrow x\log 2 + y\log 4$ or $x\log 2 + 2y\log 2$ • $\ln 2^x + \ln 4^y \rightarrow x\ln 2 + y\ln 4$ or $x\ln 2 + 2y\ln 2$ • $y = \log\left(\dfrac{1}{2^x 2\sqrt{2}}\right)$ o.e. {base of 4 omitted}		
Way 1	$2^x \times 2^{2y} = 2^{-\frac{3}{2}}$	B1	1.1b
	$2^{x+2y} = 2^{-\frac{3}{2}} \Rightarrow x + 2y = -\dfrac{3}{2} \Rightarrow y = ...$	M1	2.1
	E.g. $y = -\dfrac{1}{2}x - \dfrac{3}{4}$ or $y = -\dfrac{1}{4}(2x+3)$	A1	1.1b
		(3)	
Way 2	$\log(2^x \times 4^y) = \log\left(\dfrac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log 2^x + \log 4^y = \log\left(\dfrac{1}{2\sqrt{2}}\right)$ $\Rightarrow x\log 2 + y\log 4 = \log 1 - \log(2\sqrt{2}) \Rightarrow y = ...$	M1	2.1
	$y = \dfrac{-\log(2\sqrt{2}) - x\log 2}{\log 4} \quad \left\{ \Rightarrow y = -\dfrac{1}{2}x - \dfrac{3}{4} \right\}$	A1	1.1b
		(3)	
Way 3	$\log(2^x \times 4^y) = \log\left(\dfrac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log 2^x + \log 4^y = \log\left(\dfrac{1}{2\sqrt{2}}\right) \Rightarrow \log 2^x + y\log 4 = \log\left(\dfrac{1}{2\sqrt{2}}\right) \Rightarrow y = ...$	M1	2.1
	$y = \dfrac{\log\left(\dfrac{1}{2\sqrt{2}}\right) - \log(2^x)}{\log 4} \quad \left\{ \Rightarrow y = -\dfrac{1}{2}x - \dfrac{3}{4} \right\}$	A1	1.1b
		(3)	
Way 4	$\log_2(2^x \times 4^y) = \log_2\left(\dfrac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log_2 2^x + \log_2 4^y = \log_2\left(\dfrac{1}{2\sqrt{2}}\right) \Rightarrow x + 2y = -\dfrac{3}{2} \Rightarrow y = ...$	M1	2.1
	E.g. $y = -\dfrac{1}{2}x - \dfrac{3}{4}$ or $y = -\dfrac{1}{4}(2x+3)$	A1	1.1b
		(3)	
			(3 marks)

Question	Scheme	Marks	AOs
Way 5	$4^{\frac{1}{2}x}\times 4^{y}=4^{-\frac{3}{4}}$	B1	1.1b
	$4^{\frac{1}{2}x+y}=4^{-\frac{3}{4}}\Rightarrow \frac{1}{2}x+y=-\frac{3}{4}\Rightarrow y=...$	M1	2.1
	E.g. $y=-\frac{1}{2}x-\frac{3}{4}$ or $y=-\frac{1}{4}(2x+3)$	A1	1.1b
		(3)	

Notes for Question 1

	Way 1
B1:	Writes a correct equation in powers of 2 only
M1:	Complete process of writing a correct equation in powers of 2 only and using correct index laws to obtain *y* written as a function of *x*.
A1:	$y=-\frac{1}{2}x-\frac{3}{4}$ o.e.
	Way 2, Way 3 and Way 4
B1:	Writes a correct equation involving logarithms
M1:	Complete process of writing a correct equation involving logarithms and using correct log laws to obtain *y* written as a function of *x*.
A1:	$y=\frac{-\log(2\sqrt{2})-x\log 2}{\log 4}$ or $y=\frac{-\ln(2\sqrt{2})-x\ln 2}{\ln 4}$ or $y=\frac{\log\left(\frac{1}{2\sqrt{2}}\right)-\log(2^{x})}{\log 4}$ or $y=-\frac{1}{2}x-\frac{3}{4}$ or $y=-\frac{1}{4}(2x+3)$ o.e.
	Way 5
B1:	Writes a correct equation in powers of 4 only
M1:	Complete process of writing a correct equation in powers of 4 only and using correct index laws to obtain *y* written as a function of *x*.
A1:	$y=-\frac{1}{2}x-\frac{3}{4}$ o.e.
Note:	Allow equivalent results for A1 where *y* is written as a function of *x*
Note:	You can ignore subsequent working following on from a correct answer.
Note:	Allow B1 for $2^{x}\times 4^{y}=\frac{1}{2\sqrt{2}}\Rightarrow 4^{y}=\frac{1}{2^{x}2\sqrt{2}}\Rightarrow \log_4(4^{y})=\log_4\left(\frac{1}{2^{x}2\sqrt{2}}\right)$ followed by M1 A1 for $y=\log_4\left(\frac{1}{2^{x}2\sqrt{2}}\right)$ or $y=\log_4\left(\frac{2^{-x}}{2\sqrt{2}}\right)$ or $y=\log_4\left(\frac{\sqrt{2}}{4(2^{x})}\right)$ or $y=-\log_4\left(2^{x+\frac{3}{2}}\right)$ or $y=-\log_4(\sqrt{2}(2^{x+1}))$

Question	Scheme	Marks	AOs

Time (s)	0	5	10	15	20	25

Question	Scheme	Marks	AOs
7	£y is the total cost of making x bars of soap Bars of soap are sold for £2 each		
(a)	$y = kx + c$ {where k and c are constants}	B1	3.3
	Note: Work for (a) cannot be recovered in (b) or (c)	**(1)**	
(b) **Way 1**	Either • $x = 800 \Rightarrow y = 2(800) - 500$ {$= 1100 \Rightarrow (x, y) = (800, 1100)$} • $x = 300 \Rightarrow y = 2(300) + 80$ {$= 680 \Rightarrow (x, y) = (300, 680)$}	M1	3.1b
	Applies (800, their 1100) and (300, their 680) to give two equations $1100 = 800k + c$ and $680 = 300k + c \Rightarrow k, c = ...$	dM1	1.1b
	Solves correctly to find $k = 0.84$, $c = 428$ and states $y = 0.84x + 428$ *	A1*	2.1
	Note: the answer $y = 0.84x + 428$ must be stated in (b)	**(3)**	
(b) **Way 2**	Either • $x = 800 \Rightarrow y = 2(800) - 500$ {$= 1100 \Rightarrow (x, y) = (800, 1100)$} • $x = 300 \Rightarrow y = 2(300) + 80$ {$= 680 \Rightarrow (x, y) = (300, 680)$}	M1	3.1b
	Complete method for finding both $k = ...$ and $c = ...$ e.g. $k = \dfrac{1100 - 680}{800 - 300}$ {$= 0.84$} $(800, 1100) \Rightarrow 1100 = 800(0.84) + c \Rightarrow c = ...$	dM1	1.1b
	Solves to find $k = 0.84$, $c = 428$ and states $y = 0.84x + 428$ *	A1*	2.1
	Note: the answer $y = 0.84x + 428$ must be stated in (b)	**(3)**	
(b) **Way 3**	Either • $x = 800 \Rightarrow y = 2(800) - 500$ {$= 1100 \Rightarrow (x, y) = (800, 1100)$} • $x = 300 \Rightarrow y = 2(300) + 80$ {$= 680 \Rightarrow (x, y) = (300, 680)$}	M1	3.1b
	{$y = 0.84x + 428 \Rightarrow$} $x = 800 \Rightarrow y = (0.84)(800) + 428 = 1100$ $x = 300 \Rightarrow y = (0.84)(300) + 428 = 680$	dM1	1.1b
	Hence $y = 0.84x + 428$ *	A1*	2.1
		(3)	
(c)	Allow any of {0.84, in £s} represents • the ***cost*** of {making} each extra bar {of soap} • the direct ***cost*** of {making} a bar {of soap} • the marginal ***cost*** of {making} a bar {of soap} • the ***cost*** of {making} a bar {of soap} **(**Condone this answer) **Note: Do not allow** • {0.84, in £s} is the profit per bar {of soap} • {0.84, in £s} is the (selling) price per bar {of soap}	B1	3.4
		(1)	
(d) **Way 1**	{Let n be the least number of bars required to make a profit}		
	$2n = 0.84n + 428 \Rightarrow n = ...$ (Condone $2x = 0.84x + 428 \Rightarrow x = ...$)	M1	3.4
	Answer of 369 {bars}	A1	3.2a
		(2)	
(d) **Way 2**	• Trial 1: $n = 368 \Rightarrow y = (0.84)(368) + 428 \Rightarrow y = 737.12$ {revenue $= 2(368) = 736$ or loss $= 1.12$} • Trial 2: $n = 369 \Rightarrow y = (0.84)(369) + 428 \Rightarrow y = 737.96$ {revenue $= 2(369) = 738$ or profit $= 0.04$} leading to an answer of 369 {bars}	M1 A1	3.4 3.2a
		(2)	
			(7 marks)

Notes for Question 7	
(a)	
B1:	Obtains a correct form of the equation. E.g. $y = kx + c$; $k \neq 0, c \neq 0$. **Note:** Must be seen in (a)
Note:	Ignore how the constants are labelled – as long as they appear to be constants. e.g. *k*, *c*, *m* etc.
(b)	**Way 1**
M1:	Translates the problem into the model by finding either • $y = 2(800) - 500$ for $x = 800$ • $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** See scheme
A1:	See scheme – no errors in their working
Note	Allow 1st M1 for any of • $1600 - y = 500$ • $600 - y = -80$
(b)	**Way 2**
M1:	Translates the problem into the model by finding either $y = 2(800) - 500$ for $x = 800$ $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** See scheme
A1:	See scheme – no error in their working
(b)	**Way 3**
M1:	Translates the problem into the model by finding either $y = 2(800) - 500$ for $x = 800$ $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** Uses the model to test both points $(800,$ their $1100)$ and $(300,$ their $680)$
A1:	Confirms $y = 0.84x + 428$ is true for both $(800, 1100)$ and $(300, 680)$ and gives a conclusion
Note:	Conclusion could be "$y = 0.84x + 428$" or "QED" or "proved"
Note:	Give 1st M0 for $500 = 800k + c,\ 80 = 300k + c \Rightarrow k = \dfrac{500 - 80}{800 - 300} = 0.84$
(c)	
B1:	see scheme
Note:	Also condone B1 for "rate of change of cost", "cost of {making} a bar", "constant of proportionality for cost per bar of soap" or "rate of increase in cost",
Note:	Do not allow reasons such as "price increase or decrease", "rate of change of the bar of soap" or "decrease in cost"
Note:	Give B0 for incorrect use of units. E.g. Give B0 for "the cost of making each extra bar of soap is £84" Condone the use of £0.84p

Notes for Question 7 Continued	
(d)	**Way 1**
M1:	Using the model and constructing an argument leading to a critical value for the number of bars of soap sold. See scheme.
A1:	369 only. Do not accept decimal answers.
(d)	**Way 2**
M1:	Uses either 368 or 369 to find the cost $y = ...$
A1:	Attempts both trial 1 and trial 2 to find both the cost $y = ...$ and arrives at an answer of 369 only. Do not accept decimal answers.
Note:	You can ignore inequality symbols for the method mark in part (d)
Note:	Give M1 A1 for no working leading to 369 {bars}
Note:	Give final A0 for $x > 369$ or $x > 368$ or $x \geq 369$ without $x = 369$ or 369 stated as their final answer
Note:	Condone final A1 for **in words** "at least 369 bars must be made/sold"
Note:	**Special Case:** Assuming a profit of £1 is required and achieving $x = 370$ scores special case M1A0

Question	Scheme	Marks	AOs
	$\sum_{r=0}^{\infty} \frac{(-1)^r x^{2r}}{(2r)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$		

11 (a)	$\sqrt{\frac{1+4x}{1-x}} = (1+4x)^{0.5} \times (1-x)^{-0.5}$	B1	3.1a
	$(1+4x)^{0.5} = 1 + 0.5 \times (4x) + \frac{0.5 \times -0.5}{2} \times (4x)^2$ $(1-x)^{-0.5} = 1 + (-0.5)(-x) + \frac{(-0.5) \times (-1.5)}{2}(-x)^2$ $(1+4x)^{0.5} = 1 + 2x - 2x^2$ **and** $(1-x)^{-0.5} = 1 + 0.5x + 0.375x^2$ oe	M1 M1 A1	1.1b 1.1b 1.1b
	$(1+4x)^{0.5} \times (1-x)^{-0.5} = (1 + 2x - 2x^2 .....) \times \left(1 + \frac{1}{2}x + \frac{3}{8}x^2 ...\right)$ $= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + 2x + x^2 - 2x^2 + ...$ $= A + Bx + Cx^2$	dM1	2.1
	$= 1 + \frac{5}{2}x - \frac{5}{8}x^2 ..... *$	A1*	1.1b
		(6)	
(b)	Expression is valid $\lvert x \rvert < \frac{1}{4}$ Should not use $x = \frac{1}{2}$ as $\frac{1}{2} > \frac{1}{4}$	B1	2.3
		(1)	
(c)	Substitutes $x = \frac{1}{11}$ into $\sqrt{\frac{1+4x}{1-x}} \approx 1 + \frac{5}{2}x - \frac{5}{8}x^2$	M1	1.1b
	$\sqrt{\frac{3}{2}} = \frac{1183}{968}$	A1	1.1b
	(so $\sqrt{6}$ is) $\frac{1183}{484}$ or $\frac{2904}{1183}$	A1	2.1
		(3)	
			(10 marks)

(a)

B1: Scored for key step in setting up the process so that it can be attempted using binomial expansions

This could be achieved by $\sqrt{\frac{1+4x}{1-x}} = (1+4x)^{0.5} \times (1-x)^{-0.5}$ See end for other alternatives

It may be implied by later work.

M1: Award for an attempt at the binomial expansion $(1+4x)^{0.5} = 1 + 0.5 \times (4x) + \frac{(0.5) \times (-0.5)}{2} \times (4x)^2$

There must be three (or more terms). Allow a missing bracket on the $(4x)^2$ and a sign slip so the correct application may be implied by $1 + 2x \pm 0.5x^2$

M1: Award for an attempt at the binomial expansion $(1-x)^{-0.5} = 1 + (-0.5)(-x) + \frac{(-0.5) \times (-1.5)}{2}(-x)^2$

There must be three (or more terms). Allow a missing bracket on the $(-x)^2$ and a sign slips so the method may be awarded on $1 \pm 0.5x \pm 0.375x^2$

A1: Both correct and simplified. This may be awarded for a correct final answer if a candidate does all their simplification at the end

dM1: In the main scheme it is for multiplying their two expansions to reach a quadratic. It is for the key step in adding 'six' terms to produce the quadratic expression. Higher power terms may be seen. Condone slips on

M's

In the alternative it is for multiplying $\left(1+\frac{5}{2}x-\frac{5}{8}x^2\right)(1-x)^{0.5}$ and comparing it to $(1+4x)^{0.5}$

It is for the key step in adding 'six' terms to produce the quadratic expression.

A1*: Completes proof with no errors or omissions. In the alternative there must be some reference to the fact that both sides are equal.

(b)

B1: States that the expansion may not / is not valid when $|x|>\frac{1}{4}$

This may be implied by a statement such as $\frac{1}{2}>\frac{1}{4}$ or stating that the expansion is only valid when $|x|<\frac{1}{4}$

Condone, for this mark a candidate who substitutes $x=\frac{1}{2}$ into the $4x$ **and** states it is not valid as $2 > 1$ oe

Don't award for candidates who state that $\frac{1}{2}$ is too big without any reference to the validity of the expansion.

As a rule you should see some reference to $\frac{1}{4}$ or $4x$

(c)(i)

M1: Substitutes $x=\frac{1}{11}$ into BOTH sides $\sqrt{\frac{1+4x}{1-x}}\approx 1+\frac{5}{2}x-\frac{5}{8}x^2$ and attempts to find at least one side.

As the left hand side is $\frac{\sqrt{6}}{2}$ they may multiply by 2 first which is acceptable

A1: Finds both sides leading to a correct equation/statement $\sqrt{\frac{15}{10}}=\frac{1183}{968}$ oe $\sqrt{6}=2\times\frac{1183}{968}$

A1: $\sqrt{6}=\frac{1183}{484}$ or $\sqrt{6}=\frac{2904}{1183}$ $\quad\sqrt{6}=2\times\frac{1183}{968}=\frac{1183}{484}$ would imply all 3 marks

……

Watch for other equally valid alternatives for 11(a) including

B1: $(1+4x)^{0.5}\approx\left(1+\frac{5}{2}x-\frac{5}{8}x^2\right)(1-x)^{0.5}$ then the M's are for $(1+4x)^{0.5}$ and $(1-x)^{0.5}$

M1: $(1-x)^{0.5}=1+(0.5)(-x)+\frac{(0.5)\times(-0.5)}{2}(-x)^2$

……

Or

B1: $\sqrt{\frac{1+4x}{1-x}}=\sqrt{1+\frac{5x}{1-x}}=\left(1+5x(1-x)^{-1}\right)^{\frac{1}{2}}$ then the first M1 for one application of binomial and the second would be for both $(1-x)^{-1}$ and $(1-x)^{-2}$

……

Or

B1: $\sqrt{\frac{1+4x}{1-x}}\times\frac{\sqrt{1-x}}{\sqrt{1-x}}=\sqrt{\left(1+3x-4x^2\right)}\times(1-x)^{-1}=\left(1+\left(3x-4x^2\right)\right)^{\frac{1}{2}}\times(1-x)^{-1}$

Question	Scheme	Marks	AOs

1	$g(x)=\dfrac{2x+5}{x-3},\ x\geq 5$		
(a) Way 1	$g(5)=\dfrac{2(5)+5}{5-3}=7.5 \Rightarrow gg(5)=\dfrac{2("7.5")+5}{"7.5"-3}$	M1	1.1b
	$gg(5)=\dfrac{40}{9}\ \left(\text{or } 4\dfrac{4}{9} \text{ or } 4.\dot{4}\right)$	A1	1.1b
		(2)	
(a) Way 2	$gg(x)=\dfrac{2\left(\dfrac{2x+5}{x-3}\right)+5}{\left(\dfrac{2x+5}{x-3}\right)-3} \Rightarrow gg(5)=\dfrac{2\left(\dfrac{2(5)+5}{(5)-3}\right)+5}{\left(\dfrac{2(5)+5}{(5)-3}\right)-3}$	M1	1.1b
	$gg(5)=\dfrac{40}{9}\ \left(\text{or } 4\dfrac{4}{9} \text{ or } 4.\dot{4}\right)$	A1	1.1b
		(2)	
(b)	{Range:} $2<y\leq\dfrac{15}{2}$	B1	1.1b
		(1)	
(c) Way 1	$y=\dfrac{2x+5}{x-3} \Rightarrow\ yx-3y=2x+5 \Rightarrow yx-2x=3y+5$	M1	1.1b
	$x(y-2)=3y+5 \Rightarrow\ x=\dfrac{3y+5}{y-2}\quad \left\{\text{or } y=\dfrac{3x+5}{x-2}\right\}$	M1	2.1
	$g^{-1}(x)=\dfrac{3x+5}{x-2},\quad 2<x\leq\dfrac{15}{2}$	A1ft	2.5
		(3)	
(c) Way 2	$y=\dfrac{2x-6+11}{x-3} \Rightarrow\ y=2+\dfrac{11}{x-3} \Rightarrow y-2=\dfrac{11}{x-3}$	M1	1.1b
	$x-3=\dfrac{11}{y-2} \Rightarrow x=\dfrac{11}{y-2}+3\quad \left\{\text{or } y=\dfrac{11}{x-2}+3\right\}$	M1	2.1
	$g^{-1}(x)=\dfrac{11}{x-2}+3,\quad 2<x\leq\dfrac{15}{2}$	A1ft	2.5
		(3)	
			(6 marks)

Notes for Question 1

(a)	
M1:	Full method of attempting $g(5)$ and substituting the result into g
Note:	**Way 2:** Attempts to substitute $x=5$ into $\dfrac{2\left(\dfrac{2x+5}{x-3}\right)+5}{\left(\dfrac{2x+5}{x-3}\right)-3}$, o.e. Note that $gg(x)=\dfrac{9x-5}{14-x}$
A1:	Obtains $\dfrac{40}{9}$ or $4\dfrac{4}{9}$ or $4.\dot{4}$ or an exact equivalent
Note:	Give A0 for 4.4 or $4.444...$ without reference to $\dfrac{40}{9}$ or $4\dfrac{4}{9}$ or $4.\dot{4}$

Notes for Question 1 Continued	
(b)	
B1:	States $2 < y \leq \frac{15}{2}$ Accept any of $2 < g \leq \frac{15}{2}$, $2 < g(x) \leq \frac{15}{2}$, $\left(2, \frac{15}{2}\right]$
Note:	Accept $g(x) > 2$ **and** $g(x) \leq \frac{15}{2}$ o.e.
(c) Way 1	
M1:	Correct method of cross multiplication followed by an attempt to collect terms in x or terms in a swapped y
M1:	A complete method (i.e. as above and also factorising and dividing) to find the inverse
A1ft:	Uses correct notation to correctly define the inverse function g^{-1}, where the domain of g^{-1} stated correctly or correctly followed through (using correct notation) on the values shown in their range in part (b). Allow $g^{-1} : x \rightarrow$. Condone $g^{-1} = ...$ Do not accept $y = ...$
Note:	Correct notation is required when stating the domain of $g^{-1}(x)$. Allow $2 < x \leq \frac{15}{2}$ or $\left(2, \frac{15}{2}\right]$ Do not allow any of e.g. $2 < g \leq \frac{15}{2}$, $2 < g^{-1}(x) \leq \frac{15}{2}$
Note:	Do not allow A1ft for following through their range in (b) to give a domain for g^{-1} as $x \in \mathbb{R}$
(c) Way 2	
M1:	Writes $y = \frac{2x+5}{x-3}$ in the form $y = 2 \pm \frac{k}{x-3}$, $k \neq 0$ and rearranges to isolate y and 2 on one side of their equation. **Note:** Allow the equivalent method with x swapped with y
M1:	A complete method to find the inverse
A1ft:	As in Way 1
Note:	If a candidate scores no marks in part (c), but • states the domain of g^{-1} correctly, **or** • states a domain of g^{-1} which is correctly followed through on the values shown in their range in part (b) then give special case (SC) M1 M0 A0

Question	Scheme	Marks	AOs
	$\overrightarrow{OA} = 2\mathbf{i}+3\mathbf{j}-4\mathbf{k}$, $\overrightarrow{OB} = 4\mathbf{i}-2\mathbf{j}+3\mathbf{k}$, $\overrightarrow{OC} = q\mathbf{i}+5\mathbf{j}-2\mathbf{k}$, $q<0$		

11	$\dfrac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A+\dfrac{B}{(x-3)}+\dfrac{C}{(1-2x)}$		
(a) Way 1	$1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3) \Rightarrow B=..., C=...$	M1	2.1
	$A=3$	B1	1.1b
	Uses substitution or compares terms to find either $B=...$ or $C=...$	M1	1.1b
	$B=4$ and $C=-2$ which have been found using a correct identity	A1	1.1b
		(4)	
(a) Way 2	{long division gives} $\dfrac{1+11x-6x^2}{(x-3)(1-2x)} \equiv 3+\dfrac{-10x+10}{(x-3)(1-2x)}$		
	$-10x+10 \equiv B(1-2x)+C(x-3) \Rightarrow B=..., C=...$	M1	2.1
	$A=3$	B1	1.1b
	Uses substitution or compares terms to find either $B=...$ or $C=...$	M1	1.1b
	$B=4$ and $C=-2$ which have been found using $-10x+10 \equiv B(1-2x)+C(x-3)$	A1	1.1b
		(4)	
(b)	$f(x)=3+\dfrac{4}{(x-3)}-\dfrac{2}{(1-2x)}$ $\{=3+4(x-3)^{-1}-2(1-2x)^{-1}\}$; $x>3$		
	$f'(x)=-4(x-3)^{-2}-4(1-2x)^{-2}$ $\left\{=-\dfrac{4}{(x-3)^2}-\dfrac{4}{(1-2x)^2}\right\}$	M1	2.1
		A1ft	1.1b
	Correct $f'(x)$ **and** as $(x-3)^2>0$ and $(1-2x)^2>0$, then $f'(x)=-(+\text{ve})-(+\text{ve})<0$, so $f(x)$ is a decreasing function	A1	2.4
		(3)	
			(7 marks)

Notes for Question 11

(a)	
M1:	**Way 1:** Uses a correct identity $1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3)$ in a complete method to find values for *B* and *C*. **Note:** Allow one slip in copying $1+11x-6x^2$ **Way 2:** Uses a correct identity $-10x+10 \equiv B(1-2x)+C(x-3)$ (which has been found from long division) in a complete method to find values for *B* and *C*
B1:	$A=3$
M1:	Attempts to find the value of either *B* or *C* from their identity This can be achieved by ***either*** substituting values into their identity ***or*** by comparing coefficients and solving the resulting equations simultaneously
A1:	See scheme
Note:	**Way 1:** Comparing terms: x^2: $-6=-2A$; x: $11=7A-2B+C$; constant: $1=-3A+B-3C$ **Way 1:** Substituting: $x=3$: $-20=-5B \Rightarrow B=4$; $x=\frac{1}{2}$: $5=-\frac{5}{2}C \Rightarrow C=-2$
Note:	**Way 2:** Comparing terms: x: $-10=-2B+C$; constant: $10=B-3C$ **Way 2:** Substituting: $x=3$: $-20=-5B \Rightarrow B=4$; $x=\frac{1}{2}$: $5=-\frac{5}{2}C \Rightarrow C=-2$

Note:	$A=3, B=4, C=-2$ from no working scores M1B1M1A1
Note:	The final A1 mark is effectively dependent upon both M marks

Notes for Question 11 Continued	
(a) ctd	
Note:	Writing $1+11x-6x^2 \equiv B(1-2x)+C(x-3) \Rightarrow B=4, C=-2$ will get 1st M0, 2nd M1, 1st A0
Note:	**Way 1:** You can imply a correct identity $1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3)$ from seeing $\dfrac{1+11x-6x^2}{(x-3)(1-2x)} \equiv \dfrac{A(1-2x)(x-3)+B(1-2x)+C(x-3)}{(x-3)(1-2x)}$
Note:	**Way 2:** You can imply a correct identity $-10x+10 \equiv B(1-2x)+C(x-3)$ from seeing $\dfrac{-10x+10}{(x-3)(1-2x)} \equiv \dfrac{B(1-2x)+C(x-3)}{(x-3)(1-2x)}$
(b)	
M1:	Differentiates to give $\{f'(x)=\}\ \pm\lambda(x-3)^{-2} \pm \mu(1-2x)^{-2};\ \lambda,\ \mu \neq 0$
A1ft:	$f'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2}$, which can be simplified or un-simplified
Note:	Allow A1ft for $f'(x) = -(\text{their } B)(x-3)^{-2} + (2)(\text{their } C)(1-2x)^{-2};\ (\text{their } B), (\text{their } C) \neq 0$
A1:	$f'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2}$ or $f'(x) = -\dfrac{4}{(x-3)^2} - \dfrac{4}{(1-2x)^2}$ **and** a correct explanation e.g. $f'(x) = -(+\text{ve}) - (+\text{ve}) < 0$, so $f(x)$ is a decreasing {function}
Note:	The final A mark can be scored in part (b) from an incorrect $A=\ldots$ or from $A=0$ or no value of A found in part (a)

Notes for Question 11 Continued - Alternatives

(a)

Note: Be aware of the following alternative solutions, by initially dividing by $"(x-3)"$ or $"(1-2x)"$

- $\dfrac{1+11x-6x^2}{"(x-3)"(1-2x)} \equiv \dfrac{-6x-7}{(1-2x)} - \dfrac{20}{(x-3)(1-2x)} \equiv 3 - \dfrac{10}{(1-2x)} - \dfrac{20}{(x-3)(1-2x)}$

  $\dfrac{20}{(x-3)(1-2x)} \equiv \dfrac{D}{(x-3)} + \dfrac{E}{(1-2x)} \Rightarrow 20 \equiv D(1-2x)+E(x-3) \Rightarrow D=-4,\ E=-8$

  $\Rightarrow 3 - \dfrac{10}{(1-2x)} - \left(\dfrac{-4}{(x-3)} + \dfrac{-8}{(1-2x)}\right) \equiv 3 + \dfrac{4}{(x-3)} - \dfrac{2}{(1-2x)};\ A=3,\ B=4,\ C=-2$

- $\dfrac{1+11x-6x^2}{(x-3)"(1-2x)"} \equiv \dfrac{3x-4}{(x-3)} + \dfrac{5}{(x-3)(1-2x)} \equiv 3 + \dfrac{5}{(x-3)} + \dfrac{5}{(x-3)(1-2x)}$

  $\dfrac{5}{(x-3)(1-2x)} \equiv \dfrac{D}{(x-3)} + \dfrac{E}{(1-2x)} \Rightarrow 5 \equiv D(1-2x)+E(x-3) \Rightarrow D=-1,\ E=-2$

  $\Rightarrow 3 + \dfrac{5}{(x-3)} + \left(\dfrac{-1}{(x-3)} + \dfrac{-2}{(1-2x)}\right) \equiv 3 + \dfrac{4}{(x-3)} - \dfrac{2}{(1-2x)};\ A=3,\ B=4,\ C=-2$

(b)

Alternative Method 1:

$\mathrm{f}(x) = \dfrac{1+11x-6x^2}{(x-3)(1-2x)},\ x>3 \Rightarrow \mathrm{f}(x) = \dfrac{1+11x-6x^2}{-2x^2+7x-3};\ \begin{Bmatrix} u=1+11x-6x^2 & v=-2x^2+7x-3 \\ u'=11-12x & v'=-4x+7 \end{Bmatrix}$

$\mathrm{f}'(x) = \dfrac{(-2x^2+7x-3)(11-12x)-(1+11x-6x^2)(-4x+7)}{(-2x^2+7x-3)^2}$	Uses quotient rule to find $\mathrm{f}'(x)$	**M1**
	Correct differentiation	**A1**
$\mathrm{f}'(x) = \dfrac{-20((x-1)^2+1)}{(-2x^2+7x-3)^2}$ and a correct explanation, e.g. $\mathrm{f}'(x) = -\dfrac{(+\text{ve})}{(+\text{ve})} < 0$, so $\mathrm{f}(x)$ is a decreasing {function}		**A1**

Alternative Method 2:

Allow M1A1A1 for the following solution:

Given $\mathrm{f}(x) = 3 + \dfrac{4}{(x-3)} - \dfrac{2}{(1-2x)} = 3 + \dfrac{4}{(x-3)} + \dfrac{2}{(2x-1)}$

as $\dfrac{4}{(x-3)}$ decreases when $x>3$ and $\dfrac{2}{(2x-1)}$ decreases when $x>3$

then $\mathrm{f}(x)$ is a decreasing {function}

Question	Scheme	Marks	AOs