adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Differentiation

Question paper

13 questions

Exam questions and marking instructions remain credited to their board.

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12 June 2018 - Paper 1 - Q5

13 June 2018 - Paper 1 - Q9

11. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The value of a car, £V, is modelled by the equation

$$V = 1500 + A\,e^{-kt}$$

where A and k are positive constants and t is the age of the car in years.

Given that

- the initial value of the car was £20 000
- the value of the car was £12 000 when it was 2.5 years old

(a) find a complete equation for the model, giving the exact value of A and the value of k to 3 significant figures.

(4)

(b) Show that the rate of change in the value of the car can be expressed in the form

$$-k(V - 1500)$$

(3)

(c) State a limitation of this model.

(1)

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Question 11 continued

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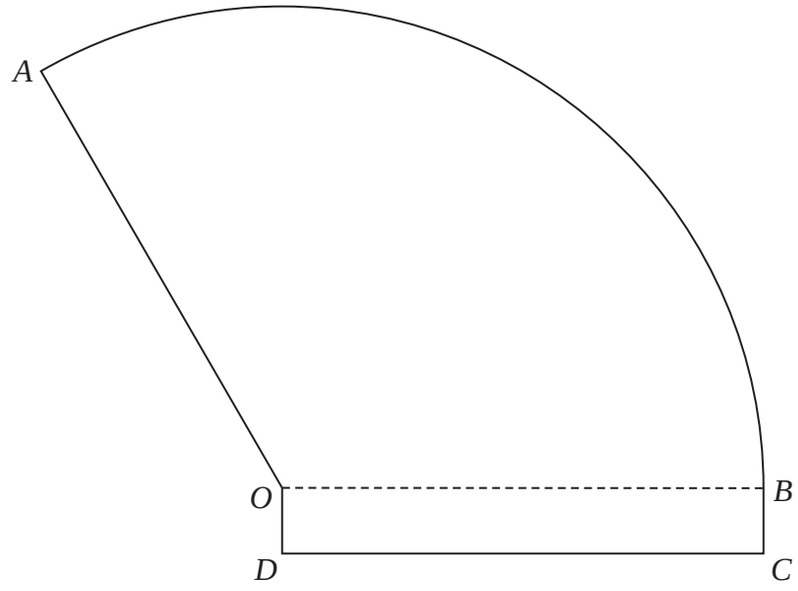


Figure 4

Figure 4 shows the plan view for the design of a stage.

The shape of this design consists of a sector of a circle *AOB* joined to a rectangle *OBCD*.

Given that

- the radius of the sector is r metres and angle AOB is θ radians
- the length and width of the rectangle are r metres and $\frac{1}{10}r$ metres respectively
- the total area of the stage is $240\,\text{m}^2$

(a) show that the perimeter of the stage, P metres, is given by

$$P = 2r + \frac{480}{r}$$

You must make your method clear.

(4)

Using algebraic differentiation,

(b) find the value of r for which P has a stationary value.

(3)

(c) Prove, by further differentiation, that this value of r gives the minimum perimeter of the stage.

(2)

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Question 15 continued

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1. $y = 4x^3 - 7x^2 + 5x - 10$

(a) Find in simplest form

(i) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(ii) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$

(3)

(b) Hence find the exact value of x when $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$

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Question 1 continued

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4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = f(x)$ where $x \in \mathbb{R}$

Given that

- $f'(x) = 2x + \frac{1}{2}\cos x$
- the curve has a stationary point with x coordinate α
- α is small

(a) use the small angle approximation for $\cos x$ to estimate the value of α to 3 decimal places.

(3)

The point $P(0, 3)$ lies on C

(b) Find the equation of the tangent to the curve at P, giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(2)

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Question 4 continued

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1.

$$f(x) = x^3 + 2x^2 - 8x + 5$$

(a) Find $f''(x)$

(2)

(b) (i) Solve $f''(x) = 0$

(ii) Hence find the range of values of x for which $f(x)$ is concave.

(2)

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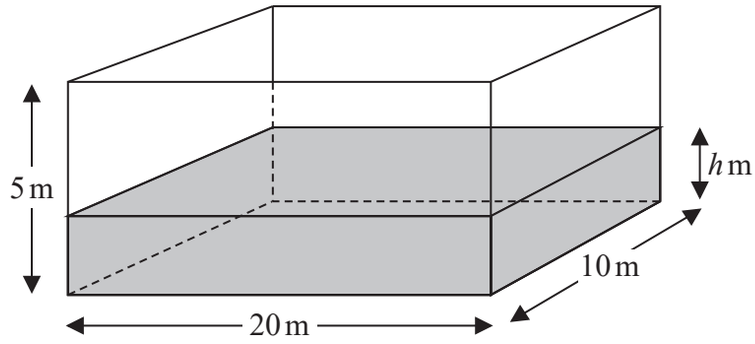


Figure 1

A tank in the shape of a cuboid is being filled with water.

The base of the tank measures 20 m by 10 m and the height of the tank is 5 m, as shown in Figure 1.

At time t minutes after water started flowing into the tank the height of the water was h m and the volume of water in the tank was V m^3

In a model of this situation

- the sides of the tank have negligible thickness
- the rate of change of V is inversely proportional to the square root of h

(a) Show that

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\lambda}{\sqrt{h}}$$

where λ is a constant.

(3)

Given that

- initially the height of the water in the tank was 1.44 m
- exactly 8 minutes after water started flowing into the tank the height of the water was 3.24 m

(b) use the model to find an equation linking h with t, giving your answer in the form

$$h^{\frac{3}{2}} = At + B$$

where A and B are constants to be found.

(5)

(c) Hence find the time taken, from when water started flowing into the tank, for the tank to be completely full.

(2)

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14. Given that

$$y = \frac{x - 4}{2 + \sqrt{x}} \qquad x > 0$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{A\sqrt{x}} \qquad x > 0$$

where A is a constant to be found.

(4)

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Question 14 continued

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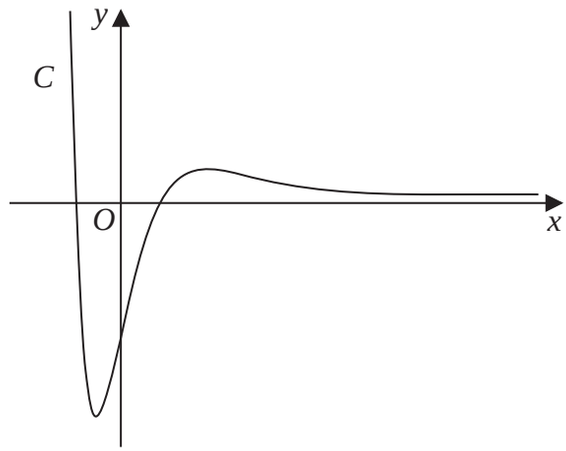


Figure 2

Figure 2 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 4\left(x^2 - 2\right)\mathrm{e}^{-2x} \qquad x \in \mathbb{R}$$

(a) Show that $\mathrm{f}'(x) = 8\left(2 + x - x^2\right)\mathrm{e}^{-2x}$

(3)

(b) Hence find, in simplest form, the exact coordinates of the stationary points of C.

(3)

The function g and the function h are defined by

$$\mathrm{g}(x) = 2\mathrm{f}(x) \qquad x \in \mathbb{R}$$

$$\mathrm{h}(x) = 2\mathrm{f}(x) - 3 \qquad x \geqslant 0$$

(c) Find (i) the range of g

(ii) the range of h

(3)

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Question 9 continued

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15. The curve C has equation

$$x^2 \tan y = 9 \qquad 0 < y < \frac{\pi}{2}$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-18x}{x^4 + 81}$$

(4)

(b) Prove that C has a point of inflection at $x = \sqrt[4]{27}$

(3)

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14. The curve C, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{dy}{dx}$ at the origin.

(2)

(b) (i) Use the small angle approximation for $\sin 2y$ to find an equation linking x and y for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i).

(2)

(c) Show that, for all points (x, y) lying on C,

$$\frac{dy}{dx} = \frac{1}{a\sqrt{b - x^2}}$$

where a and b are constants to be found.

(3)

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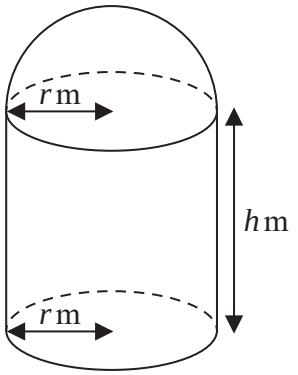


Figure 9

[A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$]

A manufacturer produces a storage tank.

The tank is modelled in the shape of a hollow circular cylinder closed at one end with a hemispherical shell at the other end as shown in Figure 9.

The walls of the tank are assumed to have negligible thickness.

The cylinder has radius r metres and height h metres and the hemisphere has radius r metres.

The volume of the tank is $6\,\text{m}^3$.

(a) Show that, according to the model, the surface area of the tank, in m^2, is given by

$$\frac{12}{r} + \frac{5}{3}\pi r^2$$

(4)

The manufacturer needs to minimise the surface area of the tank.

(b) Use calculus to find the radius of the tank for which the surface area is a minimum.

(4)

(c) Calculate the minimum surface area of the tank, giving your answer to the nearest integer.

(2)

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Question 13 continued

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5. Given that

$$y = \frac{3\sin\theta}{2\sin\theta + 2\cos\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}\theta} = \frac{A}{1 + \sin 2\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

where A is a rational constant to be found.

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Question 5 continued

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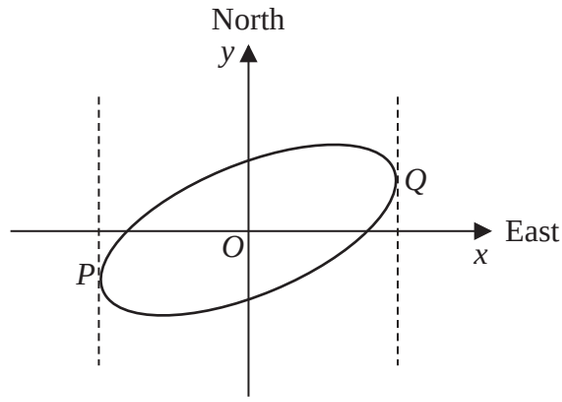


Figure 4

Figure 4 shows a sketch of the curve with equation $x^2 - 2xy + 3y^2 = 50$

(a) Show that $\frac{dy}{dx} = \frac{y - x}{3y - x}$

(4)

The curve is used to model the shape of a cycle track with both x and y measured in km.

The points P and Q represent points that are furthest west and furthest east of the origin O, as shown in Figure 4.

Using part (a),

(b) find the exact coordinates of the point P.

(5)

(c) Explain briefly how to find the coordinates of the point that is furthest north of the origin O. (You **do not** need to carry out this calculation).

(1)

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