adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Arithmetic sequences

Mark scheme

4 questions

Exam questions and marking instructions remain credited to their board.

Contents

1 June 2025 - Paper 1 - Q3

2 June 2024 - Paper 2 - Q2

3 June 2022 - Paper 1 - Q13

4 October 2021 - Paper 2 - Q1

Question	Scheme	Marks	AOs
3 (a)	Attempts to solve $10-6k=2k-10\Rightarrow k=...$	M1	3.1a
	$\left(k=\right)\frac{5}{2}$ o.e.	A1	1.1b
		(2)	
(b)	Deduces the value of "d" $=-5$	B1 ft	2.2a
	$S_{50}=\frac{50}{2}\left(2\times"15"+49\times"-5"\right)$	M1	1.1b
	$=-5375$	A1	1.1b
		(3)	
			(5 marks)

Notes:

(a) Condone using other letters for *a* and *d* (e.g. may use *r* for *d*)

M1: Attempts a valid method to solve the problem

- Uses the common difference to form a correct equation and attempts to solve to find a value for k

  e.g. Attempts to solve $10-6k=2k-10$ o.e such as $2k-6k=2\left(10-6k\right)$
- Uses 10 as the mean of $2k$ and $6k$: $\frac{2k+6k}{2}=10\Rightarrow k=...$
- Sets up correct equations $a=6k,\ a+d=10$ and $a+2d=2k,$ or may be seen as $6k+d=10$ and $10+d=2k$, **and** proceeds to find k.
- Uses the summation formula $S_3=\frac{3}{2}\left(6k+2k\right)=6k+10+2k$ **and** proceeds to find k

In each attempt the initial equation (or simultaneous equations) must be correct but do not be concerned by the mechanics of the rearrangement to find k. May be implied by $k=\frac{5}{2}$

A1: $\left(k=\right)\frac{5}{2}$ o.e.

(b) Work seen in (a) can only be scored if seen or used in (b)

B1ft: Common difference $=-5$ **or** ft on their value for k (even if k has been found from an incorrect method) e.g. $10-6\times"\frac{5}{2}"$ or e.g. $2\times"\frac{5}{2}"-10$ if only a numerical value is seen.

May be implied or seen in a term or summation formula.

Note that some candidates may work in terms of k throughout so only allow B1ft to be scored when they substitute in their numerical value for k, following $10-6k$ o.e. correctly embedded in a correct formula. They may make arithmetical slips before they substitute in their numerical value for k which can be condoned.

must be using a correct value for a and $\pm d$ (or ft on their value for k for a and d) which are correctly placed in the formula.

e.g. $\left(S_{50}=\right)\frac{50}{2}\left(2\times"6k"+49\times"\pm d"\right)$.

Alternatively, they may find the 50th term $u_{50}="15"+(50-1)\times"-5"=-230$ and use $\left(S_{50}=\right)\frac{50}{2}\left("15"+"-230"\right)$.

If working in terms of k they must substitute in their value for k

e.g. $\left(S_{50}=\right)\frac{50}{2}\left(2\times6k+49\times(10-6k)\right)=-7050k+12250=-7050\times"\frac{5}{2}"+12250$

Do not withhold this mark for omission of brackets around (-5)

e.g. $\frac{50}{2}\left(2\times15+\left(49\right)-5\right)$ scores M1

A1: -5375 cao

2(a)	$(u_{12}=)400+11\times-10=290$* or e.g. $(u_{12}=)400-110=290$* or e.g. $(u_{12}=)400+(12-1)\times-10=290$* or e.g. $(u_{12}=)410+12\times-10=290$*	B1*	1.1b
		(1)	
	Alternative 1:		
	$400+(n-1)\times-10=290$ $\Rightarrow 400-10n+10=290\Rightarrow 10n=120\Rightarrow n=12$*	B1*	1.1b
	Alternative 2:		
	$290=400+(12-1)d\Rightarrow 11d=-110\Rightarrow d=-10$*	B1*	1.1b
(b)	$8100=\frac{1}{2}N(2\times400+(N-1)\times-10)$ or e.g. $8100=\frac{1}{2}N(400+400+(N-1)\times-10)$	M1	1.1b
	$8100=\frac{1}{2}N(2\times400+(N-1)\times-10)$ $\Rightarrow 16200=800N-10N^2+10N$ or e.g. $\Rightarrow 8100=400N-5N^2+5N$ $\Rightarrow N^2-81N+1620=0$*	A1*	2.1
		(2)	
(c)	$N^2-81N+1620=0\Rightarrow(N-45)(N-36)=0\Rightarrow N=45,36$	M1	1.1b
	$(N=)$ 36	A1	2.3
		(2)	
			(5 marks)

Notes

(a)

B1*: Correct working to obtain 290. Must be a correct calculation so do not condone missing brackets unless they are recovered. E.g. $(u_{12}=)400+12-1\times-10=290$ scores B0 unless followed by $=400+11\times-10=290$. Condone $(u_{12}=)400+(12-1)-10=290$

The "£" symbol is not required but the "290" must appear.

Alternative 1:

B1*: Correct working using the 290 to obtain $n = 12$.

There must be at least one intermediate line after setting up the equation and must be correct work so do not condone missing brackets unless they are recovered (as above).

A conclusion is **not** required with this approach as long as 12 is correctly obtained.

Alternative 2:

B1*: Correct working using the 290 and 400 to obtain $d = -10$.

There must be at least one intermediate line after setting up the equation and must be correct work so do not condone missing brackets unless they are recovered (as above).

A conclusion is **not** required with this approach as long as −10 is correctly obtained.

Allow candidates to list terms and show the 12th term is 290 e.g.

400, 390, 380, 370, 360, 350, 340, 330, 320, 310, 300, 290

Must list all 12 terms which must be correct and end with 290

Condone if missing 400, 390, 380 as these are given in the question.

(b) **Mark (b) and (c) together**

M1: Uses a correct sum formula in terms of N or n with $a = 400$ and $d = -10$ or $+10$ and sets = 8100. Condone e.g. > 8100 and allow A1 if this is recovered to become "=" before the final line.

Condone $8100 = \frac{1}{2}N\left(2\times 400 + (N-1) - 10\right)$ if recovered or not.

A1*: Fully correct proof with sufficient working shown and no unrecovered errors.
Do not condone e.g. missing brackets or e.g. a missing N/n unless recovered before the final given answer.
Condone the use of n instead of N for **both** marks.
Condone terms in a different order as long as they are correct.
Condone $0 = N^2 - 81N + 1620$*
Sufficient working requires all brackets to be removed to obtain an unsimplified expanded quadratic before proceeding to the given answer including the "=0".

Alternative (further maths method): Series summation approach:

$$\sum_{r=1}^{N}(410 - 10r) = 8100 \Rightarrow 410N - 10\times\frac{1}{2}N(N+1)$$

$$\Rightarrow 410N - 5N^2 - 5N = 8100 \Rightarrow N^2 - 81N + 1620 = 0*$$

M1: Attempt to sum an appropriate series with first term 400. Condone use of +10 as in the main scheme.

A1*: As main scheme.

(c)

M1: Solves the **given** quadratic equation by any correct method including a calculator to obtain at least one value for N. See general guidance for solving a 3-term quadratic.
If values are just written down and only one value is given it must be 45 or 36.
If both values are just written down they must both be correct.

A1: Realises that the smaller value is required and so selects (N =) 36.
Ignore any units if given.
The "N =" is not required, just look for the correct value.
It must be clear that this value has been selected. This may be indicated by e.g. underlining the 36 or the omission of the 45. If the 45 is not rejected score A0.
N = 36 with no working scores M1A1

Question	Scheme	Marks	AOs

Question			
13 (i)	States that $S = a + (a+d) + \ldots\ldots\ldots\ldots\ldots + (a+(n-1)d)$	B1	1.1a
	$S = a + (a+d) + \ldots\ldots\ldots\ldots\ldots + (a+(n-1)d)$ $S = (a+(n-1)d) + (a+(n-2)d) + \ldots\ldots\ldots\ldots\ldots + a$	M1	3.1a
	Reaches $2S = n \times (2a + (n-1)d)$ And so proves that $S = \frac{n}{2}[2a + (n-1)d]$ *	A1*	2.1
		(3)	
(ii)	(a) $S = 10 + 9.20 + 8.40 + \ldots$		
	$64 = \frac{n}{2}(20 - 0.8(n-1))$ o.e	M1	3.1b
	$128 = 20n - 0.8n^2 + 0.8n$ $0.8n^2 - 20.8n + 128 = 0$ $n^2 - 26n + 160 = 0$ *	A1*	2.1
		(2)	
	(b) $n = 10, 16$	B1	1.1b
		(1)	
	(c) 10 weeks with a minimal correct reason. E.g. • He has saved up the amount by 10 weeks so he would not save for another 6 weeks • You would choose the smaller number • He starts saving negative amounts (in week 14) so 16 does not make sense	B1	2.3
		(1)	
			(7 marks)

Notes:

(i)

B1: Correctly writes down an expression for the key terms S or S_n including $S =$ or $S_n =$

Allow a minimum of 3 correct terms including the first and last terms, and no incorrect terms.

Score for S or $S_n = a + (a+d) + \ldots\ldots\ldots\ldots\ldots + (a+(n-1)d)$ with + signs, not commas

If the series contains extra terms that should not be there E.g

$S = a + (a+d) + \ldots\ldots\ldots\ldots\ldots (\boldsymbol{a + nd}) + (a+(n-1)d)$ score B0

M1: For the key step in reversing the terms and adding the two series.

Look for a minimum of two terms, including a and $a+(n-1)d$, the series reversed with evidence of adding, for example $2S =$ Condone the extra incorrect terms (see above) appearing.

Can be scored when terms are separated by commas

A1*: Shows correct work (no errors) with all steps shown leading to given answer.

There should be no incorrect terms. A minimum of 3 terms should be shown in each sum

$S = a + (a+d) + \ldots\ldots\ldots\ldots\ldots\ldots + l$

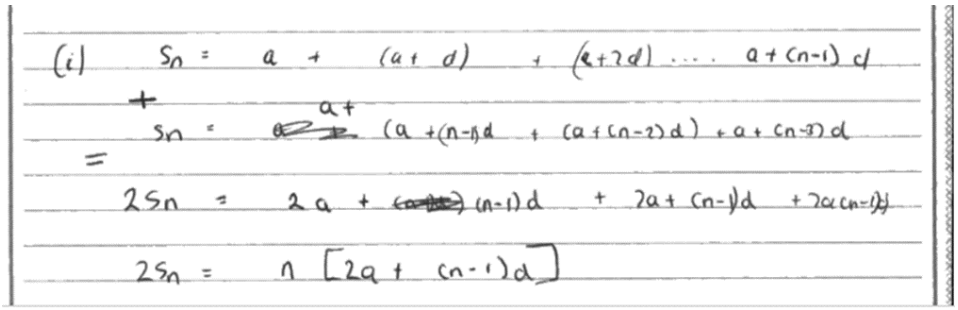
$S = l + (l-d) + \ldots\ldots\ldots\ldots\ldots\ldots + a$

$2S = n(a+l)$

$S = \frac{n}{2}(a+l) = \frac{n}{2}(a+a+(n-1)d) = \frac{n}{2}(2a+(n-1)d)$

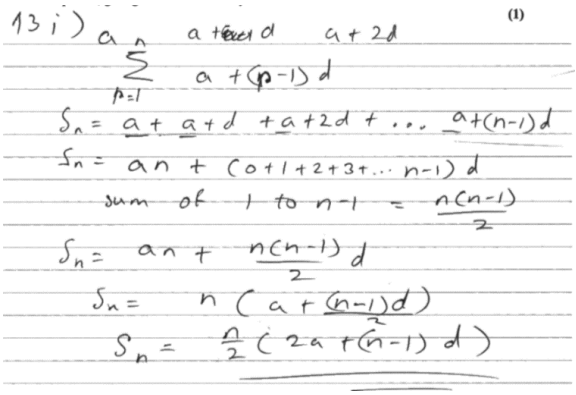
B1 and A1 are not scored until the last line, M scored on line 3

The following scores B1 M0 A0 as the terms in the second sum are not reversed



SC in (a) Scores B1 M0 A0.

They use $0+1+2+\ldots.+(n-1)=\frac{n(n-1)}{2}$ which relies on the quoted proof.



(ii) (a)

M1: Uses the information given to set up a correct equation in *n*.

The values of *S*, *a* and *d* need to be correct and used within a correct formula

Possible ways to score this include unsimplified versions $64=\frac{n}{2}(2\times10+(n-1)\times-0.8)$,

$64=\frac{n}{2}(10+10+(n-1)\times-0.8)$ or versions using pence rather than £'s $6400=\frac{n}{2}(2000+(n-1)\times-80)$

Allow recovery for both marks following $64=\frac{n}{2}(2\times10+(n-1)-0.8)$ with an invisible ×

A1*: Proceeds without error to the given answer. (Do not penalise a missing final trailing bracket)

Look for at least a line with the brackets correctly removed as well as a line with the terms in *n* correctly combined

E.g. $64=\frac{n}{2}(20+(n-1)\times-0.8)\Rightarrow64=10n-0.4n^2+0.4n\Rightarrow0.4n^2-10.4n+64=0\Rightarrow n^2-26n+160=0$

(ii)(b)

B1: $n=10,16$

(ii)(c)

B1: Chooses 10 (weeks) and gives a minimal acceptable reason. The reason must focus on why the answer is 10 (weeks) rather than 16(weeks) or alternatively why it would not be 16 weeks.

1(a)	$16+(21-1)\times d=24\Rightarrow d=...$	M1	1.1b
	$d=0.4$	A1	1.1b
	Answer only scores both marks.		
		(2)	
(b)	$S_n=\frac{1}{2}n\{2a+(n-1)d\}\Rightarrow S_{500}=\frac{1}{2}\times 500\{2\times 16+499\times "0.4"\}$	M1	1.1b
	$=57\,900$	A1	1.1b
	Answer only scores both marks		
		(2)	
	(b) Alternative using $S_n=\frac{1}{2}n\{a+l\}$		
	$l=16+(500-1)\times "0.4"=215.6\Rightarrow S_{500}=\frac{1}{2}\times 500\{16+"215.6"\}$	M1	1.1b
	$=57\,900$	A1	1.1b
			(4 marks)

Notes

(a)

M1: Correct strategy to find the common difference – must be a correct method using a = 16, and n = 21 and the 24. The method may be implied by their working.

If the AP term formula is quoted it must be correct, so use of e.g. $u_n=a+nd$ scores M0

A1: Correct value. Accept equivalents e.g. $\frac{8}{20}, \frac{4}{10}, \frac{2}{5}$ etc.

(b)

M1: Attempts to use a correct sum formula with a = 16, n = 500 and their numerical d from part (a)

If a formula is quoted it must be correct (it is in the formula book)

A1: Correct value

Alternative:

M1: Correct method for the 500th term and then uses $S_n=\frac{1}{2}n\{a+l\}$ with their l

A1: Correct value

Note that some candidates are showing implied use of $u_n=a+nd$ by showing the following:

(a) $d=\frac{24-16}{21}=\frac{8}{21}$ (b) $S_{500}=\frac{1}{2}\times 500\left\{2\times 16+499\times\frac{8}{21}\right\}=55523.80952...$

This scores (a) M0A0 (b) M1A0

Question	Scheme	Marks	AOs