adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Circles

Mark scheme

7 questions

Exam questions and marking instructions remain credited to their board.

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Question	Scheme	Marks	AOs
2 (a)	(i) Centre $(-3, 4)$	B1	1.1b
	(ii) States or implies that $r^2 = 24$ or $r = \sqrt{24}$	M1	1.1b
	$2\sqrt{6}$	A1	1.1b
		(3)	
(b)	Attempts a valid method e.g. Finds distance of centre from origin, Sets $y = 0$ and finds values of x	M1	3.1a
	Correct calculations, reason and conclusion (see notes)	A1	2.4
		(2)	
			(5 marks)

Notes:

Mark (i) and (ii) together

(a)(i)

B1: Centre $(-3, 4)$ Accept without brackets. May be written e.g. $x = -3,\ y = 4$

(a)(ii)

M1: States or implies that $r^2 = 24$ or $r = \sqrt{24}$. A final answer of $\sqrt{24}$ or $2\sqrt{6}$ implies the radius

May multiply out the brackets, collect terms ($x^2 + y^2 + 6x - 8y + 1 = 0$) and states the radius is $r^2 = \frac{6^2}{4} + \frac{(-8)^2}{4} - 1$ o.e. Do not condone slips for this mark.

A1: $2\sqrt{6}$ isw if they proceed to write as a decimal

(b) Note that if their radius is incorrect in (a) then maximum score is M1A0 unless they restart in (b)

M1: Attempts a valid method. For example

- Finds the distance (or distance ^{2}) of the centre from the origin.
  They must be attempting $(d =)\sqrt{(\pm"3")^2 + ("\pm"4")^2} = ...$ or
  $(d^2 =)(\pm"3" - 0)^2 + (\pm"4" - 0)^2 = ...$ and proceed to a value.
  May be seen as substituting the coordinates of the origin into the equation for C proceeding to a value for the left hand side e.g. 25 (to be able to compare with 24)
- Sets $y = 0$ and attempts to solve $(x+3)^2 + (-4)^2 = 24 \Rightarrow x = ...$ (at least one value)
- Sets $x = 0$ and attempts to solve $(3)^2 + (y-4)^2 = 24 \Rightarrow y = ...$ (at least one value)

In each method the starting expression or equation must be correct but do not be concerned by slips when evaluating or processing in finding the distance, the x coordinate or y coordinate.

Calculation examples	Reason examples	Conclusion examples
e.g. $\left(d^2 =\right) 25$ or e.g. $(d =) 5$	$25 > 24$ o.e. or $5 > \sqrt{24}$ o.e. (allow 4.9 or better)	e.g. origin does NOT lie within circle / origin lies outside circle / not in circle o.e.
e.g. $(x+3)^2 + (-4)^2 = 24$ $\Rightarrow (x =) -3 \pm \sqrt{8}$ (allow decimals awrt -0.2 and awrt -5.8)	roots are both negative (same signs) o.e.	
e.g. $(3)^2 + (y-4)^2 = 24$ $\Rightarrow (y =) 4 \pm \sqrt{15}$ (allow decimals awrt 0.1 and awrt 7.9)	roots are both positive (same signs) o.e.	

Note if their reasoning is incorrect e.g. referring to the radius as 24 instead of $\sqrt{24}$ then A0 but allow referencing to "the radius of *C*" provided their radius in (a) was correct.

Note if they give a reason that the origin does not lie inside the circle *C* because e.g. $25 \neq 24$ this scores M1A0 (M1 for 25 but A0 incorrect reasoning)

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
4(a)	e.g. $6.4^2 = 8^2 + 13^2 - 2(8)(13)\cos ABC$	M1	1.1b
	$(\text{angle } ABC =)\ \ \cos^{-1}\left(\dfrac{8^2+13^2-6.4^2}{2\times 8\times 13}\right) = 0.394 \text{ (radians)}$*	A1*	1.1b
		(2)	
(b)	e.g. $(\text{Area}(ABC) =)\ \ \dfrac{1}{2}(8)^2(0.394)\ \ (= \text{awrt } 12.6)$ **or** $(\text{Area}(ABD) =)\ \ \dfrac{1}{2}(8)(13)\sin 0.394\ \ (= \text{awrt } 20.0)$	M1	1.1b
	$(\text{Area}(R) =)\ \ \dfrac{1}{2}(8)(13)\sin 0.394 - \dfrac{1}{2}(8)^2(0.394) = \ldots$	dM1	3.1a
	$(\text{Area}(R) =)$ awrt 7.34 to 7.37	A1	1.1b
		(3)	
			(5 marks)

Notes

It is acceptable in this question to work in degrees and convert if necessary.
NB Angle *ABC* in degrees is 22.591…°

(a)

M1: Attempts to use the cosine rule with values correctly placed. May be implied by, e.g.,

$$\cos\theta = \frac{8^2+13^2-6.4^2}{2\times 8\times 13} \text{ or } \cos^{-1}\left(\frac{6.4^2-8^2-13^2}{-2\times 8\times 13}\right)$$

Condone slips in substitution if a correct cosine formula is seen. The placement of the values should be correct. So, condone e.g. a missing 2 once a correct cosine formula is seen.

Alt 1: attempts the cosine rule and finds angle *ADB* $(= \text{awrt } 0.5 \text{ or } 29°)$ **or** angle *BAD* $(= \text{awrt } 2.2 \text{ or } 2.3 \text{ or } 129°)$ **and** uses the sine rule correctly with angle *ABC* involved.

Alt 2: attempts the cosine rule and finds angle *ADB* $(= \text{awrt } 0.5 \text{ or } 29°)$ **and** angle *BAD* $(= \text{awrt } 2.2 \text{ or } 2.3 \text{ or } 129°)$ **and** sums angles *ADB*, *BAD*, and *ABC* to π (or 180°)

Alt 3: using Pythagoras **and** simultaneous equations to find the length of *AN* or *BN* (see diagram on the next page) **and** uses e.g. $\cos ABC = \dfrac{BN}{8}$ or $\sin ABC = \dfrac{AN}{8}$

A1*: cso Correct proof.

If starting with $6.4^2 = 8^2 + 13^2 - 2(8)(13)\cos ABC$ then there must be an intermediate line such as

- $\cos ABC = \dfrac{8^2+13^2-6.4^2}{2\times 8\times 13}$ or $\text{angle } ABC = \cos^{-1}\left(\dfrac{8^2+13^2-6.4^2}{2\times 8\times 13}\right)$
- $\cos ABC = \text{awrt } 0.92$ or $\dfrac{4801}{5200}$
- angle *ABC* = awrt 0.3943

The minimum required is e.g. $\cos^{-1}\left(\dfrac{8^2+13^2-6.4^2}{2\times 8\times 13}\right)$ or $\cos\theta = \dfrac{8^2+13^2-6.4^2}{2\times 8\times 13} \rightarrow \theta = 0.394$

both of which score M1A1*.

The final value must be 0.394 and not e.g. 0.3943. There should be no obvious incorrect statements in the proof e.g. $\cos(0.92) = 0.394$ and no obvious incorrect work.

Allow the use of, e.g., θ, *x*, or even e.g. *A* throughout. Mention of radians is not required.

Those working in degrees and achieve 22.6 can convert to 0.394 without calculation for M1A1

Attempts via verification e.g. $AD^2 = 8^2 + 13^2 - 2(8)(13)\cos 0.394 \Rightarrow AD =$ awrt 6.4 score maximum M1A0*.

(b) **Note: you may need to check the diagram for working.**

M1: Attempts the area of the sector **or** the area of the triangle *ABD* via a **correct** method. May be implied by a full attempt at the area of *R*.

The sector area may be implied by $\frac{1576}{125}$ or by $\frac{0.394}{2\pi}\times\pi\times8^2$ or $\frac{22.6}{360}\times\pi\times8^2$

The angle *ABC* is given in the question as 0.394 and should be used (or a more accurate value). Do not allow use of a different value for angle *ABC* (other than the angle in degrees).

There are many acceptable alternative approaches to find the area of triangle *ABD*, e.g. using their angle *ADB* **or** their angle *BAD* **or** using right-angled triangles

e.g. $\frac{1}{2}(13)(\text{"3.07"})$ **or** $\frac{1}{2}(\text{"7.39"})(\text{"3.07"})+\frac{1}{2}(\text{"5.614"})(\text{"3.07"})$ and could be implied by e.g. 11.3 + 8.6

Alternatively, attempts the area of the triangle *ACD* $\left(\text{e.g. }\frac{1}{2}(5)(6.4)\sin\text{"0.501"} = \text{awrt } 7.68\right)$

or the area of the segment (in sector *ABC*) $\left(\frac{1}{2}(8^2)(0.394)-\frac{1}{2}(8^2)\sin 0.394 = \text{awrt } 0.324\right)$

via a **correct** method. The area of the triangle *ABC* alone is insufficient for this mark.

Note that $\sin 0.394$ might be seen as $\sqrt{1-\left(\frac{4801}{5200}\right)^2}$

For values, see the diagram below.

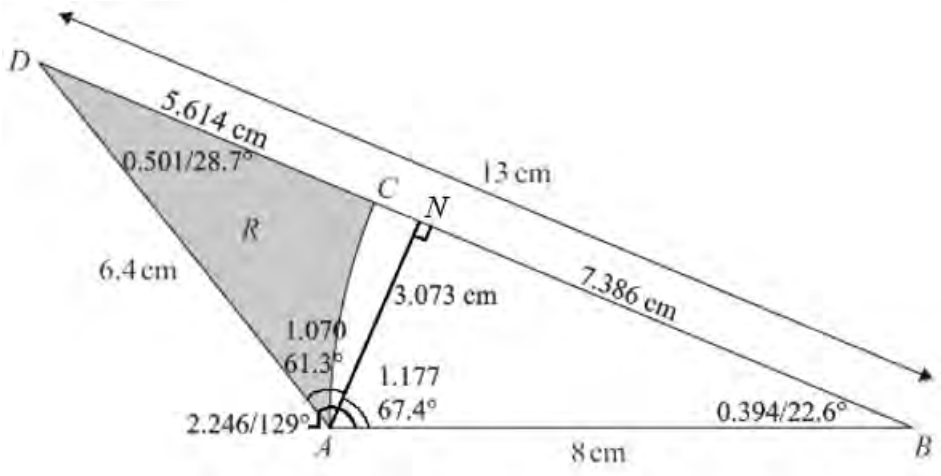
dM1: Complete and correct method for the area of *R*. Requires correct attempts at both the area of the sector **and** the area of the triangle *ABD* **and** subtraction of the two values (or expressions). Allow sector – triangle *ABD* if this is then recovered by making the area positive.

Alternatively, correct attempts at both the area of the triangle *ACD* **and** the area of the segment (in sector *ABC*) **and** subtraction of the two values (or expressions).

Condone slips in substitution provided a correct formula is seen.
May be implied by a correct answer.

A1: awrt 7.34 to 7.37 no units required but penalise incorrect units.
ISW after an acceptable answer is seen.

Helpful Diagram:



Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
11	Identifies angle $BAO = \frac{\pi}{3}$ or angle $BOC = \frac{2\pi}{3}$	B1	2.2a
	$\text{Area(segment)} = \frac{1}{2}\times 5^2 \times \frac{\pi}{3} - \frac{1}{2}\times 5^2 \sin\frac{\pi}{3}\left\{= \frac{25\pi}{6} - \frac{25\sqrt{3}}{4}\right\}$ **or** $\text{Area}(AOB) = 2\times\frac{1}{2}\times 5^2 \times \frac{\pi}{3} - \frac{1}{2}\times 5^2 \sin\frac{\pi}{3}\left\{= \frac{25\pi}{3} - \frac{25\sqrt{3}}{4}\right\}$	M1	2.1
	$\text{Area of } R = \frac{1}{2}\times 5^2 \times \frac{2\pi}{3} - \left(\frac{1}{2}\times 5^2 \times \frac{\pi}{3} - \frac{1}{2}\times 5^2 \sin\frac{\pi}{3}\right)$ **or** $\text{Area of } R = \frac{1}{2}\times \pi \times 5^2 - \left(\frac{1}{2}\times 5^2 \times \frac{\pi}{3} - \frac{1}{2}\times 5^2 \sin\frac{\pi}{3} + \frac{1}{2}\times 5^2 \times \frac{\pi}{3}\right)$	dM1	3.1a
	$= \frac{25}{4}\sqrt{3} + \frac{25}{6}\pi \left(\text{cm}^2\right)$	A1	1.1b
		(4)	
			(4 marks)

Notes

Note: Use of degrees, if used in formulae in degrees, can score full marks.

B1: Deduces angle *BAO* or angle *BOA* is $\frac{\pi}{3}$ radians or 60° or deduces that angle *BOC* is $\frac{2\pi}{3}$ radians or 120°. May be seen on the diagram or their own sketches or embedded in a formula. May be implied if they find e.g. $\frac{1}{6}\times\pi\times 5^2$ for the area of the minor sector.

M1: Uses a correct process and an angle of $\frac{\pi}{3}$ radians or 60° to find

- the area of the segment bounded by the arc *OB* and straight line *OB*
- or the area of the segment bounded by the arc *AB* and straight line *AB*
- or the area of unshaded region *AOB*.

Allow decimal values to imply the method (require 3sf rounded or truncated).
Use of 60° must be in a correct formula in degrees. $0.5\times 5^2 \times 60$ scores M0.
For reference these are the areas of the regions:

- segment $= \frac{1}{2}\times 5^2 \times \frac{\pi}{3} - \frac{1}{2}\times 5^2 \times \sin\frac{\pi}{3}\left\{= \frac{25\pi}{6} - \frac{25\sqrt{3}}{4} = 2.26\right\}$ (3 s.f.) scores M1
- $AOB = \frac{1}{2}\times 5^2 \times \frac{\pi}{3} + \frac{1}{2}\times 5^2 \times \frac{\pi}{3} - \frac{1}{2}\times 5^2 \times \sin\frac{\pi}{3}\left\{= \frac{25\pi}{3} - \frac{25\sqrt{3}}{4} = 15.35\right\}$ (2 d.p.) scores M1
- Minor sector $= \frac{1}{2}\times 5^2 \times \frac{\pi}{3}\left\{= \frac{25\pi}{6} = 13.09\right\}$ (2 d.p.) scores M0 on its own
- Major sector $= \frac{1}{2}\times 5^2 \times \frac{2\pi}{3}\left\{= \frac{25\pi}{3} = 26.18\right\}$ (2 d.p.) scores M0 on its own

- Triangle $AOB = \frac{1}{2} \times 5^2 \times \sin\frac{\pi}{3} \left\{ = \frac{25\sqrt{3}}{4} = 10.8 \right\}$ (3 s.f.) scores M0 on its own.

This mark may be implied if seen as part of a **correct** strategy for the area of *R*.

Note that the area of triangle *AOB* may also be found using Pythagoras and $\frac{1}{2}bh$ but their method must be correct.

dM1: A fully correct strategy for finding the area of *R*.
Follow through on incorrectly simplified areas but the method must be correct.
Allow decimal values to imply the method (require 3sf rounded or truncated).
For reference, the area is approximately 23.92 (2d.p.) and is likely to imply B1M1dM1 but their work should be checked.
Either:

- major sector – segment $= \frac{1}{2} \times 5^2 \times \frac{2\pi}{3} - \left(\frac{1}{2} \times 5^2 \times \frac{\pi}{3} - \frac{1}{2} \times 5^2 \sin\frac{\pi}{3} \right)$

or

- semicircle – $AOB = \frac{1}{2} \times \pi \times 5^2 - \left(\frac{1}{2} \times 5^2 \times \frac{\pi}{3} - \frac{1}{2} \times 5^2 \sin\frac{\pi}{3} + \frac{1}{2} \times 5^2 \times \frac{\pi}{3} \right)$

or

- semicircle – 2 × sector *AOB* + triangle $AOB = \frac{1}{2} \times \pi \times 5^2 - 2 \times \frac{1}{2} \times 5^2 \times \frac{\pi}{3} + \frac{1}{2} \times 5^2 \sin\frac{\pi}{3}$

or

- sector *AOB* + triangle $AOB = \frac{1}{2} \times 5^2 \times \frac{\pi}{3} + \frac{1}{2} \times 5^2 \sin\frac{\pi}{3}$ (this also implies the first M1)

If they go straight to one of these expressions then we would imply the first M1 (and the B1).

A1: Correct expression $\frac{25}{4}\sqrt{3} + \frac{25}{6}\pi$ o.e. in the correct form e.g. $\frac{50}{8}\sqrt{3} + \frac{75}{18}\pi$

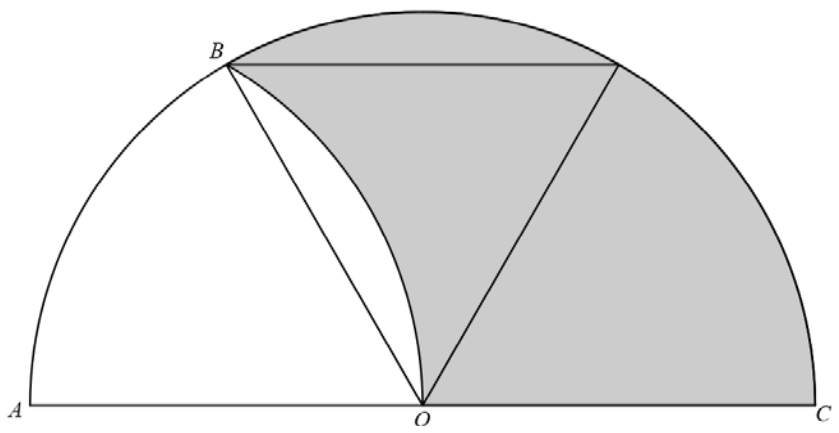
Ignore any reference to (or absence of) units.

Do not apply isw if they go on to add or subtract additional areas.

However, we can condone poor simplification e.g. $\frac{25}{12}\left(\sqrt{3} + \pi\right)$ following a correct answer.

Note: Attempts via integration for the area underneath a circle or polar coordinates are unlikely but if seen then use Review. The schemes for these approaches follow the main scheme but the first M1 may also be scored for finding half of the area of *AOB*.

You might find the following diagram helpful:



Question	Scheme	Marks	AOs

14(a)(i)	Centre is $(3, -7)$	B1	1.1b
(ii)	$(x-3)^2+(y+7)^2=49+9-33 \Rightarrow r^2=...(25)$	M1	1.1b
	$r=5$	A1	1.1b
		(3)	
(b)	Distance between centres = $\sqrt{(3+6)^2+(-7+8)^2}=\sqrt{82}$	M1 A1ft	3.1a 1.1b
	$"\sqrt{82}"-"5"$ **or** $"\sqrt{82}"+"5"$	dM1	3.1a
	$\sqrt{82}-5$ **and** $\sqrt{82}+5$	A1	2.2a
	$\{k:\sqrt{82}-5<k\}\cap\{k:k<\sqrt{82}+5\}$ or e.g. $\{k:\sqrt{82}-5<k<\sqrt{82}+5\}$	A1	2.5
		(5)	
			(8 marks)

Notes

(a)(i)

B1: Correct centre. Allow as a coordinate pair or written separately e.g. $x = 3$, $y = -7$ or as a column vector $\begin{pmatrix} 3 \\ -7 \end{pmatrix}$

Condone missing brackets e.g. 3, –7 but do **not** allow coordinates the wrong way round.

(ii)

M1: Uses a correct strategy to find the radius or radius^2

Requires an attempt at: $(x\pm3)^2+(y\pm7)^2-3^2-7^2\pm33=0\Rightarrow(x\pm3)^2+(y\pm7)^2=\alpha,\ \alpha>0$

Award for $(x\pm3)^2+(y\pm7)^2-a^2-b^2\pm33=0\Rightarrow(x\pm3)^2+(y\pm7)^2=\alpha,\ \alpha>0$ with at least one of $a = 3$ or $b = 7$ (or 9 or 49)

You may see an attempt at $"f^2+g^2-c"$ or $\sqrt{"f^2+g^2-c"}$ e.g. $"3^2+7^2\pm33"$ or $\sqrt{"3^2+7^2\pm33"}$

A1: Correct radius of 5. Do **not** allow ±5 or $\sqrt{25}$.

May be scored following $(x\pm3)^2+(y\pm7)^2=25$

Correct answers only in (a) scores B1M1A1

(b)

M1: Uses Pythagoras correctly on their centre from part (a) and the given centre to find the distance between the centres.

Look for $\sqrt{(-6-(\text{their } x))^2+(-8-(\text{their } y))^2}$ or e.g. $\sqrt{((\text{their } x)-(-6))^2+((\text{their } y)-(-8))^2}$ but condone one sign slip with their coordinates if the intention is clear.

A1ft: Correct distance or follow through their centre from part (a).

This may be implied by their value. Condone the use of decimals so allow 3sf accuracy e.g. awrt 9.06 for $\sqrt{82}$ or you may need to check their value following an incorrect centre in (a)(i). Not e.g. $\pm\sqrt{82}$ unless the positive root is subsequently used.

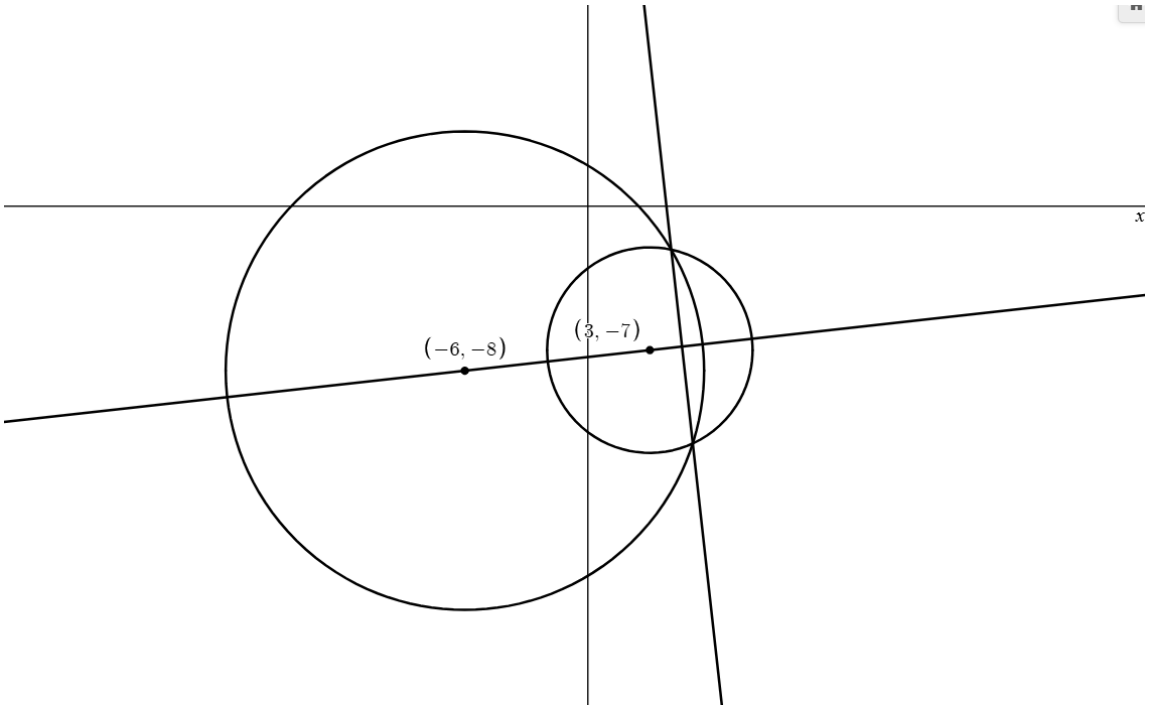
dM1: Correct strategy for one of the limits. E.g. adds or subtracts <u>their 5</u> to their distance between centres.

A1: Correct limits. There is no follow through but allow decimals to 3sf e.g. awrt 4.06 and awrt 14.1

A1: Correct answer with **exact values** using set notation.
Allow as shown in the main scheme but also allow equivalent set notation e.g.
$\{k : k \in \mathrm{R}, \sqrt{82}-5 < k < \sqrt{82}+5\}$, $\{k : \sqrt{82}-5 < k < \sqrt{82}+5\}$, $k \in (\sqrt{82}-5, \sqrt{82}+5)$
and allow "|" for ":" and allow the "k:" or "$k \in$" to be missing
e.g. $\{\sqrt{82}-5 < k < \sqrt{82}+5\}$ and $(\sqrt{82}-5, \sqrt{82}+5)$ are both acceptable.
But $\{k : k < \sqrt{82}+5\} \cup \{k : k > \sqrt{82}-5\}$ or $\{k : k < \sqrt{82}-5, k < \sqrt{82}+5\}$ score A0
Do **not** allow solutions not in set notation such as $\sqrt{82}-5 < k < \sqrt{82}+5$

Correct answers with no working should be sent to review.

Scenario for part (b) for reference:



Algebraic approach for part (b):

$$x^2+y^2-6x+14y+33=x^2+12x+36+y^2+16y+64-k^2$$

$$\Rightarrow 18x+2y+67-k^2=0 \Rightarrow y=\frac{k^2-67}{2}-9x$$

$$(x-3)^2+(y+7)^2=25 \Rightarrow x^2-6x+9+\left(\frac{k^2-67}{2}-9x+7\right)^2=25$$

$$\Rightarrow 82x^2+471x-9k^2x+\frac{k^4-106k^2+2745}{4}=0$$

When circles touch $b^2-4ac=0$

$$\Rightarrow \left(471-9k^2\right)^2-4\times 82\left(\frac{k^4-106k^2+2745}{4}\right)=0$$

$$\Rightarrow k^4-214k^2+3249=0$$

$$\Rightarrow \left(k^2-10k-57\right)\left(k^2+10k-57\right)=0$$

$$\Rightarrow k=5+\sqrt{82},\ 5-\sqrt{82},\ -5+\sqrt{82},\ -5-\sqrt{82}$$

$$k=\underline{5+\sqrt{82},\ -5+\sqrt{82}}$$

We will mark this as follows:

M1: This requires a valid strategy that:
- solves the 2 circle equations simultaneously to find y in terms of x and k, or x in terms of y and k
- substitutes for y or x into one of the circle equations to obtain an equation in x and k only, or y and k only,
- attempts $b^2-4ac=0$ or e.g. $b^2-4ac>0$ or equivalent to obtain an equation in k only. You do not need to look at the details of their algebra.

A1: Correct simplified 3TQ in k^2

dM1: Solves their 3TQ in k^2 by any correct method including a calculator to find k.

A1: Both correct values for k (exact or decimals as in the main scheme) (they may have extras which can be ignored)

A1: As main scheme (exact and in set notation)

Note that work such as

$$(x+6)^2+(y+8)^2=k^2 \Rightarrow x+6+y+8=k$$

is not a valid strategy as it greatly simplifies the problem and would generally score no marks.

Implicit differentiation approach for part (b):

$$x^2+y^2-6x+14y+33=0 \Rightarrow 2x+2y\frac{\mathrm{d}y}{\mathrm{d}x}-6+14\frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{6-2x}{2y+14}$$

$$\frac{3-x}{y+7}=-\frac{x+6}{y+8} \Rightarrow (3-x)(y+8)=-(x+6)(y+7) \Rightarrow x=9y+66$$

$$(9y+66)^2+y^2-6(9y+66)+14y+33=0 \Rightarrow 82y^2+1148y+3993=0$$

$$(\text{or } 82x^2-492x-1287=0)$$

$$y=\frac{-574\pm 5\sqrt{82}}{82} \Rightarrow x=\frac{246\pm 45\sqrt{82}}{82}$$

$$\left(\frac{246+45\sqrt{82}}{82},\frac{-574+5\sqrt{82}}{82}\right) \rightarrow \left(\frac{246+45\sqrt{82}}{82}+6\right)^2+\left(\frac{-574+5\sqrt{82}}{82}+8\right)^2=k^2$$

$$\Rightarrow k^2=107+10\sqrt{82} \Rightarrow k=5+\sqrt{82}$$

$$\text{Then the same for } \left(\frac{246-45\sqrt{82}}{82},\frac{-574-5\sqrt{82}}{82}\right) \rightarrow k=-5+\sqrt{82}$$

We will mark this as follows:

M1: This requires a valid strategy that:

- differentiates the equations of both circles implicitly and equates the derivatives to obtain an equation connecting y and x. (Note that the equation connecting y and x is the common equation through the centres which can also be found from using the coordinates of the centres)
- substitutes for x or y into the equation for C_1 to obtain an equation in one variable

A1: Correct 3TQ in y or x

dM1: This requires:

- solves their 3TQ in y or x by any correct means including a calculator and finds at least one point of intersection
- substitutes this point into C_2 and proceeds to a value for k

A1: Correct values for k (exact or decimals as in the main scheme)

A1: As main scheme (exact and in set notation)

Question	Scheme	Marks	AOs

3 (a)	(i) $x^2+y^2-10x+16y=80 \Rightarrow (x-5)^2+(y+8)^2=...$	M1	1.1b
	Centre $(5,-8)$	A1	1.1b
	(ii) Radius 13	A1	1.1b
		(3)	
(b)	Attempts $\sqrt{"5"^2+"8"^2}+"13"$	M1	3.1a
	$13+\sqrt{89}$ but ft on their centre and radius	A1ft	1.1b
		(2)	
			(5 marks)

Notes:

(a)(i)

M1: Attempts to complete the square on **both** *x* and *y* terms.

Accept $(x\pm5)^2+(y\pm8)^2=...$ or imply this mark for a centre of $(\pm5,\pm8)$

Condone $(x\pm5)^2....(y\pm8)^2=...$ where the first ... could be , or even $-$

A1: Correct centre $(5,-8)$.

Accept without brackets. May be written $x=5, y=-8$

(a)(ii)

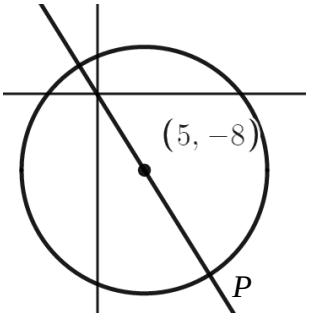
A1: 13. The M mark must have been awarded, so it can be scored following a centre of $(\pm5,\pm8)$.

Do not allow for $\sqrt{169}$ or ±13

(b)

M1: Attempts $\sqrt{"5"^2+"8"^2}+"13"$ for their centre $(5,-8)$ and their radius 13.

Award when this is given as a decimal, e.g. 22.4 for correct centre and radius. Look for $\sqrt{a^2+b^2}+r$ where centre is $(\pm a,\pm b)$ and radius is *r*



A1ft: $13+\sqrt{89}$ Follow through on their $(5,-8)$ and their 13 leading to an exact answer. ISW for example if they write $13+\sqrt{89}=22.4$

.....

There are more complicated attempts which could involve finding *P* by solving $y="-\frac{8}{5}x"$ and $x^2+y^2-10x+16y=80$ simultaneously and choosing the coordinate with the greatest modulus. The method is only scored when the distance of the largest coordinate from *O* is attempted. Such methods are unlikely to result in an exact value but can score 1 mark for the method. Condone slips

FYI. Solving $y=-\frac{8}{5}x$ and $x^2+y^2-10x+16y=80 \Rightarrow 89x^2-890x-2000=0 \Rightarrow P=(11.89,-19.02)$

Hence *OP* = $\sqrt{"11.89"^2+"19.02"^2}\ (=22.43)$ scores M1 A0 but *OP* = $\sqrt{258+26\sqrt{89}}$ is M1 A1

Question	Scheme	Marks	AOs

7(a)(i)	$(x-5)^2+(y+2)^2=...$	M1	1.1b
	$(5,-2)$	A1	1.1b
(ii)	$r=\sqrt{"5"^2+"-2"^2-11}$	M1	1.1b
	$r=3\sqrt{2}$	A1	1.1b
		(4)	
(b)	$y=3x+k \Rightarrow x^2+(3x+k)^2-10x+4(3x+k)+11=0$ $\Rightarrow x^2+9x^2+6kx+k^2-10x+12x+4k+11=0$	M1	2.1
	$\Rightarrow 10x^2+(6k+2)x+k^2+4k+11=0$	A1	1.1b
	$b^2-4ac=0 \Rightarrow (6k+2)^2-4\times10\times(k^2+4k+11)=0$	M1	3.1a
	$\Rightarrow 4k^2+136k+436=0 \Rightarrow k=...$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a
		(5)	
			(9 marks)

Notes

(a)(i)

M1: Attempts to complete the square on by halving both x and y terms.

Award for sight of $(x\pm5)^2,(y\pm2)^2=...$ This mark can be implied by a centre of (±5, ±2).

A1: Correct coordinates. (Allow $x = 5$, $y = -2$)

(a)(ii)

M1: Correct strategy for the radius or radius^2 . For example award for $r=\sqrt{"\pm5"^2+"\pm2"^2-11}$

or an attempt such as $(x-a)^2-a^2+(y-b)^2-b^2+11=0 \Rightarrow (x-a)^2+(y-b)^2=k \Rightarrow r^2=k$

A1: $r=3\sqrt{2}$. Do not accept for the A1 either $r=\pm3\sqrt{2}$ or $\sqrt{18}$

The A1 can be awarded following sign slips on $(5,-2)$ so following $r^2="\pm5"^2+"\pm2"^2-11$

(b) Main method seen

M1: Substitutes $y = 3x + k$ into the given equation (or their factorised version) and makes progress by attempting to expand the brackets. Condone lack of = 0

A1: Correct 3 term quadratic equation.

The terms must be collected but this can be implied by correct a, b and c

M1: Recognises the requirement to use $b^2 – 4ac = 0$ (or equivalent) where both b and c are expressions in k . It is dependent upon having attempted to substitute $y = 3x + k$ into the given equation

M1: Solves 3TQ in k. See General Principles.

The 3TQ in k must have been found as a result of attempt at $b^2 – 4ac$ ... 0

A1: Correct simplified values

Look carefully at the method used. It is possible to attempt this using gradients

(b) Alt 1	$x^2+y^2-10x+4y+11=0 \Rightarrow 2x+2y\frac{dy}{dx}-10+4\frac{dy}{dx}=0$	M1 A1	2.1 1.1b
	Sets $\frac{dy}{dx}=3 \Rightarrow x+3y+1=0$ and combines with equation for C $\Rightarrow 5x^2-50x+44=0$ or $5y^2+20y+11=0$ $\Rightarrow x=...$ or $y=...$	M1	3.1a
	$x=\frac{25\pm9\sqrt{5}}{5}$, $y=\frac{-10\pm3\sqrt{5}}{5}$, $k=y-3x \Rightarrow k=...$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a

M1: Differentiates implicitly condoning slips but must have two $\frac{dy}{dx}$'s coming from correct terms

A1: Correct differentiation.

M1: Sets $\frac{dy}{dx} = 3$, makes y or x the subject, substitutes back into C and attempts to solve the resulting quadratic in x or y.

M1: Uses at least one pair of coordinates and l to find at least one value for k. It is dependent upon having attempted both M's

A1: Correct simplified values

(b) Alt 2	$x^2 + y^2 - 10x + 4y + 11 = 0 \Rightarrow 2x + 2y\frac{dy}{dx} - 10 + 4\frac{dy}{dx} = 0$	M1 A1	2.1 1.1b
	Sets $\frac{dy}{dx} = 3 \Rightarrow x + 3y + 1 = 0$ and combines with equation for l $y = 3x + k, x + 3y = 1$ $\Rightarrow x = ...$ and $y = ...$ in terms of k	M1	3.1a
	$x = \frac{-3k-1}{10}, \ y = \frac{k-3}{10}, \ x^2 + y^2 - 10x + 4y + 11 = 0 \Rightarrow k = ...$	M1	1.1b
	$k = -17 \pm 6\sqrt{5}$	A1	2.2a

Very similar except it uses equation for l instead of C in mark 3

M1 A1: Correct differentiation (See alt 1)

M1: Sets $\frac{dy}{dx} = 3$, makes y or x the subject, substitutes back into l to obtain x and y in terms of k

M1: Substitutes for x and y into C and solves resulting 3TQ in k

A1: Correct simplified values

(b) Alt 3	$y = 3x + k \Rightarrow m = 3 \Rightarrow m_r = -\frac{1}{3}$	M1
	$y + 2 = -\frac{1}{3}(x - 5)$	A1
	$(x-5)^2 + (y+2)^2 = 18, \ y + 2 = -\frac{1}{3}(x-5)$ $\Rightarrow \frac{10}{9}(x-5)^2 = 18 \Rightarrow x = ...$ or $\Rightarrow 10(y+2)^2 = 18 \Rightarrow y = ...$	M1
	$x = \frac{25 \pm 9\sqrt{5}}{5}, \ y = \frac{-10 \pm 3\sqrt{5}}{5}, \ k = y - 3x \Rightarrow k = ...$	M1
	$k = -17 \pm 6\sqrt{5}$	A1

M1: Applies negative reciprocal rule to obtain gradient of radius

A1: Correct equation of radial line passing through the centre of C

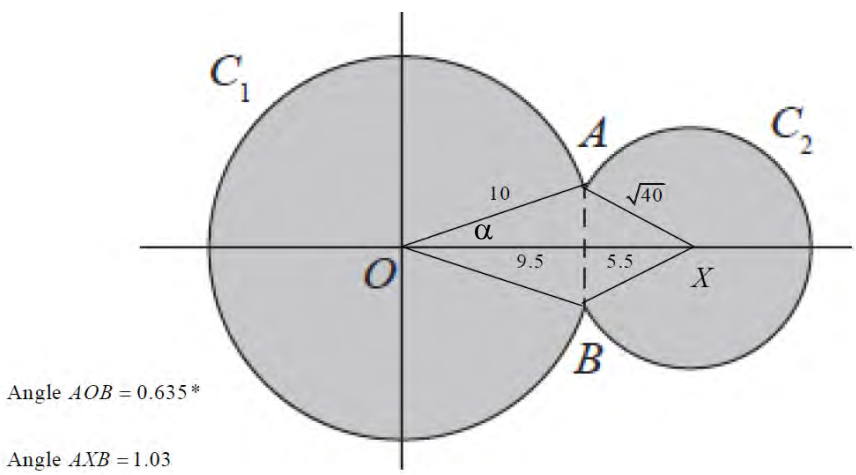
M1: Solves simultaneously to find x or y

Alternatively solves "$y = -\frac{1}{3}x - \frac{1}{3}$" and $y = 3x + k$ to get x in terms of k which they substitute in $x^2 + (3x + k)^2 - 10x + 4(3x + k) + 11 = 0$ to form an equation in k.

M1: Applies $k = y - 3x$ with at least one pair of values to find k

A1: Correct simplified values

Question	Scheme	Marks	AOs
11 (a)	Solves $x^2+y^2=100$ and $(x-15)^2+y^2=40$ simultaneously to find x or y E.g. $(x-15)^2+100-x^2=40 \Rightarrow x=...$	M1	3.1a
	Either $\Rightarrow -30x+325=40 \Rightarrow x=9.5$ Or $y=\frac{\sqrt{39}}{2}$ = awrt ± 3.12	A1	1.1b
	Attempts to find the angle AOB in circle C_1 Eg Attempts $\cos\alpha=\frac{"9.5"}{10}$ to find α then $\times 2$	M1	3.1a
	Angle AOB = $2\times\arccos\left(\frac{9.5}{10}\right)=0.635$ rads (3sf) *	A1*	2.1
		(4)	
(b)	Attempts $10\times(2\pi-0.635)$ = 56.48	M1	1.1b
	Attempts to find angle AXB or AXO in circle C_2 (see diagram) E.g. $\cos\beta=\frac{15-"9.5"}{\sqrt{40}}\Rightarrow\beta=...$ (Note AXB =1.03 rads)	M1	3.1a
	Attempts $10\times(2\pi-0.635)+\sqrt{40}\times(2\pi-2\beta)$	dM1	2.1
	= 89.7	A1	1.1b
		(4)	
			(8 marks)
Notes:			



(a)

M1: For the key step in an attempt to find either coordinate for where the two circles meet.
Look for an attempt to set up an equation in a single variable leading to a value for x or y.

A1: $x=9.5$ (or $y=\frac{\sqrt{39}}{2}$ = awrt ± 3.12)

M1: Uses the radius of the circle and correct trigonometry in an attempt to find angle AOB in circle C_1

E.g. Attempts $\cos\alpha = \frac{"9.5"}{10}$ to find α then $\times 2$

Alternatives include $\tan\alpha = \frac{\sqrt{100-"9.5"^2}}{"9.5"} = (0.3286...)$ to find α then $\times 2$

And $\cos AOB = \frac{10^2+10^2-\left(\sqrt{39}\right)^2}{2\times 10\times 10} = \frac{161}{200}$

A1*: Correct and careful work in proceeding to the given answer. Condone an answer with greater accuracy. Condone a solution where the intermediate value has been truncated, provided the trig equation is correct.

E.g. $\sin\alpha = \frac{\sqrt{39}}{20} \Rightarrow \alpha = 0.317 \Rightarrow AOB = 2\alpha = 0.635$

Condone a solution written down from awrt $36.4°$ (without the need to shown any calculation.)

E

(b)

M1: Attempts to use the formula $s = r\theta$ with $r = 10$ and $\theta = 2\pi - 0.635$

The formula may be embedded. You may see $\underline{\underline{2\pi 10}} + 2\pi\sqrt{40} \underline{\underline{-10\times 0.635}}...$ which is fine for this M1

M1: Attempts to use a correct method in order to find angle AXB or AXO in circle C_2

Amongst many other methods are $\tan\beta = \frac{"3.12"'''}{15-9.5}$ and $\cos AXB = \frac{40+40-\left(\sqrt{39}\right)^2}{2\times\sqrt{40}\times\sqrt{40}} = \frac{41}{80}$

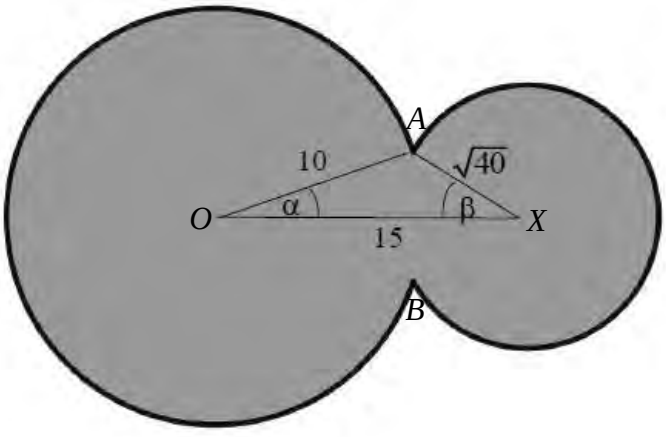
Note that many candidates believe this to be 0.635. This scores M0 dM0 A0

dM1: A full and complete attempt to find the perimeter of the region.

It is dependent upon having scored both M's.

A1: awrt 89.7

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(a)

M1: For the key step in attempting to find all lengths in triangle OAX, condoning slips

A1: All three lengths correct

M1: Attempts cosine rule to find α then $\times 2$

A1*: Correct and careful work in proceeding to the given answer

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Question	Scheme	Marks	AOs