adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

# Differential equations

## Question paper

8 questions

Exam questions and marking instructions remain credited to their board.

# Contents


1 June 2025 - Paper 2 - Q10

2 June 2024 - Paper 1 - Q7

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7 June 2019 - Paper 2 - Q14

8 June 2018 - Paper 1 - Q10

**10.** Water flows at a constant rate into a large container.

There is a tap at the bottom of the container.

At time $t$ hours after the tap was opened

- the volume of water in the container is $V\text{m}^3$
- water is flowing into the container at a constant rate of $0.45\,\text{m}^3$ per hour
- water is leaving the container through the tap at a rate of $0.3V\text{m}^3$ per hour

(a) Show that

$$20\frac{\mathrm{d}V}{\mathrm{d}t} = 9 - 6V$$

**(2)**

Given that when the tap was opened, there was $0.25\,\text{m}^3$ of water in the container,

(b) solve the differential equation to show that

$$V = P - Q\mathrm{e}^{-kt}$$

where $P$, $Q$ and $k$ are positive constants to be found.

**(5)**

Given that

- the capacity of the container is $2\,\text{m}^3$
- the tap remains open
- the water continues to flow into the tank at the same rate

(c) determine whether the container will ever become full, giving a reason for your answer.

**(2)**

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**Question 10 continued**

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**7.**

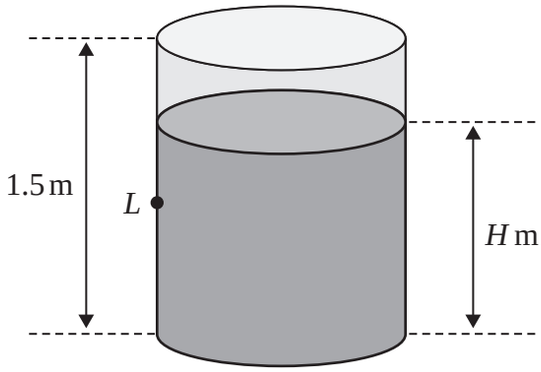



Diagram not drawn to scale.

**Figure 2**

Figure 2 shows a cylindrical tank of height 1.5 m.

Initially the tank is full of water.

The water starts to leak from a small hole, at a point $L$, in the side of the tank.

While the tank is leaking, the depth, $H$ metres, of the water in the tank is modelled by the differential equation

$$\frac{dH}{dt} = -0.12e^{-0.2t}$$

where $t$ hours is the time after the leak starts.

Using the model,

(a) show that

$$H = Ae^{-0.2t} + B$$

where $A$ and $B$ are constants to be found,

**(3)**

(b) find the time taken for the depth of the water to decrease to 1.2 m. Give your answer in hours and minutes, to the nearest minute.

**(3)**

In the long term, the water level in the tank falls to the same height as the hole.

(c) Find, according to the model, the height of the hole from the bottom of the tank.

**(2)**

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**14.** A balloon is being inflated.

In a simple model,

- the balloon is modelled as a sphere
- the rate of increase of the radius of the balloon is inversely proportional to the square root of the radius of the balloon

At time $t$ seconds, the radius of the balloon is $r$ cm.

(a) Write down a differential equation to model this situation.

**(1)**

At the instant when $t = 10$

- the radius is 16 cm
- the radius is increasing at a rate of $0.9\,\text{cm}\,\text{s}^{-1}$

(b) Solve the differential equation to show that

$$r^{\frac{3}{2}} = 5.4t + 10$$

**(5)**

(c) Hence find the radius of the balloon when $t = 20$

Give your answer to the nearest millimetre.

**(2)**

(d) Suggest a limitation of the model.

**(1)**

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**12.** (a) Express $\dfrac{1}{V(25-V)}$ in partial fractions.

**(2)**

The volume, $V$ microlitres, of a plant cell $t$ hours after the plant is watered is modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{1}{10}V(25-V)$$

The plant cell has an initial volume of 20 microlitres.

(b) Find, according to the model, the time taken, in minutes, for the volume of the plant cell to reach 24 microlitres.

**(5)**

(c) Show that

$$V = \frac{A}{\mathrm{e}^{-kt} + B}$$

where $A$, $B$ and $k$ are constants to be found.

**(3)**

The model predicts that there is an upper limit, $L$ microlitres, on the volume of the plant cell.

(d) Find the value of $L$, giving a reason for your answer.

**(2)**

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**14.** (a) Express $\dfrac{3}{(2x-1)(x+1)}$ in partial fractions.

**(3)**

When chemical *A* and chemical *B* are mixed, oxygen is produced.

A scientist mixed these two chemicals and measured the total volume of oxygen produced over a period of time.

The total volume of oxygen produced, $V\,\text{m}^3$, $t$ hours after the chemicals were mixed, is modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{3V}{(2t-1)(t+1)} \qquad V \geqslant 0 \qquad t \geqslant k$$

where $k$ is a constant.

Given that exactly 2 hours after the chemicals were mixed, a total volume of $3\,\text{m}^3$ of oxygen had been produced,

(b) solve the differential equation to show that

$$V = \frac{3(2t-1)}{(t+1)}$$

**(5)**

The scientist noticed that

- there was a **time delay** between the chemicals being mixed and oxygen being produced
- there was a **limit** to the total volume of oxygen produced

Deduce from the model

(c) (i) the **time delay** giving your answer in minutes,

(ii) the **limit** giving your answer in $\text{m}^3$

**(2)**

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**14.** A large spherical balloon is deflating.

At time $t$ seconds the balloon has radius $r$ cm and volume $V$ cm$^3$

The volume of the balloon is modelled as decreasing at a constant rate.

(a) Using this model, show that

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{k}{r^2}$$

where $k$ is a positive constant.

**(3)**

Given that

- the initial radius of the balloon is 40 cm
- after 5 seconds the radius of the balloon is 20 cm
- the volume of the balloon continues to decrease at a constant rate until the balloon is empty

(b) solve the differential equation to find a complete equation linking $r$ and $t$.

**(5)**

(c) Find the limitation on the values of $t$ for which the equation in part (b) is valid.

**(2)**

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**14.** (a) Use the substitution $u = 4 - \sqrt{h}$ to show that

$$\int \frac{\mathrm{d}h}{4 - \sqrt{h}} = -8\ln\left|4 - \sqrt{h}\right| - 2\sqrt{h} + k$$

where $k$ is a constant

**(6)**

A team of scientists is studying a species of slow growing tree.

The rate of change in height of a tree in this species is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{t^{0.25}\left(4 - \sqrt{h}\right)}{20}$$

where $h$ is the height in metres and $t$ is the time, measured in years, after the tree is planted.

(b) Find, according to the model, the range in heights of trees in this species.

**(2)**

One of these trees is one metre high when it is first planted.

According to the model,

(c) calculate the time this tree would take to reach a height of 12 metres, giving your answer to 3 significant figures.

**(7)**

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**TOTAL FOR PAPER IS 100 MARKS**

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**10.** The height above ground, $H$ metres, of a passenger on a roller coaster can be modelled by the differential equation

$$\frac{dH}{dt} = \frac{H\cos(0.25t)}{40}$$

where $t$ is the time, in seconds, from the start of the ride.

Given that the passenger is 5 m above the ground at the start of the ride,

(a) show that $H = 5e^{0.1\sin(0.25t)}$

**(5)**

(b) State the maximum height of the passenger above the ground.

**(1)**

The passenger reaches the maximum height, for the second time, $T$ seconds after the start of the ride.

(c) Find the value of $T$.

**(2)**

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