adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Coordinate geometry

Mark scheme

10 questions

Exam questions and marking instructions remain credited to their board.

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Question	Scheme	Marks	AOs
8(a)	$x^4 \to \ldots x^3$ or $\ldots x^3 \to \ldots x^2$ or $\ldots x^2 \to \ldots x$ or $ax \to a$	M1	1.1b
	$\left(\mathrm{f}'(x)=\right)\ 4x^3+x^2-16x+a$	A1	1.1b
	$\left(\mathrm{f}'\left(-\frac{1}{4}\right)=\right)\ 4\left(-\frac{1}{4}\right)^3+\left(\pm\frac{1}{4}\right)^2-16\left(-\frac{1}{4}\right)+a=0$ $\Rightarrow a=\ldots$	dM1	3.1a
	$a=-4$ *	A1*	2.1
		(4)	
(b)	$\frac{4735}{768}$	B1	1.1b
		(1)	

Notes

(a) The first two marks are the same for all approaches.

M1: Reduces the power by one for any term in $\mathrm{f}(x)$. Look for $x^n \to x^{n-1}$ and allow for $ax \to a$ including $ax^1 \to ax^0$ or $-4x \to -4$

A1: Correct differentiation $\left(\mathrm{f}'(x)=\right)\ 4x^3+x^2-16x+a$ or $4x^3+x^2-16x-4$

Ignore the absence of the LHS. May be in terms of *a* or have –4 substituted at this point as above.

Main Scheme:

dM1: Substitutes $x=-\frac{1}{4}$ into their $\mathrm{f}'(x)$, sets = 0 (= 0 may be implied) and solves for *a*.

Condone $4+a=0$ as evidence of substitution for the dM1 only.

This mark is not available if their $\mathrm{f}'(x)$ does not have a term with *a* in it using this approach.

A1*: cso Requires:

- a correct derivative
- correct substitution of $x=-\frac{1}{4}$ into $\mathrm{f}'(x)$ and set = 0 (now the = 0 must be seen somewhere for this mark). Each term does not need to be evaluated.
- achieves $a = -4$

For the A1*, evidence of correct substitution might be $-\frac{1}{16}+\frac{1}{16}+4+a=0$ but not $4+a=0$

Condone invisible brackets around $-\frac{1}{4}$ provided this is recovered before the given answer e.g.

$-\frac{1}{4}^2$ recovered to $+\frac{1}{16}$

Alt 1: Verification

dM1: Substitutes $x=-\frac{1}{4}$ and $a = -4$ into their $\mathrm{f}'(x)$, and finds a value.

The $a = -4$ may already have been substituted into f (*x*) as above.

A1*: cso Requires:

- a correct derivative

- correct substitution of $x = -\frac{1}{4}$ into $f'(x)$ and use of $a = -4$ (at some stage) to achieve $f'\left(-\frac{1}{4}\right) = ... = 0$. The substitution must be embedded in $f'(x)$ **or** evaluated in an intermediate step e.g. $-\frac{1}{16} + \frac{1}{16} + 4 - 4$ before the = 0
- a minimal conclusion e.g. "hence proven" or "∴ $a = -4$" or e.g. a tick.
  but **not** e.g. "max = $-\frac{1}{4}$" or just "0 = 0"

Alt 2: Algebraic division

dM1: Attempts to divide their $f'(x)$ by $(4x+1)$ $\left(\text{or}\left(x+\frac{1}{4}\right)\right)$, achieves a linear remainder in a only, sets = 0 and achieves a value for a. Condone slips in their calculations.
As a minimum, expect to see $x^2 + \lambda x + \mu$ $\left(\text{or } 4x^2 + \lambda x + \mu\right)$ as their quotient (with $\mu \neq 0$) leading to a linear remainder in a only set = 0 leading to a value for a.

A1*: cso Requires:
- a correct derivative
- correct quotient $x^2 - 4$ $\left(\text{or}\left(4x^2 - 16\right)\right)$ and correct remainder $a + 4$ set = 0
- achieves $a = -4$

(b)

B1: $\frac{4735}{768}$ cao Allow exact equivalents e.g. $6\frac{127}{768}$ or the recurring decimal $6.165364583\dot{}$
There is no need to see a calculation. Decimal approximations e.g. 6.1653645833... score B0.

Question	Scheme	Marks	AOs
8(c)	$\left(\mathrm{f}'(x)=\right)\ 4x^3+x^2-16x-4=(4x+1)\left(x^2+...\right)$	M1	3.1a
	$=(4x+1)\left(x^2-4\right)$ **or** e.g. $\dfrac{4x^3+x^2-16x-4}{4x+1}=x^2-4$	A1	1.1b
	$\mathrm{f}("-2")=...(-5)$ $\Rightarrow "-5"<k<"\dfrac{4735}{768}"$	M1	2.1
	$\left\{k\in\mathbb{R}:-5<k<"\dfrac{4735}{768}"\right\}$	A1ft	2.5
		(4)	
			(9 marks)

Notes

(c) **Note: Candidates who do not score the first M mark may score maximum M0A0M1A1ft via the special case at the end of the mark scheme for part (c).**

M1: For the key step in using the factor theorem to take $(4x+1)$ or $\left(x+\dfrac{1}{4}\right)$ out as a factor of their $\mathrm{f}'(x)$, **not** $\mathrm{f}(x)$. Look for $(4x+1)\left(x^2+\lambda x-4\right)$ or $(4x+1)\left(x^2\pm\mu\right)$ or $\left(x+\dfrac{1}{4}\right)\left(4x^2\pm\mu\right)$ or $\left(x+\dfrac{1}{4}\right)\left(4x^2\pm\lambda x-16\right)$ (where the λ or μ is a constant) either by inspection or division.

If they attempt division in (a) then they must use their quotient in (c) to score the marks.

They may "spot" that one of f'(±2) = 0 and factor out $(x\pm2)\left(4x^2+\lambda x\pm2\right)$ without evidence that f'(±2) = 0

There must be some factorisation or algebraic division present – not just roots stated from a calculator.

A1: Correct factorisation of the cubic to a linear and quadratic product, so

$(4x+1)\left(x^2-4\right)$ or $\left(x+\dfrac{1}{4}\right)\left(4x^2-16\right)$ or $(x-2)\left(4x^2+9x+2\right)$ or $(x+2)\left(4x^2-7x-2\right)$

or for the correct quadratic seen e.g. x^2-4 or $4x^2-16$ (which may be the quotient in their algebraic division and may come from their work in (a) provided it is used in (c)).

There is no need to complete the long division if the appropriate quotient is found.

Note that proceeding directly to $(4x+1)(x-2)(x+2)$ does not score either mark without sight of the e.g. $(4x+1)\left(x^2-4\right)$ but they can score the final 2 marks via the special case.

Similarly, proceeding directly to $\left(x+\dfrac{1}{4}\right)(2x-4)(2x+4)$ or $4\left(x+\dfrac{1}{4}\right)(x-2)(x+2)$ does not score either mark without sight of $\left(x+\dfrac{1}{4}\right)\left(4x^2-16\right)$ but they can score marks via the SC.

M1: Solves their quadratic factor = 0, substitutes one of these solutions (not $x=-\dfrac{1}{4}$) into $\mathrm{f}(x)$ to find a value **and** selects the inside region between this and their answer to (b).

If their quadratic is of the form Ax^2-B then they can write down their solution for *x* but if it is a 3TQ then they must find their value of *x* using the usual non-calculator rules.

Condone "incorrect" factorisation following the correct quadratic leading to *x* = ±2

e.g. $\left(x+\frac{1}{4}\right)\left(4x^2-16\right)\rightarrow\left(x+\frac{1}{4}\right)(x-2)(x+2)=0\rightarrow x=\pm 2$

y (or k) = –5 (or $-\frac{47}{3}$) with nothing else can imply substitution of x = –2 (or 2)

Note that if they "spot" that f'(±2) = 0 earlier then they can go straight to substitution.

Condone the use of y but not x in their region. Condone e.g. $k>-5, k<"\frac{4735}{768}"$ for this mark.

Condone the use of ≤ in place of < for this mark. Note that $\left(2,-\frac{47}{3}\right)$ is the other minimum.

A1ft: $\left\{k:-5<k<"\frac{4735}{768}"\right\}$ Requires correct use of set notation but condone absence of $\in\mathbb{R}$

Follow through on their upper limit from (b) which may be a decimal but must be positive. Must be strict inequalities i.e. not include the end points and must now be in terms of k.

Other acceptable notation includes: $k\in\left(-5,"\frac{4735}{768}"\right)$, $\{k\,,k>-5\}\cap\left\{k\,,k<"\frac{4735}{768}"\right\}$

Condone $\left\{k:k>-5\cap k<"\frac{4735}{768}"\right\}$ or $\left\{-5<k<"\frac{4735}{768}"\right\}$

Do not allow e.g. $\{k:k>-5\}\cup\left\{k:k<"\frac{4735}{768}"\right\}$ or just $-5<k<"\frac{4735}{768}"$

SC: Candidates that have not scored the first M1 may go on to score the final two marks by finding an acceptable range between –5 and $"\frac{4735}{768}"$.

Use of their "–5" or "$-\frac{47}{3}$", coming from x = ±2, (or –5 or $-\frac{47}{3}$ directly from a calculator) with $"\frac{4735}{768}"$ will score M1 if the inside region is selected. A correct $\left\{k:-5<k<"\frac{4735}{768}"\right\}$ will score A1ft

There is no need to see any working to achieve the –5 (or $-\frac{47}{3}$).

All the relevant guidance regarding the region in the M1 and A1ft notes above continues to apply. e.g. for the M1 we will condone the use of y but not x in their region while for the A1ft we require use of k.

For example:

- $\left\{k:-5<k<"\frac{4735}{768}"\right\}$ without scoring the first M scores M0A0M1A1ft
- $"-5"<k<"\frac{4735}{768}"$ without scoring the first M scores M0A0M1A0ft
- $\left\{k:"-\frac{47}{3}"<k<"\frac{4735}{768}"\right\}$ without scoring the first M scores M0A0M1A0ft

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
10(a)	$8(4)-4^{\frac{5}{2}}=32-32=0$	B1	1.1b
		(1)	
(b)	$8-\frac{5}{2}x^{\frac{3}{2}}$	B1	1.1b
	$x=4\Rightarrow\left\{\frac{dy}{dx}=\right\}8-\frac{5}{2}\times 8=-12$ $\Rightarrow y\{-0\}=\text{"}-12\text{"}(x-4)$	M1	1.1b
	$12x+y=48$ *	A1*	1.1b
		(3)	
(c)	Attempts to find one of the coordinates of the point of intersection $y=8x,\ 12x+y=48\Rightarrow y=19.2$ (or $x=2.4$)	M1	1.1b
	Triangle area is $\frac{1}{2}\times 4\times\text{"19.2"}\left(=38.4 \text{ or } \frac{192}{5}\right)$ or $\int_0^{\text{"2.4"}} 8x\ dx+\int_{\text{"2.4"}}^{4}\text{"}(48-12x)\text{"}\ dx$	dM1	3.1a
	$\int\left(8x-x^{\frac{5}{2}}\right)dx=4x^2-\frac{2}{7}x^{\frac{7}{2}}$	B1	1.1b
	$A=38.4-\left[\text{"}4x^2-\frac{2}{7}x^{\frac{7}{2}}\text{"}\right]_0^4=38.4-64+\frac{256}{7}$	ddM1	3.1a
	$=\frac{384}{35}$	A1	1.1b
		(5)	
			(9 marks)

Notes

(a)

B1: Substitutes $x=4$ into the equation of the curve and verifies that $y=0$. Accept "$8(4)-4^{\frac{5}{2}}=0$"

Alternatively, sets $8x-x^{\frac{5}{2}}=0$ and solves with correct processing to achieve $x=4$.

As a minimum accept e.g. $8x-x^{\frac{5}{2}}=0\ \Rightarrow\ x^{\frac{3}{2}}=8\ \Rightarrow\ \{x=\}\ 4$ which may follow factorisation.

(b)

B1: Correct differentiation. The $\frac{dy}{dx}=$ need not be present.

M1: Correct method for finding the equation of the tangent at $A(4, 0)$.

Requires substitution of $x=4$ into their $\frac{dy}{dx}$ and an attempt at the equation of the line using this gradient. If using $y-y_1=m(x-x_1)$ then condone the omission of the – 0.

If $y=mx+c$ is used they must proceed as far as $c=\ldots$

Accept $\frac{dy}{dx}=-12$ or $m=-12$ without explicit substitution of $x=4$ provided $8-\frac{5}{2}x^{\frac{3}{2}}$ is seen.

A1*: Correct work leading to the given equation having scored B1M1.
Condone $y+12x=48$ and apply isw once seen.
Do not condone $12x+y-48=0$ (unless a correct equation = 48 is seen).

(c) **Note:** Condone poor notation such as missing dx or spurious $\int$ symbols throughout.

M1: Attempts to find either the x or y coordinate of the intersection of line l_1 and line l_2
You may need to check the diagram or limits to their integrals.

dM1: Correct method for the area of the triangle. e.g. Triangle area is $\frac{1}{2}\times 4\times "19.2"\left(=38.4 \text{ or } \frac{192}{5}\right)$

If integration is attempted then condone slips in their rearrangement of $12x+y=48$ to $y=48-12x$ and note that their integrals do need not to be evaluated, so for example

look for $\int_0^{"2.4"} 8x\ \mathrm{d}x+\int_{"2.4"}^{4} "(48-12x)"\ \mathrm{d}x \qquad \left\{=\frac{576}{25}+\frac{384}{25}=23.04+15.36\right\}$

B1: Correct integration of **curve** ignoring limits, i.e. $4x^2-\frac{2}{7}x^{\frac{7}{2}}$ but condone e.g. $\frac{8x^{1+1}}{2}-\frac{x^{\frac{5}{2}+1}}{\frac{7}{2}}$

ddM1: Fully correct strategy including substitution which would lead to an **exact** area.
Does not need to reach a value. Dependent on both previous M marks.
Implied by $38.4-\frac{192}{7}$ or a correct final answer $\frac{384}{35}$
Note that the decimal approximation that might be seen is 10.97142857 and implies ddM0 unless there is evidence of substitution (which need not be evaluated).

A1: Correct exact value. Either $\frac{384}{35}$ or $10\frac{34}{35}$

Alternative using lines – curve:

M1: Attempts to find either the x or y coordinate of the intersection of line l_1 and line l_2
You may need to check the diagram or limits to their integrals.

dM1: Correct method for at least one part (0 to "2.4" **or** "2.4" to 4) of the area of R including limits.
Condone slips in their rearrangement of $12x+y=48$ to $y=48-12x$ and note that their integrals do need not to be evaluated, so for example

look for $\int_0^{"2.4"} 8x-\left(8x-x^{\frac{5}{2}}\right)\mathrm{d}x$ or $\int_{"2.4"}^{4} "(48-12x)"-\left(8x-x^{\frac{5}{2}}\right)\ \mathrm{d}x$ (or a sum of both)

B1: Correct integration of **both** regions ignoring limits. May be completed as a sum or separately.

Condone e.g. $\frac{x^{\frac{5}{2}+1}}{\frac{7}{2}}$ in place of $\frac{2}{7}x^{\frac{7}{2}}$ Note that each integral may have been simplified.

$\int_{\dots}^{\dots} x^{\frac{5}{2}}\ \mathrm{d}x$ and {+} $\int_{\dots}^{\dots} 48-20x+x^{\frac{5}{2}}\ \mathrm{d}x \rightarrow \left[\frac{2}{7}x^{\frac{7}{2}}\right]_{\dots}^{\dots}$ and {+} $\left[48x-10x^2+\frac{2}{7}x^{\frac{7}{2}}\right]_{\dots}^{\dots}$

ddM1: Fully correct strategy including substitution which would lead to an **exact** area.
Does not need to reach a value. Dependent on both previous M marks.
This approach requires:

- substitution of 0, their 2.4 and 4 in the correct places
- the $\frac{2}{7}(2.4)^{\frac{7}{2}}-\frac{2}{7}(2.4)^{\frac{7}{2}}$ to be cancelled (may be implied by a correct final answer $\frac{384}{35}$)

Note that the decimal approximation that might be seen is 10.97142857 and implies ddM0 unless there is evidence that the $\frac{2}{7}(2.4)^{\frac{7}{2}}-\frac{2}{7}(2.4)^{\frac{7}{2}}$ has been cancelled e.g. ~~6.118...~~ – ~~6.118...~~

A1: Correct exact value. Either $\frac{384}{35}$ or $10\frac{34}{35}$

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
11 (a)	Substitutes $x=\frac{1}{2}$ into $y=2x^3+10$ **and** $y=42x-15x^2-7$ and finds the *y* values for both	M1	1.1b
	Achieves $\frac{41}{4}$ o.e. for both and makes a valid conclusion. *	A1*	2.4
		(2)	
(b)	Sets $42x-15x^2-7=2x^3+10 \Rightarrow 2x^3+15x^2-42x+17=0$	M1	1.1b
	Deduces that $(2x-1)$ is a factor and attempts to divide	dM1	2.1
	$2x^3+15x^2-42x+17=(2x-1)\left(x^2+8x-17\right)$	A1	1.1b
	Solves their $x^2+8x-17=0$ using suitable method	M1	1.1b
	Deduces $x=-4+\sqrt{33}$ (see note)	A1	2.2a
		(5)	
			(7 marks)

Notes:

(a)

M1: Substitutes $x=\frac{1}{2}$ into both $y=2x^3+10$ and $y=42x-15x^2-7$ and finds *y* values

Sight of just the *y* values at each is sufficient for this mark only.

Alternative: Sets $42x-15x^2-7=2x^3+10 \Rightarrow$ cubic and substitutes $x=\frac{1}{2}$ into the expression, attempts $\text{f}\left(\frac{1}{2}\right)$ or else attempts to divide the cubic = 0 by $(2x-1)$ or $\left(x-\frac{1}{2}\right)$. Condone $\text{f}\left(\frac{1}{2}\right)=0$ without calculations for this mark only.

A1*: Correct calculations must be seen with a minimal conclusion that curves intersect (at $x=\frac{1}{2}$).

E.g. $2\left(\frac{1}{2}\right)^3+10=10.25$ and $42\left(\frac{1}{2}\right)-15\left(\frac{1}{2}\right)^2-7=10.25$ so curves intersect.

Acceptable alternatives are:

$\text{f}(x)=42x-15x^2-7-2x^3-10, \text{f}\left(\frac{1}{2}\right)=42\left(\frac{1}{2}\right)-15\left(\frac{1}{2}\right)^2-7-2\left(\frac{1}{2}\right)^3-10=0 \Rightarrow$ so curves intersect

$\text{f}(x)=2x^3+15x^2-42x+17 \Rightarrow \left(x-\frac{1}{2}\right)\left(2x^2+16x-34\right)$ so $x=\frac{1}{2}$ is a root so curves intersect

$\text{f}(x)=2x^3+15x^2-42x+17 \Rightarrow (2x-1)\left(x^2+8x-17\right)$ so $(2x-1)$ is a factor hence curves intersect

Only accept verified, QED etc if there is a preamble mentioning intersection about how it will be shown.

Special case: Scores M1 A0 with or without a conclusion

This is presumably done using a calculator and requires all three roots exact or correct to 3sf

$\text{f}(x)=2x^3+15x^2-42x+17=0$

$\Rightarrow x=0.5, 1.74, -9.74$

Answers without working will not score any marks

A method must be seen which could be from part (a) which must then be continued in (b)

M1: Sets $42x-15x^2-7=2x^3+10$ and proceeds to 4 term cubic equation.

Condone slips, e.g. signs. Terms do not have to be on one side of the equation.

dM1: For the key step in attempting to "divide" the cubic by $(2x-1)$

If attempted via inspection look for correct first and last terms

E.g. $2x^3+15x^2-42x+17=(2x-1)\left(x^2+...\pm 17\right)$ if cubic expression is correct

If attempted via division look for correct first and second terms

$$2x-1\overline{)2x^3+15x^2-42x+17}\quad x^2+8x$$

if cubic expression is correct

It is acceptable for an attempt to divide by $\left(x-\frac{1}{2}\right)$. It is easily marked using the same guidelines, e.g. $2x^3+15x^2-42x+17=\left(x-\frac{1}{2}\right)\left(2x^2+16x...\right)$

A1: $2x^3+15x^2-42x+17=(2x-1)\left(x^2+8x-17\right)$ o.e. $\left(x-\frac{1}{2}\right)\left(2x^2+16x-34\right)$

This may be implied by sight of $\left(x^2+8x-17\right)$ or $\left(2x^2+16x-34\right)$ in a "division" sum.

M1: Solves their quadratic $x^2+8x-17=0$ using a suitable method including calculator. You may need to check this. It is not completely dependent upon the previous M's but an attempt at a full method must have been seen. So look for

- the two equations being set equal to each other and some attempt made to combine
- some attempt to "divide" the result by $(2x-1)$ o.e. allowing for flaws in the method

A1: Gives $x=-4+\sqrt{33}$ o.e. only. The $x=-4-\sqrt{33}$ must not be included in the final answer.

Allow exact unsimplified equivalents such as $x=\frac{-8+\sqrt{132}}{2}$. ISW for instance if they then put this in decimal form.

16(a)	Attempts $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{4\sec^2 t\tan t}{2\sec^2 t}\left(=2\tan t\right)$	M1 A1	1.1b 1.1b
	At $t=\frac{\pi}{4}$, $\frac{dy}{dx}=2, x=3, y=7$	M1	2.1
	Attempts equation of normal $y-7=-\frac{1}{2}(x-3)$	M1	1.1b
	$y=-\frac{1}{2}x+\frac{17}{2}$ *	A1*	2.1
		(5)	
(b)	Attempts to use $\sec^2 t=1+\tan^2 t\Rightarrow\frac{y-3}{2}=1+\left(\frac{x-1}{2}\right)^2$	M1	3.1a
	$\Rightarrow y-3=2+\frac{(x-1)^2}{2}\Rightarrow y=\frac{1}{2}(x-1)^2+5$ *	A1*	2.1
		(2)	
	(b) Alternative 1:		
	$y=\frac{1}{2}(x-1)^2+5=\frac{1}{2}(2\tan t+1-1)^2+5$ $=\frac{1}{2}4\tan^2 t+5=2\left(\sec^2 t-1\right)+5$	M1	3.1a
	$=2\sec^2 t+3=y*$	A1	2.1
	(b) Alternative 2:		
	$x=2\tan t+1\Rightarrow t=\tan^{-1}\left(\frac{x-1}{2}\right)\Rightarrow y=2\sec^2\left(\tan^{-1}\left(\frac{x-1}{2}\right)\right)+3$ $\Rightarrow y=2\left(1+\tan^2\left(\tan^{-1}\left(\frac{x-1}{2}\right)\right)\right)+3$	M1	3.1a
	$\Rightarrow y=2\left(1+\left(\frac{x-1}{2}\right)^2\right)+3=\frac{1}{2}(x-1)^2+5*$	A1	2.1
	(b) Alternative 3:		
	$\frac{dy}{dx}=2\tan t=x-1\Rightarrow y=\int(x-1)dx=\frac{x^2}{2}-x+c$ $(3,7)\rightarrow 7=\frac{3^2}{2}-3+c\Rightarrow c=\frac{11}{2}$	M1	3.1a
	$\frac{x^2}{2}-x+\frac{11}{2}=\frac{1}{2}\left(x^2-2x\right)+\frac{11}{2}=\frac{1}{2}(x-1)^2-\frac{1}{2}+\frac{11}{2}=\frac{1}{2}(x-1)^2+5*$	A1	2.1

(c)	**Attempts the lower limit for *k*:** $\frac{1}{2}(x-1)^2+5=-\frac{1}{2}x+k \Rightarrow x^2-x+(11-2k)=0$ $b^2-4ac=1-4(11-2k)=0 \Rightarrow k=...$	M1	2.1
	$(k=)\frac{43}{8}$	A1	1.1b
	Attempts the upper limit for *k*: $(x, y)_{t=-\frac{\pi}{4}}: t=-\frac{\pi}{4} \Rightarrow x=2\tan\left(-\frac{\pi}{4}\right)+1=-1,\ y=2\sec^2\left(-\frac{\pi}{4}\right)+3=7$ $(-1,\ 7), y=-\frac{1}{2}x+k \Rightarrow 7=\frac{1}{2}+k \Rightarrow k=...$	M1	2.1
	$(k=)\frac{13}{2}$	A1	1.1b
	$\frac{43}{8}<k\leqslant\frac{13}{2}$	A1	2.2a
		(5)	
			(12 marks)

Notes:

(a) **Must use parametric differentiation to score any marks in this part and not e.g. Cartesian form**

M1: For the key step of attempting $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}$. There must be some attempt to differentiate both parameters however poor and divide the right way round so using $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y}{x}$ scores M0.

This may be implied by e.g. $\frac{\mathrm{d}x}{\mathrm{d}t}=2\sec^2 t,\ \frac{\mathrm{d}y}{\mathrm{d}t}=4\sec^2 t\tan t,\ t=\frac{\pi}{4}\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}t}=4,\ \frac{\mathrm{d}y}{\mathrm{d}t}=8\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=2$

A1: $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{4\sec^2 t\tan t}{2\sec^2 t}$. Correct expression in any form. May be implied as above.

Condone the confusion with variables as long as the intention is clear e.g.

$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}=\frac{4\sec^2 x\tan x}{2\sec^2 x}\left(=2\tan x\right)$ and allow subsequent marks if this is interpreted correctly

M1: For attempting to find the values of *x*, *y* and the gradient at $t=\frac{\pi}{4}$ **AND** getting at least two correct.

Follow through on their $\frac{\mathrm{d}y}{\mathrm{d}x}$ so allow for any two of $x=3,\ y=7,\ \frac{\mathrm{d}y}{\mathrm{d}x}=2\left(\text{or their}\frac{\mathrm{d}y}{\mathrm{d}x}\text{at}\ t=\frac{\pi}{4}\right)$

Note that the $x=3,\ y=7$ may be seen as e.g. (3, 7) on the diagram. There must be a non-trivial $\frac{\mathrm{d}y}{\mathrm{d}x}$ for this mark e.g. they must have a $\frac{\mathrm{d}y}{\mathrm{d}x}$ to substitute into.

M1: For a correct attempt at the normal equation using their *x* and *y* at $t=\frac{\pi}{4}$ with the negative reciprocal of their $\frac{\mathrm{d}y}{\mathrm{d}x}\text{at}\ t=\frac{\pi}{4}$ having made some attempt at $\frac{\mathrm{d}y}{\mathrm{d}x}$ and all correctly placed.

For attempts using $y = mx + c$ they must reach as far as a value for *c* using their *x* and *y* at $t=\frac{\pi}{4}$ with the negative reciprocal of their $\frac{\mathrm{d}y}{\mathrm{d}x}\text{at}\ t=\frac{\pi}{4}$ all correctly placed.

A1*: Proceeds with a clear argument to the given answer with no errors.

M1: Attempts to use $\sec^2 t = 1 + \tan^2 t$ oe to obtain an equation involving y and $(x-1)^2$

E.g. as above or e.g. $y = 2\sec^2 t + 3 = 2\left(1+\tan^2 t\right)+3 = 2\left(1+\left(\frac{x-1}{2}\right)^2\right)+3$ for M1 and then

$y = \frac{1}{2}(x-1)^2 + 5^*$ for A1

A1*: Proceeds with a clear argument to the given answer with no errors

Alternative 1:

M1: Uses the given result, substitutes for x and attempts to use $\sec^2 t = 1 + \tan^2 t$ oe

A1: Proceeds with a clear argument to the y parameter and makes a (minimal) conclusion e.g. " = y" QED, hence proven etc.

Alternative 2:

M1: Uses the x parameter to obtain t in terms of arctan, substitutes into y and attempts to use $\sec^2 t = 1 + \tan^2 t$ oe

A1: Proceeds with a clear argument to the given answer with no errors

Alternative 3:

M1: Uses $\frac{dy}{dx}$ from part (a) to express $\frac{dy}{dx}$ in terms of x, integrates and uses (3, 7) to find "c" to reach a Cartesian equation.

A1: Proceeds with a clear argument to the given answer with no errors

Allow the marks for (b) to score anywhere in their solution e.g. if they find the Cartesian equation in part (a)

(c)

M1: A full attempt to find the **lower** limit for k.

$\frac{1}{2}(x-1)^2 + 5 = -\frac{1}{2}x + k \Rightarrow x^2 - x + (11-2k) = 0 \Rightarrow b^2 - 4ac = 1 - 4(11-2k) = 0 \Rightarrow k = ...$

Score **M1** for setting $\frac{1}{2}(x-1)^2 + 5 = -\frac{1}{2}x + k,$ rearranging to 3TQ form and attempts $b^2 - 4ac ... 0$ e.g. $b^2 - 4ac > 0$ or e.g. $b^2 - 4ac < 0$ correctly to find a value for k.

A1: $k = \frac{43}{8}$ oe. Look for this **value** e.g. may appear in an inequality e.g. $k > \frac{43}{8}$, $k < \frac{43}{8}$

An alternative method using calculus for lower limit:

$$y = \frac{1}{2}(x-1)^2 + 5 \Rightarrow \frac{dy}{dx} = x - 1,\ x - 1 = -\frac{1}{2} \Rightarrow x = \frac{1}{2}$$

$$x = \frac{1}{2} \Rightarrow y = \frac{1}{2}\left(\frac{1}{2} - 1\right)^2 + 5 = \frac{41}{8}$$

$$y = -\frac{1}{2}x + k \Rightarrow \frac{41}{8} = -\frac{1}{4} + k \Rightarrow k = ...$$

Score **M1** for $\frac{dy}{dx}$ = "a linear expression in x", sets = $-\frac{1}{2},$ solves a linear equation to find x and then substitutes into the given result in (b) to find y and then uses $y = -\frac{1}{2}x + k$ to find a value for k. **A1**: $k = \frac{43}{8}$ oe. Look for this **value** e.g. may appear in an inequality e.g. $k > \frac{43}{8}$, $k < \frac{43}{8}$

An alternative method using parameters for lower limit:

$$y = -\frac{1}{2}x + k \Rightarrow 2\sec^2 t + 3 = -\frac{1}{2}(2\tan t + 1) + k$$

$$\Rightarrow 2\left(1 + \tan^2 t\right) + 3 = -\frac{1}{2}(2\tan t + 1) + k \Rightarrow 2\tan^2 t + \tan t + 5.5 - k = 0$$

$$b^2 - 4ac = 0 \Rightarrow 1 - 4 \times 2(5.5 - k) = 0 \Rightarrow k = \frac{43}{8}$$

Score **M1** for substituting parametric form of *x* and *y* into $y = -\frac{1}{2}x + k$, uses $\sec^2 t = 1 + \tan^2 t$ rearranges to 3TQ form and attempts $b^2 - 4ac ... 0$ or e.g. $b^2 - 4ac > 0$ or $b^2 - 4ac < 0$ correctly to find a value for *k*.

A1: $k = \frac{43}{8}$ oe. Look for this **value** e.g. may appear in an inequality e.g. $k > \frac{43}{8}$, $k < \frac{43}{8}$

M1: A full attempt to find the **upper** limit for *k*. This requires an attempt to find the value of *x* and the value of *y* using $t = -\frac{\pi}{4}$, the substitution of these values into $y = -\frac{1}{2}x + k$ and solves for *k*.

A1: $k = \frac{13}{2}$. Look for this value e.g. may appear in an inequality.

A1: Deduces the correct range for *k*: $\frac{43}{8} < k \leqslant \frac{13}{2}$

Allow equivalent notation e.g. $\left(k \leqslant \frac{13}{2} \text{ and } k > \frac{43}{8}\right)$, $\left(k \leqslant \frac{13}{2} \cap k > \frac{43}{8}\right)$, $\left(\frac{43}{8}, \frac{13}{2}\right]$

But not e.g. $\left(k \leqslant \frac{13}{2},\ k > \frac{43}{8}\right)$, $\left(k \leqslant \frac{13}{2} \cup k > \frac{43}{8}\right)$, $\left(k \leqslant \frac{13}{2} \text{ or } k > \frac{43}{8}\right)$ and do not allow if in terms of *x*.

Allow equivalent exact values for $\frac{43}{8}$, $\frac{13}{2}$

There may be other methods for finding the upper limit which are valid. If you are in any doubt if a method deserves credit then use Review.

7(a)	$y = x^3 - 10x^2 + 27x - 23 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 20x + 27$	B1	1.1b
	$\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)_{x=5} = 3\times 5^2 - 20\times 5 + 27 \left(= 2\right)$	M1	1.1b
	$y + 13 = 2\left(x - 5\right)$	M1	2.1
	$y = 2x - 23$	A1	1.1b
		(4)	
(b)	Both C and l pass through $(0, -23)$ and so C meets l again on the y-axis	B1	2.2a
		(1)	
(c)	$\pm\int \left(x^3 - 10x^2 + 27x - 23 - \left(2x - 23\right)\right) \mathrm{d}x$ $= \pm\left(\dfrac{x^4}{4} - \dfrac{10}{3}x^3 + \dfrac{25}{2}x^2\right)$	M1 A1ft	1.1b 1.1b
	$\left[\dfrac{x^4}{4} - \dfrac{10}{3}x^3 + \dfrac{25}{2}x^2\right]_0^5$ $= \left(\dfrac{625}{4} - \dfrac{1250}{3} + \dfrac{625}{2}\right)(-0)$	dM1	2.1
	$= \dfrac{625}{12}$	A1	1.1b
		(4)	
	(c) Alternative:		
	$\pm\int \left(x^3 - 10x^2 + 27x - 23\right) \mathrm{d}x$ $= \pm\left(\dfrac{x^4}{4} - \dfrac{10}{3}x^3 + \dfrac{27}{2}x^2 - 23x\right)$	M1 A1	1.1b 1.1b
	$\left[\dfrac{x^4}{4} - \dfrac{10}{3}x^3 + \dfrac{27}{2}x^2 - 23x\right]_0^5 + \dfrac{1}{2}\times 5\left(23 + 13\right)$ $= -\dfrac{455}{12} + 90$	dM1	2.1
	$= \dfrac{625}{12}$	A1	1.1b
			(9 marks)

Notes

(a)

B1: Correct derivative

M1: Substitutes $x = 5$ into their derivative. This may be implied by their value for $\frac{\mathrm{d}y}{\mathrm{d}x}$

M1: Fully correct straight line method using $(5, -13)$ and their $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 5$

A1: cao. Must see the full equation in the required form.

(b)

B1: Makes a suitable deduction.

Alternative via equating l and C and factorising e.g.

$$x^3 - 10x^2 + 27x - 23 = 2x - 23$$

$$x^3 - 10x^2 + 25x = 0$$

$$x\left(x^2 - 10x + 25\right) = 0 \Rightarrow x = 0$$

So they meet on the y-axis

(c)

M1: For an attempt to integrate $x^n \to x^{n+1}$ for $\pm$" $C - l$"

A1ft: Correct integration in any form which may be simplified or unsimplified. (follow through their equation from (a))

If they attempt as 2 separate integrals e.g. $\int \left(x^3 - 10x^2 + 27x - 23\right) \mathrm{d}x - \int \left(2x - 23\right) \mathrm{d}x$ then award this mark for the correct integration of the curve as in the alternative.

If they combine the curve with the line first then the subsequent integration must be correct or a correct ft for their line and allow for $\pm$" $C - l$"

dM1: Fully correct strategy for the area. Award for use of 5 as the limit and condone the omission of the "– 0". **Depends on the first method mark.**

A1: Correct exact value

Alternative:

M1: For an attempt to integrate $x^n \to x^{n+1}$ for $\pm C$

A1: Correct integration for $\pm C$

dM1: Fully correct strategy for the area e.g. correctly attempts the area of the trapezium and subtracts the area enclosed between the curve and the x-axis. Need to see the use of 5 as the limit condoning the omission of the "– 0" **and** a correct attempt at the trapezium **and** the subtraction.

May see the trapezium area attempted as $\int \left(2x - 23\right) \mathrm{d}x$ in which case the integration and use of the limits needs to be correct or correct follow through for their straight line equation.

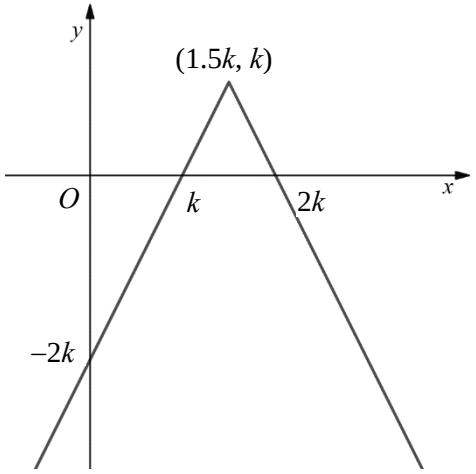
Depends on the first method mark.

A1: Correct exact value

Note if they do $l - C$ rather than $C - l$ and the working is otherwise correct allow full marks if their final answer is given as a positive value. E.g. correct work with $l - C$ leading to $-\frac{625}{12}$ and then e.g. hence area is $\frac{625}{12}$ is acceptable for full marks.

If the answer is left as $-\frac{625}{12}$ then score A0

Question	Scheme	Marks	AOs

11(a)			
	∧ shape in any position	B1	1.1b
	Correct x-intercepts or coordinates	B1	1.1b
	Correct y-intercept or coordinates	B1	1.1b
	Correct coordinates for the vertex of a ∧ shape	B1	1.1b
		(4)	
(b)	$x = k$	B1	2.2a
	$k-\left(2x-3k\right)=x-k \Rightarrow x=...$	M1	3.1a
	$x=\frac{5k}{3}$	A1	1.1b
	Set notation is required here for this mark $\left\{x : x<\frac{5k}{3}\right\}\cap\left\{x : x>k\right\}$	A1	2.5
		(4)	
(c)	$x = 3k$ **or** $y = 3 - 5k$	B1ft	2.2a
	$x = 3k$ **and** $y = 3 - 5k$	B1ft	2.2a
		(2)	
			(10 marks)

Notes

(a) **Note that the sketch may be seen on Figure 4**
B1: See scheme
B1: Correct x-intercepts. Allow as shown or written as $(k, 0)$ and $(2k, 0)$ and condone coordinates written as $(0, k)$ and $(0, 2k)$ as long as they are in the correct places.
B1: Correct y-intercept. Allow as shown or written as $(0, -2k)$ or $(-2k, 0)$ as long as it is in the correct place. Condone $k - 3k$ for $-2k$.
B1: Correct coordinates as shown
Note that the marks for the intercepts and the maximum can be seen away from the sketch but the coordinates must be the right way round or e.g. as $y = 0$, $x = k$ etc. These marks can be awarded without a sketch but if there is a sketch, such points must not contradict the sketch.
(b)
B1: Deduces the correct critical value of $x = k$. May be implied by e.g. $x > k$ or $x < k$
M1: Attempts to solve $k - (2x - 3k) = x - k$ or an equivalent equation/inequality to find the other critical value. Allow this mark for reaching $k = \ldots$ or $x = \ldots$ as long as they are solving the required equation.
A1: Correct value
A1: Correct answer using the correct set notation

Allow e.g. $\left\{x : x \in \mathbb{R}, k < x < \frac{5k}{3}\right\}$, $\left\{x : k < x < \frac{5k}{3}\right\}$, $x \in \left(k, \frac{5k}{3}\right)$ and allow $<$ for $\leqslant$

But $\left\{x : x < \frac{5k}{3}\right\} \cup \{x : x > k\}$ scores A0 $\quad$ $\left\{x : k < x,\ x < \frac{5k}{3}\right\}$ scores A0

(c)

B1ft: Deduces one correct coordinate. Follow through their maximum coordinates from (a) so allow $x = 2\times$"$1.5k$" or $y = 3 - 5\times$"k" but must be in terms of k.

Allow as coordinates or $x = .., y = \ldots$

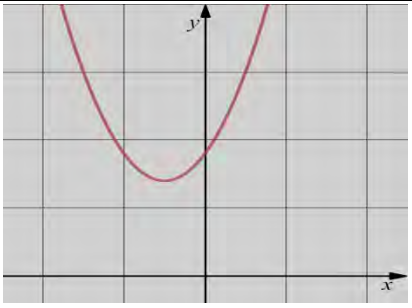
B1ft: Deduces both correct coordinates. Follow through their maximum coordinates from (a) so allow $x = 2\times$"$1.5k$" and $y = 3 - 5\times$"k" but must be in terms of k.

Allow as coordinates or $x = .., y = \ldots$

If coordinates are given the wrong way round and not seen correctly as $x = \ldots, y = \ldots$

e.g. $(3 - 5k, 3k)$ this is B0B0

Question	Scheme	Marks	AOs

5 (a)	$2x^2+4x+9=2(x\pm k)^2\pm....$ $\quad a=2$	B1	1.1b
	Full method $2x^2+4x+9=2(x+1)^2\pm...$ $\quad a=2\ \&\ b=1$	M1	1.1b
	$2x^2+4x+9=2(x+1)^2+7$	A1	1.1b
		(3)	
(b)	 U shaped curve any position but not through (0,0)	B1	1.2
	y - intercept at $(0,9)$	B1	1.1b
	Minimum at $(-1,7)$	B1ft	2.2a
		(3)	
(c)	(i) Deduces translation with one correct aspect.	M1	3.1a
	Translate $\begin{pmatrix}2\\-4\end{pmatrix}$	A1	2.2a
	(ii) $h(x)=\dfrac{21}{"2(x+1)^2+7"}\Rightarrow$ (maximum) value $\dfrac{21}{"7"}(=3)$	M1	3.1a
	$0<h(x)\leqslant 3$	A1ft	1.1b
		(4)	
			(10 marks)

(a)

B1: Achieves $2x^2+4x+9=2(x\pm k)^2\pm....$ or states that $a=2$

M1: Deals correctly with first two terms of $2x^2+4x+9$.

Scored for $2x^2+4x+9=2(x+1)^2\pm...$ or stating that $a=2$ and $b=1$

A1: $2x^2+4x+9=2(x+1)^2+7$

Note that this may be done in a variety of ways including equating $2x^2+4x+9$ with the expanded form of $a(x+b)^2+c\equiv ax^2+2abx+ab^2+c$

B1: For a U-shaped curve in any position not passing through $(0,0)$. Be tolerant of slips of the pen but do not allow if the curve bends back on itself

B1: A curve with a y - intercept on the +ve y axis of 9. The curve cannot just stop at $(0,9)$

Allow the intercept to be marked 9, $(0,9)$ but not $(9,0)$

B1ft: For a minimum at $(-1,7)$ in quadrant 2. This may be implied by -1 and 7 marked on the axes in the correct places. The curve must be a U shape and not a cubic say.

Follow through on a minimum at $(-b,c)$, marked in the correct quadrant, for their $a(x+b)^2+c$

(c)(i)

M1: Deduces translation with one correct aspect or states $\begin{pmatrix}2\\-4\end{pmatrix}$ with no reference to 'translate'.

Allow instead of the word translate, shift or move. $g(x)=f(x-2)-4$ can score M1

For example, possible methods of arriving at this deduction are:

- $f(x)\to g(x)$ is $2x^2+4x+9\to 2(x-2)^2+4(x-2)+5$ So $g(x)=f(x-2)-4$
- $g(x)=2(x-1)^2+3$ New curve has its minimum at $(1,3)$ so $(-1,7)\to(1,3)$
- Using a graphical calculator to sketch $y=g(x)$ and compares to the sketch of $y=f(x)$

In almost all cases you will not allow if the candidate gives two **different types of** transformations. Eg, stretch and .....

A1: Requires both 'translate' and '$\begin{pmatrix}2\\-4\end{pmatrix}$' Allow 'shift' or move' instead of translate.

So condone " Move shift 2 (units) to the right and move 4 (units) down

However, for M1 A1, it is possible to reflect in $x=0$ and translate $\begin{pmatrix}0\\-4\end{pmatrix}$, so please consider all responses.

SC: If the candidate writes translate $\begin{pmatrix}-2\\4\end{pmatrix}$ or " move 2 (units) to the left and 4 (units) up" score M1 A0

(c)(ii)

M1: Correct attempt at finding the maximum value (although it may not be stated as a maximum)

- Uses part (a) to write $h(x)=\dfrac{21}{"2(x+1)^2+7"}$ and attempts to find $\dfrac{21}{\text{their "7"}}$
- Attempts to differentiate, sets $4x+4=0\to x=-1$ and substitutes into $h(x)=\dfrac{21}{2x^2+4x+9}$
- Uses a graphical calculator to sketch $y=h(x)$ and establishes the 'maximum' value $(...,3)$

A1ft: $0<h(x)\leqslant 3$ Allow for $0<h\leqslant 3$ $(0,3]$ and $0<y\leqslant 3$ but not $0<x\leqslant 3$

Follow through on their $a(x+b)^2+c$ so award for $0<h(x)\leqslant\dfrac{21}{c}$

Question	Scheme	Marks	AOs

Question	Scheme		Marks	AOs
4	$C_1: x=10\cos t,\ y=4\sqrt{2}\sin t,\ \ 0\le t<2\pi;\ \ C_2: x^2+y^2=66$			
Way 1	$(10\cos t)^2+(4\sqrt{2}\sin t)^2=66$		M1	3.1a
	$100(1-\sin^2 t)+32\sin^2 t=66$	$100\cos^2 t+32(1-\cos^2 t)=66$	M1	2.1
			A1	1.1b
	$100-68\sin^2 t=66 \Rightarrow \sin^2 t=\frac{1}{2}$ $\Rightarrow \sin t=...$	$68\cos^2 t+32=66 \Rightarrow \cos^2 t=\frac{1}{2}$ $\Rightarrow \cos t=...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the *x*-coordinate and value of the corresponding *y*-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S=(5\sqrt{2},-4)$ or $x=5\sqrt{2},\ y=-4$ or $S=$(awrt 7.07, −4)		A1	3.2a
			(6)	
Way 2	$\{\cos^2 t+\sin^2 t=1\Rightarrow\}\ \ \left(\frac{x}{10}\right)^2+\left(\frac{y}{4\sqrt{2}}\right)^2=1\ \ \{\Rightarrow 32x^2+100y^2=3200\}$		M1	3.1a
	$\frac{x^2}{100}+\frac{66-x^2}{32}=1$	$\frac{66-y^2}{100}+\frac{y^2}{32}=1$	M1	2.1
			A1	1.1b
	$32x^2+6600-100x^2=3200$ $x^2=50 \Rightarrow x=...$	$2112-32y^2+100y^2=3200$ $y^2=16 \Rightarrow y=...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the corresponding *x*-coordinate or *y*-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S=(5\sqrt{2},-4)$ or $x=5\sqrt{2},\ y=-4$ or $S=$(awrt 7.07, −4)		A1	3.2a
			(6)	
Way 3	$\{C_2: x^2+y^2=66\Rightarrow\}\ \ x=\sqrt{66}\cos\alpha,\ y=\sqrt{66}\sin\alpha$ $\{C_1=C_2\ \Rightarrow\}\ \ 10\cos t=\sqrt{66}\cos\alpha,\ \ 4\sqrt{2}\sin t=\sqrt{66}\sin\alpha$ $\{\cos^2\alpha+\sin^2\alpha=1\Rightarrow\}\ \ \left(\frac{10\cos t}{\sqrt{66}}\right)^2+\left(\frac{4\sqrt{2}\sin t}{\sqrt{66}}\right)^2=1$		M1	3.1a
	then continue with applying the mark scheme for Way 1			
Way 4	$(10\cos t)^2+(4\sqrt{2}\sin t)^2=66$		M1	3.1a
	$100\left(\frac{1+\cos 2t}{2}\right)+32\left(\frac{1-\cos 2t}{2}\right)=66$		M1	2.1
			A1	1.1b
	$50+50\cos 2t+16-16\cos 2t=66 \Rightarrow 34\cos 2t+66=66$ $\Rightarrow \cos 2t=...$		dM1	1.1b
	Substitutes their solution back into the original equation(s) to get the value of the *x*-coordinate and value of the *y*-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S=(5\sqrt{2},-4)$ or $x=5\sqrt{2},\ y=-4$ or $S=$(awrt 7.07, −4)		A1	3.2a
			(6)	
	Note: Give final A0 for writing $x=5\sqrt{2},\ y=-4$ followed by $S=(-4,5\sqrt{2})$			
				(6 marks)
Notes for Question 4				

	Way 1
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 1: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.
M1:	Uses the identity $\sin^2 t + \cos^2 t \equiv 1$ to achieve an equation in $\sin^2 t$ only or $\cos^2 t$ only
A1:	A correct equation in $\sin^2 t$ only or $\cos^2 t$ only
dM1:	**dependent on both the previous M marks** Rearranges to make $\sin t = ...$ where $-1 \leq \sin t \leq 1$ or $\cos t = ...$ where $-1 \leq \cos t \leq 1$
Note:	Condone 3rd M1 for $\sin^2 t = \frac{1}{2} \Rightarrow \sin t = \frac{1}{4}$
M1:	See scheme
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$
	Way 2
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 2: A complete process of using $\cos^2 t + \sin^2 t \equiv 1$ to convert the parametric equation for C_1 into a Cartesian equation for C_1
M1:	Complete valid attempt to write an equation in terms of x only or y only not involving trigonometry
A1:	A correct equation in x only or y only not involving trigonometry
dM1:	**dependent on both the previous M marks** Rearranges to make $x = ...$ or $y = ...$
Note:	their x^2 or their y^2 must be >0 for this mark
M1:	See scheme
Note:	their x^2 and their y^2 must be >0 for this mark
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$
	Way 3
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 3: A complete process of writing C_2 in parametric form, combining the parametric equations of C_1 and C_2 and applying $\cos^2 \alpha + \sin^2 \alpha \equiv 1$ to give an equation in one variable (i.e. t) only.
	then continue with applying the mark scheme for Way 1
	Way 4
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 4: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.
M1:	Uses the identities $\cos 2t \equiv 2\cos^2 t - 1$ and $\cos 2t \equiv 1 - 2\sin^2 t$ to achieve an equation in $\cos 2t$ only
Note:	At least one of $\cos 2t \equiv 2\cos^2 t - 1$ or $\cos 2t \equiv 1 - 2\sin^2 t$ must be correct for this mark.
A1:	A correct equation in $\cos 2t$ only
dM1:	**dependent on both the previous M marks** Rearranges to make $\cos 2t = ...$ where $-1 \leq \cos 2t \leq 1$
M1:	See scheme
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$

Question	Scheme	Marks	AOs

4	$C_1: x=10\cos t,\ y=4\sqrt{2}\sin t,\ \ 0\le t<2\pi;\ \ C_2: x^2+y^2=66$		
Way 5	$(10\cos t)^2+(4\sqrt{2}\sin t)^2=66$	M1	3.1a
	$(10\cos t)^2+(4\sqrt{2}\sin t)^2=66(\sin^2 t+\cos^2 t)$	M1	2.1
		A1	1.1b
	$100\cos^2 t+32\sin^2 t=66\sin^2 t+66\cos^2 t \Rightarrow 34\cos^2 t=34\sin^2 t$ $\Rightarrow \tan t=...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the *x*-coordinate and value of the corresponding *y*-coordinate. **Note:** These may not be in the correct quadrant	M1	1.1b
	$S=(5\sqrt{2},-4)$ or $x=5\sqrt{2},\ y=-4$ or $S=$(awrt 7.07, -4)	A1	3.2a
		(6)	

	Way 5
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 5: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. *t*) only.
M1:	Uses the identity $\sin^2 t+\cos^2 t\equiv 1$ to achieve an equation in $\sin^2 t$ only and $\cos^2 t$ only **with no constant term**
A1:	A correct equation in $\sin^2 t$ and $\cos^2 t$ containing no constant term
dM1:	**dependent on both the previous M marks** Rearranges to make $\tan t=...$
M1:	See scheme
A1:	Selects the correct coordinates for *S* Allow either $S=(5\sqrt{2},-4)$ or $S=$(awrt 7.07, -4) or $S=(\sqrt{50},-4)$ or $S=\left(\frac{10}{\sqrt{2}},-4\right)$

Question	Scheme	Marks	AOs
	$\left\{\sum_{r=1}^{9}\right\}\int_{1}^{9}$		

Question	Scheme	Marks	AOs
6 (a)	Deduces that gradient of PA is $-\frac{1}{2}$	M1	2.2a
	Finding the equation of a line with gradient "$-\frac{1}{2}$" and point $(7,5)$ $y-5=-\frac{1}{2}(x-7)$	M1	1.1b
	Completes proof $2y+x=17$ *	A1*	1.1b
		(3)	
(b)	Solves $2y+x=17$ and $y=2x+1$ simultaneously	M1	2.1
	$P=(3,7)$	A1	1.1b
	Length $PA=\sqrt{(3-7)^2+(7-5)^2}=(\sqrt{20})$	M1	1.1b
	Equation of C is $(x-7)^2+(y-5)^2=20$	A1	1.1b
		(4)	
(c)	Attempts to find where $y=2x+k$ meets C using $\overrightarrow{OA}+\overrightarrow{PA}$	M1	3.1a
	Substitutes their $(11,3)$ in $y=2x+k$ to find k	M1	2.1
	$k=-19$	A1	1.1b
		(3)	
			(10 marks)
(c)	Attempts to find where $y=2x+k$ meets C via simultaneous equations proceeding to a 3TQ in x (or y) FYI $5x^2+(4k-34)x+k^2-10k+54=0$	M1	3.1a
	Uses $b^2-4ac=0$ oe and proceeds to $k=...$	M1	2.1
	$k=-19$	A1	1.1b
		(3)	

Notes:

(a)

M1: Uses the idea of perpendicular gradients to deduce that gradient of PA is $-\frac{1}{2}$. Condone $-\frac{1}{2}x$ if followed by correct work. You may well see the perpendicular line set up as $y=-\frac{1}{2}x+c$ which scored this mark

M1: Award for the method of finding the equation of a line with a changed gradient and the point $(7,5)$

So sight of $y-5=\frac{1}{2}(x-7)$ would score this mark

If the form $y=mx+c$ is used expect the candidates to proceed as far as $c=...$ to score this mark.

(b)

M1: Awarded for an attempt at the key step of finding the coordinates of point *P*. ie for an attempt at solving $2y+x=17$ and $y=2x+1$ simultaneously. Allow any methods (including use of a calculator) but it must be a valid attempt to find both coordinates. Do not allow where they start $17-x=2x+1$ as they have set $2y=y$ but condone bracketing errors, eg $2\times 2x+1+x=17$

A1: $P=(3,7)$

M1: Uses Pythagoras' Theorem to find the radius or radius ^{2} using their $P=(3,7)$ and $(7,5)$. There must be an attempt to find the difference between the coordinates in the use of Pythagoras

A1: $(x-7)^2+(y-5)^2=20$. Do not accept $(x-7)^2+(y-5)^2=\left(\sqrt{20}\right)^2$

(c)

M1: Attempts to find where $y=2x+k$ meets *C*.

Awarded for using $\overrightarrow{OA}+\overrightarrow{PA}$. $(11,3)$ or one correct coordinate of $(11,3)$ is evidence of this award.

M1: For a full method leading to k. Scored for either substituting their $(11,3)$ in $y=2x+k$

or**, in the alternative**, for solving their $(4k-34)^2-4\times 5\times\left(k^2-10k+54\right)=0\Rightarrow k=...$ Allow use of a calculator here to find roots. Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=-19$ only

.....

Alternative I

M1: For solving $y=2x+k$ with their $(x-7)^2+(y-5)^2=20$ and creating a quadratic eqn of the form $ax^2+bx+c=0$ **where both *b* and *c* are dependent upon *k***. The terms in x^2 and *x* must be collected together or implied to have been collected by their correct use in "b^2-4ac"

FYI the correct quadratic is $5x^2+(4k-34)x+k^2-10k+54=0$

M1: For using the discriminant condition $b^2-4ac=0$ to find k. It is not dependent upon the previous M and may be awarded from only one term in *k*.

$(4k-34)^2-4\times 5\times\left(k^2-10k+54\right)=0\Rightarrow k=...$ Allow use of a calculator here to find roots.

Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=-19$ only

.....

Alternative II

M1: For solving $2y+x=17$ with their $(x-7)^2+(y-5)^2=20$, creating a 3TQ and solving.

M1: For substituting their $(11,3)$ into $y=2x+k$ and finding *k*

A1: $k=-19$ only

……

Other method are possible using trigonometry.

Question	Scheme	Marks	AOs

8 (a) **Way 1**	$H = Ax(40-x)$ {or $H = Ax(x-40)$}	M1	3.3
	$x=20,\ H=12 \Rightarrow 12 = A(20)(40-20) \Rightarrow A = \frac{3}{100}$	dM1	3.1b
	$H = \frac{3}{100}x(40-x)$ or $H = -\frac{3}{100}x(x-40)$	A1	1.1b
		(3)	
(a) **Way 2**	$H = 12-\lambda(x-20)^2$ {or $H = 12+\lambda(x-20)^2$}	M1	3.3
	$x=40,\ H=0 \Rightarrow 0 = 12-\lambda(40-20)^2 \Rightarrow \lambda = \frac{3}{100}$	dM1	3.1b
	$H = 12-\frac{3}{100}(x-20)^2$	A1	1.1b
		(3)	
(a) **Way 3**	$H = ax^2+bx+c$ (or deduces $H = ax^2+bx$) **Both** $x=0,\ H=0 \Rightarrow 0=0+0+c \Rightarrow c=0$ **and either** $x=40,\ H=0 \Rightarrow 0 = 1600a+40b$ **or** $x=20,\ H=12 \Rightarrow 12 = 400a+20b$ **or** $\frac{-b}{2a} = 20$ $\{\Rightarrow b=-40a\}$	M1	3.3
	$b=-40a \Rightarrow 12 = 400a+20(-40a) \Rightarrow a=-0.03$ so $b=-40(-0.03)=1.2$	dM1	3.1b
	$H = -0.03x^2+1.2x$	A1	1.1b
		(3)	
(b)	{$H=3 \Rightarrow$} $3 = \frac{3}{100}x(40-x) \Rightarrow x^2-40x+100=0$ **or** {$H=3 \Rightarrow$} $3 = 12-\frac{3}{100}(x-20)^2 \Rightarrow (x-20)^2 = 300$	M1	3.4
	e.g. $x = \frac{40 \pm \sqrt{1600-4(1)(100)}}{2(1)}$ or $x = 20 \pm \sqrt{300}$	dM1	1.1b
	{chooses $20+\sqrt{300} \Rightarrow$} greatest distance = awrt 37.3 m	A1	3.2a
		(3)	
(c)	Gives a limitation of the model. Accept e.g. • the ground is horizontal • the ball needs to be kicked from the ground • the ball is modelled as a particle • the horizontal bar needs to be modelled as a line • there is no wind or air resistance on the ball • there is no spin on the ball • no obstacles in the trajectory (or path) of the ball • the trajectory of the ball is a perfect parabola	B1	3.5b
		(1)	
			(7 marks)

Notes for Question 8	
(a)	
M1:	Translates the situation given into a suitable equation for the model. E.g. **Way 1:** {Uses $(0, 0)$ and $(40, 0)$ to write} $H = Ax(40-x)$ o.e. {or $H = Ax(x-40)$ }
	Way 2: {Uses $(20, 12)$ to write} $H = 12-\lambda(x-20)^2$ or $H = 12+\lambda(x-20)^2$
	Way 3: Writes $H = ax^2+bx+c$, **and** uses $(0, 0)$ to deduce $c = 0$ **and** an attempt at using either $(40, 0)$ or $(20, 12)$ **Special Case:** Allow SC M1dM0A0 for not deducing $c = 0$ but attempting to apply both $(40, 0)$ and $(20, 12)$
dM1:	Applies a complete strategy with appropriate constraints to find all constants in their model. **Way 1:** Uses $(20, 12)$ on their model and finds $A = ...$ **Way 2:** Uses either $(40, 0)$ or $(0, 0)$ on their model to find $\lambda = ...$ **Way 3:** Uses $(40, 0)$ and $(20, 12)$ on their model to find $a = ...$ and $b = ...$
A1:	Finds a correct equation linking H to x E.g. $H = \frac{3}{100}x(40-x)$, $H = 12-\frac{3}{100}(x-20)^2$ or $H = -0.03x^2+1.2x$
Note:	Condone writing y in place of H for the M1 and dM1 marks.
Note:	Give final A0 for $y = -0.03x^2+1.2x$
Note:	Give special case M1dM0A0 for writing down any of $H = 12-(x-20)^2$ or $H = x(40-x)$ or $H = x(x-40)$
Note:	Give M1 dM1 for finding $-0.03x^2+1.2x$ or $a = -0.03, b = 1.2, c = 0$ in an implied ax^2+bx or ax^2+bx+c (with no indication of $H = ...$)
(b)	
M1:	Substitutes $H = 3$ into their quadratic equation and proceeds to obtain a 3TQ or a quadratic in the form $(x \pm \alpha)^2 = \beta$; $\alpha, \beta \neq 0$
Note:	E.g. $1.2x-0.03x^2 = 3$ or $40x-x^2 = 100$ are acceptable for the 1st M mark
Note:	Give M0 dM0 A0 for (their A)$x^2 = 3 \Rightarrow x = ...$ or their (their A)x^2 + (their k) $= 3 \Rightarrow x = ...$
dM1:	Correct method of solving their quadratic equation to give ***at least one solution***
A1:	Interprets their solution in the original context by selecting the larger correct value ***and states correct units for their value***. E.g. Accept awrt 37.3 m or $\left(20+\sqrt{300}\right)$ m or $\left(20+10\sqrt{3}\right)$ m
Note:	Condone the use of inequalities for the method marks in part (b)
(c):	
B1:	See scheme
Note:	Give no credit for the following reasons • H (or the height of ball) is negative when $x > 40$ • Bounce of the ball should be considered after hitting the ground • Model will not be true for a different rugby ball • Ball may not be kicked in the same way each time

Question	Scheme	Marks	AOs
	$\frac{d}{dx}(\sin x)=\lim_{h\to 0}\frac{\sin h}{h}\quad\lim_{h\to 0}\frac{\cos h-1}{h}$		