adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Exponentials and logarithms

Mark scheme

22 questions

Exam questions and marking instructions remain credited to their board.

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Question	Scheme	Marks	AOs
3	$3^x = 7^y \rightarrow x\log 3 = y\log 7$	M1	3.1a
	e.g. $\left(\frac{x}{y} =\right) \frac{\log 7}{\log 3}$	A1	1.1b
		(2)	
			(2 marks)

Notes

Note: Condone absence of reference to the value of $\frac{x}{y}$ being undefined when $x = y = 0$

M1: For the key step in attempting to take logarithms with the same base of both sides **and** apply the power law to both sides, e.g., $x = y\log_3 7$ or $y = x\log_7 3$ or $x\ln 3 = y\ln 7$

Condone e.g. $x = \log_3 7y$ but not $x = \log_3(7y)$

Alternatively, takes a root of both sides (either x or y), takes logs with the same base of both sides and applies the power law e.g., $3^x = 7^y \rightarrow 3^{\frac{x}{y}} = 7 \rightarrow \frac{x}{y}\log 3 = \log 7$

May be implied by a correct answer.

A1: $\left(\frac{x}{y} =\right) \frac{\log 7}{\log 3}$ or equivalent, e.g, $\frac{\ln 7}{\ln 3}$ or $\log_3 7$ or $\frac{1}{\log_7 3}$ Condone e.g. $\frac{\ln|7|}{\ln|3|}$

Correct answer only scores both marks provided there is no incorrect log work.

Any base k may be used for e.g. $\frac{\log_k 7}{\log_k 3}$ provided it is the same base in both logs, although watch out for e.g. $\left(\frac{\log_3 7}{\log_7 3}\right)^{\frac{1}{2}}$ or $\frac{\log_3 k}{\log_7 k}$ for constant $k > 0$ which are exceptions and correct.

There is no need to see $\frac{x}{y} =$ but it should be clear what their answer is.

Do not ISW if they incorrectly apply log laws e.g. $\frac{\log 7}{\log 3} = \log\frac{7}{3}$ or $= \log 7 - \log 3$ or $= \log(7-3)$ all of which score M1A0.

You may ISW after a correct answer if they go on to provide a decimal approximation.
A decimal approximation with no correct log work seen (1.7712…) scores no marks.

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
9(a)	$5000-5000\mathrm{e}^{-0.075\times3}=\ldots$	M1	3.4
	1007	A1	1.1b
		(2)	
(b)	$5000-5000\mathrm{e}^{-0.075T}=3000\rightarrow 5000\mathrm{e}^{-0.075T}=2000$	M1	1.1b
	$\Rightarrow T=\dfrac{1}{-0.075}\ln\dfrac{2000}{5000}$	dM1	1.1b
	$\Rightarrow T=12.22$	A1	1.1b
		(3)	
(c)	$\left(\dfrac{\mathrm{d}N}{\mathrm{d}t}=\right)\ -5000\times-0.075\mathrm{e}^{-0.075\times3}=\ldots$	M1	3.4
	299 (car sales per month)	A1	1.1b
		(2)	
(d)	Change the constant 5000 to 6500.	B1	3.5c
		(1)	
			(8 marks)

Notes

(a)

M1: Substitutes $t = 3$ into the given model and proceeds to a value.
Implied by awrt 1007, 1008 or 1010
Condone copying slips e.g. –0.75 or –0.0075 in place of –0.075 or 500 in place of 5000 – in such cases you may need to check their calculation if the substitution isn't shown.

A1: 1007 but allow 1008 or 1010 (3sf). Must be a whole number.
Correct answer only scores both marks. A0 for e.g. 1007.4
Do not ISW if they go on to sum their values of N from $t = 1$ to 3

(b)

M1: Sets $5000-5000\mathrm{e}^{-0.075T}$ equal to 3000 and proceeds to either $A\mathrm{e}^{-0.075T}=B$ or $\mathrm{e}^{-0.075T}=k$ where A, B, k are constants with no restrictions for this mark.
May use t instead of T. Condone slips.
Condone copying slips e.g. –0.75 or –0.0075 in place of –0.075 or 500 in place of 5000.

dM1: Uses the correct order of operations and correct log work from an equation of the form $A\mathrm{e}^{-0.075T}=B$ with $AB > 0$ **or** $\mathrm{e}^{-0.075T}=k$ with $k > 0$ to proceed to a value for T or t which may be a numerical expression.

e.g. $A\mathrm{e}^{-0.075T}=B \Rightarrow \ln A-0.075T=\ln B \Rightarrow T=\dfrac{\ln A-\ln B}{0.075}$ or $=\dfrac{\ln\left(A/B\right)}{0.075}$

Condone log being used in place of ln unless there is clear evidence that they have used an inconsistent base.

May not be scored if "recovered" from e.g. $\mathrm{e}^{-0.075T}=-\dfrac{2}{5}$

Condone copying slips e.g. –0.75 or –0.0075 in place of –0.075 or 500 in place of 5000.

A1: 12.22 cao but must see a correct equation following use of logs, e.g., $-0.075T=\ln\dfrac{2}{5}$ or

$T=\dfrac{1}{-0.075}\times-0.916$ which may have intermediate rounding. Ignore any units given.

This mark is available following the occasional slip in writing –0.075 as –0.75 or –0.0075 provided it is not consistently used.

M1: Differentiates once to the form $\lambda e^{-0.075t}$, λ a constant (even ±5000), and substitutes in $t = 3$
Award for an expression of the form $ke^{-0.075\times3}$ with constant k if no incorrect working is seen.
The substitution of $t = 3$ may be implied by a correct value for their derivative of the required form, provided the derivative is seen. Do not be concerned with what they call their derivative.

A1: awrt 299 (car sales per month). Answer only scores no marks. Condone 300 following 299.4
Requires a correct derivative to be seen, e.g., $375e^{-0.075t}$ o.e., with or without the substitution of $t = 3$ present.
Units may be omitted, but score A0 if an incorrect time frame is given e.g. cars per week

(d)

B1: Acceptable refinement. Some examples:

- Change the 5000 to 6500 (condone ambiguity about which 5000)
- Change the first 5000 to 6500
- Change both 5000s to 6500s
- Multiply the model by 1.3 (or e.g. $\frac{65}{50}$ or $\frac{13}{10}$)
- Add 1500 to the model
- Change the constant to 6500

The statement of an acceptable refined model e.g. $(N =)\ 6500 - 6500e^{-0.075t}$
or e.g. $(N =)\ 6500 - 5000e^{-0.075t}$ scores B1.
Ignore any extra unnecessary refinements such as increase/decrease the 0.075.

The following score B0:

- Change 5000 in the model (not specific enough – they haven't said to 6500)
- Change the second 5000 to 6500 (incorrect)

Question	Scheme	Marks	AOs

6(a)(i)	$\left(\mathrm{f}'(x)=\right)8x\mathrm{e}^{4x^2-1}$ or e.g. $\dfrac{8x\mathrm{e}^{4x^2}}{\mathrm{e}}$ oe	B1	1.1b
(ii)	$\left(\mathrm{g}'(x)=\right)\dfrac{8}{x}$ or e.g. $8x^{-1}$ oe	B1	1.2
		(2)	
(b)	$8x\mathrm{e}^{4x^2-1}=\dfrac{8}{x}\Rightarrow \mathrm{e}^{4x^2-1}=\dfrac{1}{x^2}\Rightarrow 4x^2-1=\ln\dfrac{1}{x^2}$	M1	1.1b
	$4x^2-1=\ln\dfrac{1}{x^2}\Rightarrow 4x^2-1=-2\ln x$ $\Rightarrow 4x^2+2\ln x-1=0$*	A1*	2.1
		(2)	
(c)(i)	$x_1=0.6\Rightarrow x_2=\sqrt{\dfrac{1-2\ln 0.6}{4}}$	M1	1.1b
	$\left(x_2=\right)0.7109$	A1	1.1b
(ii)	$\left(\alpha=\right)0.6706$	B1 (A1 In ePEN)	1.1b
		(3)	
			(7 marks)

(a)(i)

B1: Correct derivative in any form. "$\mathrm{f}'(x)=$" is not required. Apply isw if necessary.

(ii)

B1: Correct derivative in any form. "$\mathrm{g}'(x)=$" is not required. Apply isw if necessary.

(b)

M1: Eliminates e by setting their $\mathrm{f}'(x)=$ their $\mathrm{g}'(x)$ where $\mathrm{f}'(x)=Ax\mathrm{e}^{4x^2-1}$ oe and $\mathrm{g}'(x)=\dfrac{B}{x}$ oe with $A\times B>0$ and proceeds via $\mathrm{e}^{4x^2-1}=\dfrac{\cdots}{x^2}$ or equivalent work (see below) to obtain $4x^2-1=\ln\dfrac{\cdots}{x^2}$ oe e.g. $\ln x+4x^2-1=\ln\dfrac{1}{x}$

Allow if they use α for x.

Note that there are various alternatives for this mark but the derivatives must be of the form defined above and the processing must be correct with coefficient/sign slips only.

Examples of equivalent work:

$$8x\mathrm{e}^{4x^2-1}=\frac{8}{x}\Rightarrow x^2\mathrm{e}^{4x^2-1}=1\Rightarrow \ln x^2+\ln\mathrm{e}^{4x^2-1}=0\Rightarrow \ln\mathrm{e}^{4x^2-1}=-\ln x^2\Rightarrow 4x^2-1=-2\ln x$$

$$\frac{8x\mathrm{e}^{4x^2}}{\mathrm{e}}=\frac{8}{x}\Rightarrow \frac{1}{\mathrm{e}}\mathrm{e}^{4x^2}=\frac{1}{x^2}\Rightarrow \mathrm{e}^{4x^2}=\frac{\mathrm{e}}{x^2}\Rightarrow \ln\mathrm{e}^{4x^2}=\ln\frac{\mathrm{e}}{x^2}\Rightarrow 4x^2=\ln\frac{\mathrm{e}}{x^2}=1-2\ln x$$

A1*: Obtains the printed answer with sufficient working and no errors.

Sufficient work would require the "e" eliminated before the given answer.

Must follow correct derivatives in part (a).

Condone $4x^2+2\ln|x|-1=0$ and condone $4\alpha^2+2\ln\alpha-1=0$ or $4\alpha^2+2\ln|\alpha|-1=0$

Note that if both derivatives in (a) **<u>are correct</u>** we will allow fully correct work using the equation in (b) to work backwards to verify that $pf'(x) = qg'(x)$ for M1 then obtains $f'(x) = g'(x)$ with a minimal conclusion for A1

If either derivative in (a) is incorrect or missing, candidates who work backwards score no marks in (b).

(c)(i)/(ii)

M1: Attempts to use the iterative formula with $x_1 = 0.6$

Award this mark for e.g. $(x_2 =)\sqrt{\dfrac{1-2\ln 0.6}{4}}$ or may be implied by awrt 0.71 provided no incorrect working is seen.

Candidates sometimes find x_3 (or possibly subsequent terms) rather than x_2 in which case the M1 can be implied. (See table below for first few iterations)

A1: (x_2 =) awrt 0.7109

Sight of (x_2 =) awrt 0.7109 scores M1A1

B1(A1 on ePEN): (α =) 0.6706 (4dp)

Must be <u>this value</u> and **not** awrt 0.6706

For reference:

x_1	0.6
x_2	0.7109239143
x_3	0.6485329086
x_4	0.6830236199
x_5	0.6637868021
x_6	0.6744606223
.	.
.	.
.	.
α	0.6706416243

Question	Scheme	Marks	AOs

13(a)	$\log_{10} b = 0.0054 \Rightarrow b = 10^{0.0054}$ **or** $\log_{10} a = 0.81 \Rightarrow a = 10^{0.81}$	M1	3.1a
	$b = 1.01$ **or** $a = 6.46$	A1	1.1b
	$\log_{10} b = 0.0054 \Rightarrow b = 10^{0.0054}$ **and** $\log_{10} a = 0.81 \Rightarrow a = 10^{0.81}$	M1	2.1
	$b = 1.013$ **and** $a = 6.457$	A1	1.1b
		(4)	
(b)(i)	e.g. The world population **in billions** in 2004	B1ft	3.2a
(ii)	b = 1.013 represents the scale factor of the yearly increase in the world population	B1ft	3.2a
		(2)	
(c)	$P = 6.457...(1.013...)^{26}$ or e.g. $\log P = 0.81 + 26 \times 0.0054 \Rightarrow P = ...$	M1	3.4
	awrt 9 billion	A1	2.2b
		(2)	
(d)	Not reliable since the data used for the model covered the years 2004 – 2007 and it would not be sensible to assume that the model still holds in 2030	B1	3.2b
		(1)	
			(9 marks)

Notes

(a) Must be using base 10 in (a). Ignore any units associated with a and b in part (a).

M1: Correct strategy to get a numerical expression or value for a **or** b e.g. $a = 10^{0.81}$ **or** $b = 10^{0.0054}$. This may be implied by a = awrt 6.46 or b = awrt 1.01 if no incorrect work is seen.

A1: Correct value for a or b. Allow 3 sf for **this mark** so allow a = awrt 6.46 or b = awrt 1.01.
May be seen embedded in their formula.

M1: Correct strategy to get a numerical expression or value for a **and** b e.g. $a = 10^{0.81}$ **and** $b = 10^{0.0054}$. This may be implied by a = awrt 6.46 and b = awrt 1.01 if no incorrect work is seen.

A1: Correct values. This requires a = awrt 6.457 and b = awrt 1.013 for this mark.
May be seen embedded in their formula.
Isw once correct answers are seen.

Special case: Constants the wrong way round:

$a = 1.013$ **and** $b = 6.457$ with or without working scores M1A1M1A0 unless the equation is formed correctly in which case the final A mark can be recovered.

Note that having found the value of a, it is possible to find b by substituting e.g. t = 1 as follows:

$$a = 10^{0.81} = 6.457 \quad t = 1 \Rightarrow P = ab \Rightarrow b = \frac{P}{a}$$

$$t = 1 \Rightarrow \log_{10} P = 0.0054 + 0.81 = 0.8154 \Rightarrow P = 10^{0.8154} \Rightarrow b = \frac{P}{a} = \frac{10^{0.8154}}{6.457} = 1.013$$

Note that a misread of 0.0054 as 0.054 is quite common and may score 1110 as it does not simplify the question.

(b)(i) Follow through their a.

B1ft: Correct interpretation for a but must reference "billions".

Allow equivalent alternatives e.g.

- The original/initial population **in billions**
- The population in 2004 was "6.46" **billion**

(b)(ii) Follow through their b.

B1ft: Correct interpretation for b but must reference "each year" or e.g. "yearly" oe

Allow equivalent alternatives e.g.

- The proportional increase/change in each year.
- The population will rise by "1.3%" each year. Must follow their value for b.
- The rate/factor at which the population is rising/increasing/changing per annum.
- "1.013" is the multiplier representing the year on year increase.

Do **not** accept

- The amount it is rising
- How much it is rising
- The rate the population increases
- The percentage increase each year
- The rate of increase in billions annually

If they are not labelled (b)(i) and (b)(ii) mark in the order given but accept any way round as long as clearly labelled " *a* is..... " and "*b* is ....."

(c)

M1: Substitutes $t = 25$ or 26 or 27 into their model to find a value for P

Must be using **their** a and b correctly in $P = ab^t$

May be implied by sight of "9" or 9 billion if no incorrect working is seen.

A1: Correct value including units (allow awrt 9 billion) **from a correct model but condone incorrect/premature rounding or truncating in an otherwise correct model that leads to the correct value of awrt 9 billion.**

Allow e.g. awrt 9 000 000 000 or e.g. awrt 9×10^9

Just awrt 9 without the "billions" is A0

(d)

B1: The response must refer to the fact that the answer is unreliable together with a reference to the fact that the data used for the model is a long way from 2030

Examples:

- Not good as 2030 is a long way from 2004 – 2007
- Unreliable as based on old data
- Questionable as it has been extrapolated over a long time
- Not reliable due to how far out we have extrapolated
- By the time 2030 arrives it will be unreliable

But not e.g.

- Unreliable, extrapolation
- Not good as outside the range
- Not good as the population rises 101.3% each year
- Disease may happen
- Reliable as based on old data

Question	Scheme	Marks	AOs

6(a)	$\frac{1}{2}a$	B1	2.2a
		(1)	
(b)	$\log_2 x(x+8) \Rightarrow \log_2 x + \log_2(x+8)$	M1	1.2
	$= a + b$	A1	2.2a
		(2)	
(c)	e.g. $8 + \frac{64}{x} = \frac{8x+64}{x}$	B1	1.1b
	$\log_2 \frac{8}{x}(x+8) = 3 - \log_2 x + \log_2(x+8)$	M1	1.1b
	$3 + b - a$	A1	2.2a
		(3)	
			(6 marks)

Notes

Condone omission of base 2 in all parts.

(a)

B1: $\frac{1}{2}a$ or $\frac{a}{2}$ or $0.5a$ isw

(b)

M1: Takes a factor of x out of the bracket to achieve $\log_2 x(x+8)$ and attempts to apply the addition law of logs, usually leading to $\log_2 x + \log_2(x+8)$. Condone missing brackets or omission of base 2. May be implied by a correct answer. Allow this mark to be scored if they write $\log_2 x + \log_2 x + \log_2 8$ (an answer of $2a+3$ can score M1A0)

$\log_2 x \times \log_2(x+8)$ on its own is M0 but allow the mark to be scored if they proceed to $a+b$

A1: $a+b$ or simplified equivalent (a correct answer with no incorrect log work seen scores M1A1) isw

Note $\log_2 x \times \log_2(x+8) = a+b$ is M1A0 (allow the answer to imply the correct method but withhold the final mark)

(c)

B1: Writes $8 + \frac{64}{x}$ as a single fraction e.g. $\frac{8x+64}{x}$ or $\frac{8}{x}(x+8)$ or $8x^{-1}(x+8)$ or $8\left(\frac{x^2+8x}{x^2}\right)$ which may be implied by later work e.g. $\log_2 8 - \log_2 x + \log_2(x+8)$

M1: Attempts to apply the laws of logs, uses $\log_2 8 = 3$ and proceeds to $3 \pm \log_2 x \pm \log_2(x+8)$ (or equivalent since $\pm\log_2 x$ may appear as $\pm\log_2 \frac{1}{x}$ or $\pm\log_2 x^{-1}$)

May be implied by $3 \pm b \pm a$ and condone invisible brackets around $x+8$ and condone the omission of base 2.

Note that if they write $\log_2(x+8)$ as $\log_2 x + \log_2 8$ this is M0

A1: $3+b-a$ or simplified equivalent (a correct answer with no incorrect log work seen is B1M1A1) isw

Note $\log_2 \frac{8}{x}(x+8) = 3 \div \log_2 x \times \log_2(x+8) \Rightarrow 3 - a + b$ is B1M1A0 (allow the answer to imply the correct method but withhold the final mark)

Note: You may see attempts to work backwards to the answer.

Question	Scheme	Marks	AOs

11(a)	$\log_{10} V = 3 \Rightarrow V = 10^3$	M1	1.1b
	$(V =)$ £1000	A1	3.4
		(2)	
(b)	e.g. $(\log_{10} b =) \dfrac{2.79-3}{10-0} = -0.021$ **or** $\log_{10} V = 3 - 0.021t$ **or** $10^{2.79} = "1000" b^{10}$	M1	1.1b
	e.g. $b = 10^{-0.021}$ $(= 0.952796....)$ **or** $V = 10^3 \times 10^{-0.021t}$ **or** $b = \sqrt[10]{"0.61659..."}$	M1	3.1b
	$V = 1000 \times 0.953^t$	A1ft	3.3
		(3)	
(c)	e.g. $V = 1000 \times "0.953"^{24}$ $(= £315)$ or e.g. $\log_{10} V = 3 - "0.021" \times 24 \Rightarrow V = ... (= £313)$ which is close (to £320) so it is a suitable model	M1	3.4
		A1	3.2b
		(2)	
			(7 marks)

Notes

(a)

M1: Sets $\log_{10} V = 3$ and attempts to find a value for *a* or an expression for *V* when $t = 0$. Score for sight of 10^3 or implied by the correct answer.
There may be more complicated routes to finding the initial value. e.g. finding a complete equation such as $\left(\log_{10} V = 3 + \dfrac{2.79-3}{10} t \Rightarrow \log_{10} V = 3 \Rightarrow V = \right) 10^3$

This mark can also be scored for the equation $V = 10^3 \times 10^{"-0.021"t}$ or $V = 1000 \times (....)^t$ but not $V = 10^{3-"0.021"t}$ (the 10^3 has not been split up from $10^{"-0.021"t}$)

A1: £1000 cao (including units) do not accept $£10^3$

(b) Mark (b) and (c) together. Note work seen in (a) must be used in (b) to score

M1: Either

- finds the gradient between the two points. Score for the expression $\dfrac{2.79-3}{10-0}$ o.e. e.g. -0.021
  Do not condone sign slips for this mark. May be implied by later work such as sight of $10^{-0.021}$.
- finds the equation for $\log_{10} V$ in terms of *t* e.g. $\log_{10} V = 3 - "0.021"t$ which may be unsimplified.
- forms the equation $10^{2.79} (= 616.5...) = "1000" b^{10}$ o.e. such as $2.79 = 3 + 10 \log b$

M1: Attempts to find the value or an expression for *b* using their gradient or their equation
Score for either:

- the expression $10^{"-0.021"}$ o.e such as $10^{\frac{2.79-3}{10-0}}$ or may be implied by a correct value using their gradient. You may need to check this on your calculator.
- correctly proceeding from $\log_{10} V = 3 - "0.021"t$ to $V = 10^{3-"0.021"t}$ and splitting this into $V = 10^3 \times 10^{-"0.021"t}$

- attempting to equate coefficients:
$$\log_{10} V = \log_{10} a + (\log_{10} b)t \Leftrightarrow \log_{10} V = 3 - "0.021"t \Rightarrow \log_{10} b = "-0.021" \Rightarrow b = 10^{"-0.021"}$$
- using their equation $10^{2.79} = "1000" b^{10}$ or $2.79 = "3" + 10\log b$ and proceeding to e.g. $b = \sqrt[10]{"0.61659..."}$ or $b = 10^{"-0.021"}$

A1ft: Complete correct equation, follow through on their "1000" so score for $V = "1000" \times (\text{awrt } 0.953)^t$ or accept $V = "10^3" \times (\text{awrt } 0.953)^t$. Just stating the values of *a* and *b* is A0ft, but if the equation is written in (c) before substituting in $t = 24$ then this mark can be awarded.

(c) Mark (b) and (c) together

M1: A full and valid attempt to:
- either substitute $t = 24$ into their model of the form $V = ab^t$ where *a* is positive and finds a value for *V*
- or substitutes $t = 24$ into their model of the form $\log_{10} V = p + qt$ where *p* is positive and finds a value for *V* (if they only proceed as far as $\log_{10} V$ they would also have to find the value of $\log_{10} 320$)
- or substitutes $V = 320$ into their $V = 1000 \times 0.953^t$ o.e. and finds a value for *t*

(to enable the candidate to compare real life data with that of the model.)
Do not be too concerned with the mechanics of the solution but they must be attempting to find two values which can be compared (e.g. usually 320 and a value for *V*, but they could proceed to find $\log_{10} 320$ and compare with $\log_{10} V = 2.496$ when $t = 24$, or a value for *t* to compare with $t = 24$)
In cases with no working you will need to check the calculation.

A1: Compares their awrt £313-£315 with £320 or their awrt $t = 23.5 - 23.7$ with $t = 24$ or $\log_{10} 320 = 2.505...$ with 2.496 and makes a valid conclusion with a reason.
For this mark you require:
- correct calculations (if using percentage error allow this to be rounded to compare awrt £313-£315 with £320 then it will be in the range (1.4, 2.4). For £314.94 this is $= \text{awrt } 1.6\%$)
- a reason such as "the values are close", "the values are similar", "the values are approximately equal". Allow use of "$\approx$". Allow the calculation of the % error as reason.
- a statement that it is a "good" or "accurate" model or similar wording.

Note: Condone as a minimum e.g. "£314.94 and £320 so good model" (we accept the two values being stated here as a comparison that they are similar)

Do not allow incorrect statements such as the model is incorrect as it does not give £320.
Do not allow just "the model gives an underestimate of the true value" (does not comment sufficiently on whether the model is reliable)
Do not allow comments suggesting that the model is not reliable.
Note using the full value for *b* leads to 313.3285724…

Question	Scheme	Marks	AOs

3(a)	Uses one correct log law e.g. $\log_2(x+3)+\log_2(x+10)=\log_2(x+3)(x+10)$ $2=\log_2 4,\ \ 2\log_2 x=\log_2 x^2$	**M1**	1.1b
	$(x+3)(x+10)=4x^2$ oe	**dM1**	2.1
	$\Rightarrow 3x^2-13x-30=0$ *	**A1***	1.1b
		(3)	
(b)(i)	$(x=)\ 6,\ -\frac{5}{3}$	**B1**	1.1b
(ii)	$x\neq -\frac{5}{3}$ because $\log_{(2)}\left(-\frac{5}{3}\right)$ is not real	**B1**	2.4
		(2)	
			(5 marks)

Notes

(a)

M1: Uses one correct log law. The base does not need to be seen for this mark.
This mark is independent of any other errors they make.
Examples: $\log_2(x+3)+\log_2(x+10)=\log_2(x+3)(x+10)$, $2=\log_2 4$, $2\log_2 x=\log_2 x^2$

dM1: Fully correct work leading to a correct equation not containing logs that is not the printed answer.
Depends on the first mark. Condone a spurious base e.g. 10 or e, so long as the log work is otherwise correct (i.e., they recover the base 2).
Examples (depending on their method): $(x+3)(x+10)=4x^2$, $\frac{(x+3)(x+10)}{x^2}=4$, $\frac{x+3}{x^2}=\frac{4}{x+10}$
or $1+\frac{13}{x}+\frac{30}{x^2}=4$
Allow recovery from invisible brackets but **not** from incorrect work e.g.

$$\log_2(x+3)+\log_2(x+10)=2+2\log_2 x\Rightarrow \log_2(x+3)(x+10)-\log_2 x^2=\log_2 4$$

$$\Rightarrow \frac{\log_2(x+3)(x+10)}{\log_2 x^2}=\log_2 4\Rightarrow \frac{(x+3)(x+10)}{x^2}=4$$

This scores M1dM0A0

A1*: Obtains $3x^2-13x-30=0$ with no processing errors but condone a spurious base e.g. 10 or e, so long as the log work is otherwise correct (i.e., they recover the base 2) and allow recovery from invisible brackets.

Note the following alternative which can follow the main scheme:

$\log_2(x+3)+\log_2(x+10)=2+2\log_2 x=2+\log_2 x^2$ **M1**

$2^{\log_2(x+3)+\log_2(x+10)}=2^{2+\log_2 x^2}\Rightarrow 2^{\log_2(x+3)}\times 2^{\log_2(x+10)}=2^2\times 2^{\log_2 x^2}\Rightarrow (x+3)(x+10)=4x^2$ **dM1**

$\Rightarrow 3x^2-13x-30=0$ * **A1**

Special Cases:

1. $(x+3)(x+10)=4x^2$ with no working leading to the correct answer scores **M1dM1A0**

2. $\log_2(x+3)+\log_2(x+10)=2+2\log_2 x\Rightarrow 2^{\log_2(x+3)+\log_2(x+10)}=2^{2+2\log_2 x}\Rightarrow (x+3)(x+10)=4x^2$

$\Rightarrow 3x^2-13x-30=0$ *

Also scores **M1(implied)dM1A0** (lack of working)

(b)(i)

B1: Both values correct: $(x=)\ 6,\ -\frac{5}{3}$

(b)(ii)

B1: e.g. $(x\neq)-\frac{5}{3}$ and $\log_{(2)}\left(-\frac{5}{3}\right)$ is not real

This mark requires the identification of the **correct** negative root **and** an acceptable explanation.

For the identification of the root allow e.g. $x\neq-\frac{5}{3},\ x=-\frac{5}{3},\ -\frac{5}{3}$ etc. as long as it is clear they have identified the correct value. Requires the correct negative root $\left(-\frac{5}{3}\right)$ but the other may not be 6 but must be positive.

Some examples for the explanation:

- you get $\log_{(2)}\left(-\frac{5}{3}\right)$ which is not possible
- $\log-\frac{5}{3}$ is not possible, can't be found, gives a math error, is not real, is undefined
- if $\left\{k=\log_2\left(-\frac{5}{3}\right),\right\}\ 2^k=-\frac{5}{3}$ which is not possible
- you get log of a negative number
- negative numbers can't be "logged"
- log of negative does not work

Do not allow e.g.

- you can't have a negative log, logs can't be negative (unless clarified further)
- "you get a math error" in isolation
- a log cannot have a negative value
- logs cannot be negative
- $-\frac{5}{3}$ is not a valid input (unless clarified further)
- "it doesn't work in the logs"
- log graph isn't negative
- log graph does not cross negative *x*–axis
- *x* is only positive & negative answer does not work

Allow an implied correct answer if they say e.g. 6 is the root because $\log_{(2)}\left(-\frac{5}{3}\right)$ is not possible

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
4 (a)	$\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x} \delta x = \int_{2.1}^{6.3} \frac{2}{x} \mathrm{d}x$	B1	1.2
		(1)	
(b)	$= \left[2 \ln x\right]_{2.1}^{6.3} = 2 \ln 6.3 - 2 \ln 2.1$	M1	1.1b
	$= \ln 9$ CSO	A1	1.1b
		(2)	
			(3 marks)
Notes:			

Mark (a) and (b) as one

(a)

B1: States that $\int_{2.1}^{6.3} \frac{2}{x} \mathrm{d}x$ or equivalent such as $2\int_{2.1}^{6.3} x^{-1} \mathrm{d}x$ but must include the limits and the dx.

Condone $\mathrm{d}x \leftrightarrow \delta x$ as it is very difficult to tell one from another sometimes

(b)

M1: Know that $\int \frac{1}{x} \mathrm{d}x = \ln x$ and attempts to apply the limits (either way around)

Condone $\int \frac{2}{x} \mathrm{d}x = p \ln x$ (including $p = 1$) or $\int \frac{2}{x} \mathrm{d}x = p \ln qx$ as long as the limits are applied.

Also be aware that $\int \frac{2}{x} \mathrm{d}x = \ln x^2$, $\int \frac{2}{x} \mathrm{d}x = 2 \ln \left|x\right| + c$ and $\int \frac{2}{x} \mathrm{d}x = 2 \ln cx$ o.e. are also correct

$\left[p \ln x\right]_{2.1}^{6.3} = p \ln 6.3 - p \ln 2.1$ is sufficient evidence to award this mark

A1: CSO $\ln 9$. Also answer = $\ln 3^2$ so $k = 9$ is fine. Condone $\ln \left|9\right|$

The method mark must have been awarded. Do not accept answers such as $\ln \frac{39.69}{4.41}$

Note that solutions appearing from "rounded" decimal work when taking lns should not score the final mark. It is a "show that" question

E.g. $\left[2 \ln x\right]_{2.1}^{6.3} = 2 \ln 6.3 - 2 \ln 2.1 = 2.197 = \ln k \Rightarrow k = \mathrm{e}^{2.197} = 8.998 = 9$

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
10 (a)	265 thousand	B1	3.4
		(1)	
(b)	Attempts $\frac{\mathrm{d}N_b}{\mathrm{d}t} = 11\mathrm{e}^{0.05t}$	M1	1.1b
	Substitutes $t = 10$ into their $\frac{\mathrm{d}N_b}{\mathrm{d}t}$	M1	3.4
	$\frac{\mathrm{d}N_b}{\mathrm{d}t} =$ awrt 18.1 which is approximately 18 thousand per year *	A1*	2.1
		(3)	
(c)	Sets $45 + 220\,\mathrm{e}^{0.05t} = 10 + 800\,\mathrm{e}^{-0.05t} \Rightarrow 220\,\mathrm{e}^{0.05t} + 35 - 800\,\mathrm{e}^{-0.05t} = 0$	M1	3.1b
	Correct quadratic equation $\Rightarrow 220\left(\mathrm{e}^{0.05t}\right)^2 + 35\mathrm{e}^{0.05t} - 800 = 0$	A1	1.1b
	$\mathrm{e}^{0.05t} = 1.829, \left(-1.988\right) \Rightarrow 0.05t = \ln(1.829)$	M1	2.1
	$T = 12.08$	A1	1.1b
		(4)	
			(8 marks)
Notes:			

(a) May be seen in the question so watch out.

B1: Accept 265 thousand or 265 000 or equivalent such as 265 k but not just 265.

(b)

M1: Differentiates to a form $k\,\mathrm{e}^{0.05t}$, $k > 0, k \neq 220$. Do not be too concerned about the lhs.

M1: Substitutes $t = 10$ into a changed function that was formed from an attempt at differentiation.

The left hand side must have implied differentiation. E.g. Rate = , N', $\frac{dN_b}{dt}$, $\frac{dN}{dt}$ or even $\frac{\mathrm{d}y}{\mathrm{d}x}$

A1*: Full and complete proof that requires

- some correct lhs seen at some point. E.g. "Rate = , " $\frac{dN_b}{dt}$, $\frac{dN}{dt}$ but condone N'.
- an intermediate line/answer of either $11\mathrm{e}^{0.05\times10}$ or awrt 18.1 before a minimal conclusion which must be referencing the 18 000 or 18 thousand

(c)

M1: Attempts to set both equations equal to each other and simplify the constant terms.

Look for $220\,e^{0.05t}+35=800\,e^{-0.05t}$ o.e but condone slips

It is also possible to set $\frac{N-45}{220}=\left(e^{0.05t}=\right)\frac{800}{N-10}$ and form an equation in N

A1: Correct quadratic form.

Look for $220\left(e^{0.05t}\right)^2+35e^{0.05t}-800=0$ or $220\,e^{0.1t}+35e^{0.05t}-800=0$ but allow with terms in different order such as $220\,e^{0.1t}+35e^{0.05t}=800$

FYI the equation in N is $N^2-55N-175550=0$

M1: Full attempt to find the value of t (or a constant multiple of t)

This involves the key step of recognising and solving a 3TQ in $e^{0.05t}$ followed by the use of lns.

If the answers to the quadratic just appear (from a calculator) you will need to check.

Accuracy should be to 3sf.

You may see different variables used such as x

$x=e^{0.05t}$, $220\,e^{0.1t}+35e^{0.05t}=800\Rightarrow 220x^2+35x=800\Rightarrow x=1.82...\Rightarrow t=20\ln 1.82...$

Allow use of calculator for solving the quadratic and for $e^{0.05t}=1.82..\Rightarrow t=12.08$

Via the N route it will involve substituting the positive solution to their quadratic into either equation to find a value for t/T using same rules as above.

A1: AWRT 12.08

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Answers with limited or no working in (b) and (c)

(b) A derivative in the correct form must be seen

(c) Candidates who state $45+220\,e^{0.05t}=10+800\,e^{-0.05t}$ followed by awrt 12.08 (presumably from using num-solv on their calculators) can score SC 1100. Rubric on the front of the paper states that "Answers without working may not gain full credit" so we demand a method in this part.

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Question	Scheme	Marks	AOs

Question			
12	$\int x^3 \ln x \, \mathrm{d}x = \frac{x^4}{4}\ln x - \int \frac{x^4}{4} \times \frac{1}{x}\,\mathrm{d}x$	M1	1.1b
	$= \frac{x^4}{4}\ln x - \frac{x^4}{16}(+c)$	M1 A1	1.1b 1.1b
	$\int_1^{\mathrm{e}^2} x^3 \ln x \, \mathrm{d}x = \left[\frac{x^4}{4}\ln x - \frac{x^4}{16}\right]_1^{\mathrm{e}^2} = \left(\frac{\mathrm{e}^8}{4}\ln \mathrm{e}^2 - \frac{\mathrm{e}^8}{16}\right) - \left(-\frac{1^4}{16}\right)$	M1	2.1
	$= \frac{7}{16}\mathrm{e}^8 + \frac{1}{16}$	A1	1.1b
		(5)	
			(5 marks)

Notes:

M1: Integrates by parts the right way round.

Look for $kx^4 \ln x - \int kx^4 \times \frac{1}{x}\,\mathrm{d}x$ o.e. with $k > 0$. Condone a missing dx

M1: Uses a correct method to integrate an expression of the form $\int kx^4 \times \frac{1}{x}\,\mathrm{d}x \to c\,x^4$

A1: $\int x^3 \ln x \, \mathrm{d}x = \frac{x^4}{4}\ln x - \frac{x^4}{16}$ (+ c) which may be left unsimplified

M1: Attempts to substitute 1 and e^2 into an expression of the form $\pm px^4 \ln x \pm qx^4$, subtracts and uses $\ln \mathrm{e}^2 = 2$ (which may be implied).

A1: $\frac{7}{16}\mathrm{e}^8 + \frac{1}{16}$ o.e. Allow $0.4375\mathrm{e}^8 + 0.0625$ or uncancelled fractions. NOT ISW: $7\mathrm{e}^8 + 1$ is A0

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You may see attempts where substitution has been attempted.

E.g. $u = \ln x \Rightarrow x = \mathrm{e}^u$ and $\frac{\mathrm{d}x}{\mathrm{d}u} = \mathrm{e}^u$

M1: Attempts to integrate the correct way around condoning slips on the coefficients

$$\int x^3 \ln x \, \mathrm{d}x = \int \mathrm{e}^{4u} u \, \mathrm{d}u = \frac{\mathrm{e}^{4u}}{4}u - \int \frac{\mathrm{e}^{4u}}{4}\,\mathrm{d}u$$

M1 A1: $\int x^3 \ln x \, \mathrm{d}x = \frac{\mathrm{e}^{4u}}{4}u - \frac{\mathrm{e}^{4u}}{16}(+c)$

M1 A1: Substitutes 0 and 2 into an expression of the form $\pm pu\mathrm{e}^{4u} \pm q\mathrm{e}^{4u}$ and subtracts

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It is possible to use integration by parts "the other way around"

To do this, candidates need to know or use $\int \ln x \, \mathrm{d}x = x\ln x - x$

FYI $I = \int x^3 \ln x \, \mathrm{d}x = x^3(x\ln x - x) - \int (x\ln x - x) \times 3x^2 \, \mathrm{d}x = x^3(x\ln x - x) - 3I + \frac{3}{4}x^4$

Hence $4I = x^4 \ln x - \frac{1}{4}x^4 \Rightarrow I = \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4$

Score M1 for a full attempt at line 1 (condoning bracketing and coefficient slips) followed by M 1 for line 2 where terms in I o.e. to form the answer.

4(a)	$f'(x) = 2x + \dfrac{4x-4}{2x^2-4x+5}$	M1 A1	1.1b 1.1b
	$2x + \dfrac{4x-4}{2x^2-4x+5} = 0 \Rightarrow 2x\left(2x^2-4x+5\right)+4x-4=0$	dM1	1.1b
	$2x^3-4x^2+7x-2=0$*	A1*	2.1
		(4)	
(b)	(i) $x_2 = \dfrac{1}{7}\left(2+4(0.3)^2-2(0.3)^3\right)$	M1	1.1b
	$x_2 = 0.3294$	A1	1.1b
	(ii) $x_4 = 0.3398$	A1	1.1b
		(3)	
(c)	$h(x) = 2x^3-4x^2+7x-2$ $h(0.3415) = 0.00366...$ $h(0.3405) = -0.00130...$	M1	3.1a
	States: • there is a change of sign • $f'(x)$ is continuous • α = 0.341 to 3dp	A1	2.4
		(2)	
			(9 marks)
	Notes		

(a)

M1: Differentiates $\ln\left(2x^2-4x+5\right)$ to obtain $\frac{g(x)}{2x^2-4x+5}$ where g(x) could be 1

A1: For $f'(x)=2x+\frac{4x-4}{2x^2-4x+5}$

dM1: Sets their f′(x) $=ax+\frac{g(x)}{2x^2-4x+5}$ = 0 and uses "**correct**" algebra, condoning slips, to obtain a cubic equation. E.g Look for $ax\left(2x^2-4x+5\right)\pm g(x)=0$ o.e. , condoning slips, followed by some attempt to simplify

A1*: Achieves $2x^3-4x^2+7x-2=0$ with no errors. (The dM1 mark must have been awarded)

(b)(i)

M1: Attempts to use the iterative formula with $x_1=0.3$. If no method is shown award for $x_2=$ awrt 0.33

A1: $x_2=\text{awrt } 0.3294$ Note that $\frac{1153}{3500}$ is correct

Condone an incorrect suffix if it is clear that a correct value has been found

(b)(ii)

A1: $x_4=\text{awrt } 0.3398$ Condone an incorrect suffix if it is clear that a correct value has been found

(c)

M1: Attempts to substitute x = 0.3415 and x = 0.3405 into a suitable function and gets one value correct (rounded or truncated to 1 sf). It is allowable to use a tighter interval that contains the root 0.340762654

Examples of suitable functions are $2x^3-4x^2+7x-2$, $x-\frac{1}{7}\left(4x^2-2x^3+2\right)$ and f '(x) as this has been found in part (a) with f '(0.3405)= - 0.00067.., f '(0.3415)= (+) 0.0018

There must be sufficient evidence for the function, which would be for example, a statement such as $h(x)=2x^3-4x^2+7x-2$ or sight of embedded values that imply the function, not just a value or values even if both are correct. Condone $h(x)$ being mislabelled as f

$h(0.3415)=2\times0.3415^3-4\times0.3415^2+7\times0.3415-2$

A1: Requires

- both calculations correct (rounded or truncated to 1sf)
- a statement that there is a change in sign and that the function is continuous
- a minimal conclusion e.g. ✓, proven, α = 0.341, root

Question	Scheme	Marks	AOs

8(a)	$A = 1000$	B1	3.4
	$2000 = 1000e^{5k}$ or $e^{5k} = 2$	M1	1.1b
	$e^{5k} = 2 \Rightarrow 5k = \ln 2 \Rightarrow k = ...$	M1	2.1
	$N = 1000e^{\left(\frac{1}{5}\ln 2\right)t}$ or $N = 1000e^{0.139t}$	A1	3.3
		(4)	
(b)	$\frac{dN}{dt} = 1000 \times \left(\frac{1}{5}\ln 2\right)e^{\left(\frac{1}{5}\ln 2\right)t}$ or $\frac{dN}{dt} = 1000 \times 0.139e^{0.139t}$ $\left(\frac{dN}{dt}\right)_{t=8} = 1000 \times \left(\frac{1}{5}\ln 2\right)e^{8\times\frac{1}{5}\ln 2}$ or $\left(\frac{dN}{dt}\right)_{t=8} = 1000 \times 0.139e^{0.139\times 8}$	M1	3.1b
	= awrt 420	A1	1.1b
		(2)	
(c)	$500e^{1.4\times\left(\frac{1}{5}\ln 2\right)T} = 1000e^{\left(\frac{1}{5}\ln 2\right)T}$ or $500e^{1.4\times"0.139"t} = 1000e^{"0.139"t}$	M1	3.4
	Correct method of getting a linear equation in T E.g. $0.08T\ln 2 = \ln 2$ or $1.4\times"0.339"T = \ln 2 + "0.339"t$	M1	2.1
	$T = 12.5$ hours	A1	1.1b
		(3)	
			(9 marks)

Notes

Mark as one complete question. Marks in (a) can be awarded from (b)

(a)

B1: Correct value of A for the model. Award if equation for model is of the form $N = 1000e^{...t}$

M1: Uses the model to set up a correct equation in k. Award for substituting $N = 2000, t = 5$ following through on their value for A.

M1: Uses correct ln work to solve an equation of the form $ae^{5k} = b$ and obtain a value for k

A1: Correct equation of model. Condone an ambiguous $N = 1000e^{\frac{1}{5}\ln 2t}$ unless followed by something incorrect. Watch for $N = 1000 \times 2^{\frac{1}{5}t}$ which is also correct

(b)

M1: Differentiates αe^{kt} to βe^{kt} and substitutes $t = 8$ (Condone $\alpha = \beta$ so long as you can see an attempt to differentiate)

A1: For awrt 420 (2sf).

(c)

M1: Uses both models to set up an equation in T using their value for k, but also allow in terms of k

M1: Uses correct processing using lns to obtain a linear equation in T (or t)

A1: Awrt 12.5

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Answers to (b) and (c) appearing without working (i.e. from a calculator).

It is important that candidates show sufficient working to make their methods clear.

(b) If candidate has for example $N = 1000e^{0.139t}$, and then writes at $t = 8$ $\frac{dN}{dt} =$ awrt 420 award both marks. Just the answer from a correct model equation score SC 1,0.

(c) The first M1 should be seen E.g $500e^{1.4\times"0.139"t} = 1000e^{"0.139"t}$

If the answer $T = 12.5$ appears without any further working score SC M1 M1 A0

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Question	Scheme	Marks	AOs

10(a)	$T = al^b \Rightarrow \log_{10} T = \log_{10} a + \log_{10} l^b$	M1	2.1
	$\Rightarrow \log_{10} T = \log_{10} a + b\log_{10} l$ * or $\Rightarrow \log_{10} T = b\log_{10} l + \log_{10} a$ *	A1*	1.1b
		(2)	
(b)	$b = 0.495$ or $b = \frac{45}{91}$	B1	2.2a
	$0 = "0.495" \times -0.7 + \log_{10} a \Rightarrow a = 10^{0.346...}$ or $0.45 = "0.495" \times 0.21 + \log_{10} a \Rightarrow a = 10^{0.346...}$	M1	3.1a
	$T = 2.22l^{0.495}$	A1	3.3
		(3)	
(c)	The time taken for one swing of a pendulum of length 1 m	B1	3.2a
		(1)	
			(6 marks)

Notes

(a)

M1: Takes logs of both sides and shows the addition law.

Implied by $T = al^b \Rightarrow \log_{10} a + \log_{10} l^b$

A1*: Uses the power law to obtain the given equation with no errors. Allow the bases to be missing in the working but they must be present in the final answer.
Also allow *t* rather than *T* and *A* rather than *a*.

Allow working backwards e.g.

$$\log_{10} T = b\log_{10} l + \log_{10} a \Rightarrow \log_{10} T = \log_{10} l^b + \log_{10} a$$
$$\Rightarrow \log_{10} T = \log_{10} al^b \Rightarrow T = al^b *$$

M1: Uses the given answer and uses the power law and addition law correctly
A1: Reaches the given equation with no errors as above

(b)

B1: Deduces the correct value for *b* (Allow awrt 0.495 or $\frac{45}{91}$)

M1: Correct strategy to find the value of *a*.
E.g. substitutes one of the given points and their value for *b* into $\log_{10} T = \log_{10} a + b\log_{10} l$ and uses correct log work to identify the value of *a*. Allow slips in rearranging their equation but must be correct log work to find *a*.
Alternatively finds the equation of the straight line and equates the constant to $\log_{10} a$ and uses correct log work to identify the value of *a*.
E.g. $y - 0.45 = "0.495"(x - 0.21) \Rightarrow y = "0.495"x + 0.346 \Rightarrow a = 10^{0.346} = ...$

A1: Complete equation $T = 2.22l^{0.495}$ or $T = 2.22l^{\frac{45}{91}}$
(Allow awrt 2.22 and awrt 0.495 or $\frac{45}{91}$)

Must see the <u>equation</u> not just correct values as it is a requirement of the question.

(c)

B1: Correct interpretation

2	$4^{3p-1}=5^{210} \Rightarrow (3p-1)\log 4 = 210\log 5$	M1	1.1b
	$\Rightarrow 3p = \dfrac{210\log 5}{\log 4}+1 \Rightarrow p = ...$	dM1	2.1
	$p =$ awrt 81.6	A1	1.1b
		(3)	
			(3 marks)
Notes:			

M1: Takes logs of both sides and uses the power law on **each** side.
Condone a missing bracket on lhs and slips.
Award for any base including ln but the logs must be the same base.

dM1: A full method leading to a value for *p*.
It is dependent upon the previous M mark and there must be an attempt to change the subject of the equation in the correct order.

Look for $(3p-1)\log 4 = 210\log 5 \Rightarrow 3p = \dfrac{210\log 5}{\log 4} \pm 1 \Rightarrow p = ...$ condoning slips.

You may see numerical versions E.g. $(3p-1)\times 0.60 = 210\times 0.7 \Rightarrow 1.8p - 0.6 = 147 \Rightarrow p = 82$

Use of incorrect log laws would be dM0. E.g $(3p-1)\log 4 = 210\log 5 \Rightarrow 3p = 210\log\dfrac{5}{4} \pm 1$

A1: awrt 81.6 following a correct method. Bracketing errors can be recovered for full marks
A correct answer with no working scores 0 marks. The demand in the question is clear.

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There are alternatives:

E.g. A starting point could be $4^{3p-1}=5^{210} \Rightarrow \dfrac{4^{3p}}{4} = 5^{210}$

but the first M mark is still for using the power law correctly on each side

In such a method the dM1 mark is for using **all** log rules correctly and proceeding to a value for *p*.

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Using base 4 or 5

E.g. $4^{3p-1}=5^{210} \Rightarrow (3p-1) = \log_4 5^{210}$

The M mark is not scored until $(3p-1) = 210\log_4 5$

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Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
7 (a)	Uses a model $V = Ae^{\pm kt}$ oe (See next page for other suitable models)	M1	3.3
	Eg. Substitutes $t = 0, V = 20\,000 \Rightarrow A = 20\,000$	M1	1.1b
	Eg. Substitutes $t = 1, V = 16\,000 \Rightarrow 16\,000 = 20\,000e^{-1k} \Rightarrow k = ..$	dM1	3.1b
	$V = 20\,000e^{-0.223t}$	A1	1.1b
		(4)	
(b)	Substitutes $t = 10$ in their $V = 20\,000e^{-0.223t} \Rightarrow V = (£\,2150)$	M1	3.4
	Eg. The model is reliable as £2150 ≈ £2000	A1	3.5a
		(2)	
(c)	Make the "-0.223" less negative. Alt: Adapt model to for example $V = 18\,000e^{-0.223t} + 2000$	B1ft	3.3
		(1)	
			(7 marks)

(**a**) **Option 1**

M1: For $V = Ae^{\pm kt}$ Do not allow if k is fixed, eg $k = -0.5$

Condone different variables $V \leftrightarrow y$ $t \leftrightarrow x$ for this mark, but for A1 V and t must be used.

M1: Substitutes $t = 0 \Rightarrow A = 20\,000$ into their exponential model

Candidates may start by simply writing $V = 20\,000e^{kt}$ which would be M1 M1

dM1: Substitutes $t = 1 \Rightarrow 16\,000 = 20\,000e^{-1k} \Rightarrow k = ..$ via the correct use of logs.

It is dependent upon both previous M's.

A1: $V = 20\,000e^{-0.223t}$ (with accuracy to at least 3sf) or $V = 20\,000e^{t\ln 0.8}$

A correct linking formula with correct constants must be seen somewhere in the question

(b)

M1: Uses a model of the form $V = Ae^{\pm kt}$ to find the value of V when $t = 10$.

Alternatively substitutes $V = 2000$ into their model and finds t

A1: This can only be scored from an acceptable model with correct constants with accuracy to at least 2sf .

Compares V = (£) 2150 with (£) 2 000 and states "reliable as $2150 \approx 2000$ " or "reasonably good as they are close" or ""OK but a little high".

Allow a candidate to argue that it is unreliable as long as they state a suitable reason. Eg. ''It is too far away from £2000'' or ''It is over £100 away, so it is not good''

Do not allow ''it is not a good model because it is not the same''

In the alternative it is for comparing their value of t with 10 and making a suitable comment as to the reliability of their model with a reason.

$V = 20\,000e^{-0.223t} \Rightarrow 2000 = 20\,000e^{-0.223t} \Rightarrow t = 10.3$ years.

Deduction Reliable model as the time is approximately the same as 10 years. A candidate can argue that the model is unreliable if they can give a suitable reason.

(c)

B1ft: For a correct statement. Eg states that the value of their '-0.223' should become less negative.

Alt states that the value of their '0.223' should become smaller. If they refer to k then refer to the model and apply the same principles.

Condone the fact that they don't state their -0.223 doesn't lie in the range $(-0.223, 0)$

M1: For $V = Ar^t$ or equivalent such as $V = kr^{t-1}$

Condone different variables $V \leftrightarrow y$ $t \leftrightarrow x$ for this mark, but for A1 V and t must be used.

M1: Uses $t = 0 \Rightarrow A = 20\,000$ in their model. Alternatively uses $(0, 20000)$ and $(1, 16000)$ to give $r = \frac{4}{5}$ oe

You may award if one of the number pair $(0, 20000)$ or $(1, 16000)$ works in an allowable model

dM1: $t = 1 \Rightarrow 16\,000 = 20\,000r^1 \Rightarrow r = ..$ Dependent upon both previous M's

In the alternative it would be for using $r = \frac{4}{5}$ with one of the points to find $A = 20\,000$

You may award if both number pairs $(0, 20000)$ or $(1, 16000)$ work in an allowable model

A1: $V = 20000 \times 0.8^t$ Note that $V = 20000 \times 1.25^{-t}$ $V = 16000 \times 0.8^{t-1}$ and is also correct

(b)

M1: Uses a model of the form $V = Ar^t$ oe to find the value of V when $t = 10$. Eg. 20000×0.8^{10}

Alternatively substitutes $V = 2000$ into their model and finds t

A1: This can only be scored from an acceptable model with correct constants also allowing an accuracy to 2sf.

Compares $(£)2147$ with (£) 2 000 and states "reliable as $2147 \approx 2000$ " or "reasonably good as they are close" or ""OK but a little high".

Allow a candidate to argue that it is unreliable as long as they state a suitable reason. Eg. ''It is too far away from £2000'' or ''It is over £100 away, so it is not good''

Do not allow ''it is not a good model because it is not the same''

(c)

B1ft: States a value of r in the range $(0.8, 1)$ or states would increase the value of "0.8"

They do not need to state that "0.8" must lie in the range $(0.8, 1)$

Condone increase the 0.8. Also allow decrease the "1.25" for $V = 20000 \times 1.25^{-t}$

.....

(a) Option 3

M1: They may suggest an exponential model with a lower bound. For example, for $V = Ae^{\pm kt} + 2000$ The bound must be stated but do not allow k to be fixed . Allow as long as the bound < 10 000

M1: $t = 0, V = 20\,000 \Rightarrow A = 18\,000$

dM1: $t = 1, V = 16\,000 \Rightarrow 16\,000 = 2\,000 + 18\,000e^k \Rightarrow k = ..$ Dependent upon both previous M's

A1: $V = 18\,000 \times e^{-0.251t} + 2000$

(b)

M1: Uses their model to find the value of V when $t = 10$.

Alternatively substitutes $V = 2000$ into their model and finds t

A1: For $V = 18\,000 \times e^{-0.251 \times 10} + 2000 = £3462.83$ Deduction: Unreliable model as $£3462.83$ is not close to £2 000 This can only be scored from an acceptable model with correct constants

(c)

B1: States make the value of k or the -0.251 greater (or less negative) so that it lies in the range $(-0.251, 0)$

Condone 'make the value of k or the -0.251 greater (or less negative)'

.....

It is entirely possible that they start part (a) from a differential equation.

M1: $\frac{dV}{dt} = kV \Rightarrow \int \frac{dV}{V} = \int k dt \Rightarrow \ln V = kt + c$ M1: $\ln 20000 = c$

dM1: Using $t = 1, V = 16\,000 \Rightarrow k = ..$ A1: $\ln V = -\ln\left(\frac{5}{4}\right)t + \ln 20000$

Question	Scheme	Marks	AOs

13 (a)	(i) **Explains** $2x-q=0$ when $x=2$ oe Hence $q=4$ *	B1*	2.4
	(ii) Substitutes $\left(3,\frac{1}{2}\right)$ into $y=\frac{p-3x}{(2x-4)(x+3)}$ and solves	M1	1.1b
	$\frac{1}{2}=\frac{p-9}{(2)\times(6)}\Rightarrow p-9=6\Rightarrow p=15$*	A1*	2.1
		(3)	
(b)	Attempts to write $\frac{15-3x}{(2x-4)(x+3)}$ in PF's and integrates using lns between 3 and another value of x.	M1	3.1a
	$\frac{15-3x}{(2x-4)(x+3)}=\frac{A}{(2x-4)}+\frac{B}{(x+3)}$ leading to A and B	M1	1.1b
	$\frac{15-3x}{(2x-4)(x+3)}=\frac{1.8}{(2x-4)}-\frac{2.4}{(x+3)}$ or $\frac{0.9}{(x-2)}-\frac{2.4}{(x+3)}$ oe	A1	1.1b
	$I=\int\frac{15-3x}{(2x-4)(x+3)}dx=m\ln(2x-4)+n\ln(x+3)+(c)$	M1	1.1b
	$I=\int\frac{15-3x}{(2x-4)(x+3)}dx=0.9\ln(2x-4)-2.4\ln(x+3)$ oe	A1ft	1.1b
	Deduces that Area Either $\int_3^5\frac{15-3x}{(2x-4)(x+3)}dx$ Or $[.....]_3^5$	B1	2.2a
	Uses correct ln work seen at least once for $\ln 6=\ln 2+\ln 3$ **or** $\ln 8=3\ln 2$ $[0.9\ln(6)-2.4\ln(8)]-[0.9\ln(2)-2.4\ln(6)]$ $=3.3\ln 6-7.2\ln 2-0.9\ln 2$	dM1	2.1
	$=3.3\ln 3-4.8\ln 2$	A1	1.1b
		(8)	
			(11marks)

B1*: Is able to link $2x-q=0$ and $x=2$ to explain why $q=4$

Eg "The asymptote $x=2$ is where $2x-q=0$ so $4-q=0\Rightarrow q=4$"

"The curve is not defined when $2\times 2-q=0\Rightarrow q=4$"

There **must be some words** explaining why $q = 4$ and in most cases, you should see a reference to either "the asymptote $x=2$", "the curve is not defined at $x=2$", 'the denominator is 0 at $x=2$"

M1: Substitutes $\left(3,\frac{1}{2}\right)$ into $y=\frac{p-3x}{(2x-4)(x+3)}$ and solves

Alternatively substitutes $\left(3,\frac{1}{2}\right)$ into $y=\frac{15-3x}{(2x-4)(x+3)}$ and shows $\frac{1}{2}=\frac{6}{(2)\times(6)}$ oe

A1*: Full proof showing all necessary steps $\frac{1}{2}=\frac{p-9}{(2)\times(6)}\Rightarrow p-9=6\Rightarrow p=15$

In the alternative there would have to be some recognition that these are equal eg ✓ hence $p=15$

(b)

M1: Scored for an overall attempt at using PF's and integrating with lns seen with sight of limits 3 and another value of x.

M1: $\frac{15-3x}{(2x-4)(x+3)}=\frac{A}{(2x-4)}+\frac{B}{(x+3)}$ leading to A and B

A1: $\frac{15-3x}{(2x-4)(x+3)}=\frac{1.8}{(2x-4)}-\frac{2.4}{(x+3)}$, or for example $\frac{0.9}{(x-2)}-\frac{2.4}{(x+3)}$, $\frac{9}{(10x-20)}-\frac{12}{(5x+15)}$ oe

Must be written in PF form, not just for correct A and B

M1: Area $R=\int\frac{15-3x}{(2x-4)(x+3)}\mathrm{d}x=m\ln(2x-4)+n\ln(x+3)$

OR $\int\frac{15-3x}{(2x-4)(x+3)}\mathrm{d}x=m\ln(x-2)+n\ln(x+3)$

Note that $\int\frac{l}{(x-2)}\mathrm{d}x\to l\ln(kx-2k)$ and $\int\frac{m}{(x+3)}\mathrm{d}x\to m\ln(nx+3n)$

A1ft: $=\int\frac{15-3x}{(2x-4)(x+3)}\mathrm{d}x=0.9\ln(2x-4)-2.4\ln(x+3)$ oe. FT on their A and B

B1: Deduces that the limits for the integral are 3 and 5. It cannot just be awarded from 5 being marked on Figure 4. So award for sight of $\int_3^5\frac{15-3x}{(2x-4)(x+3)}(\mathrm{d}x)$ or $\left[.....\right]_3^5$ having performed an integral which may be incorrect

dM1: Uses correct ln work seen at least once eg $\ln 6=\ln 2+\ln 3$, $\ln 8=3\ln 2$ or $m\ln 6k-m\ln 2k=m\ln 3$

This is an attempt to get either of the above ln's in terms of ln2 and/or ln3

It is dependent upon the correct limits and having achieved $m\ln(2x-4)+n\ln(x+3)$ oe

A1: $=3.3\ln 3-4.8\ln 2$ oe

Question	Scheme	Marks	AOs

8 (i) Way 1	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = 20\left(\frac{1}{2}\right)^4 + 20\left(\frac{1}{2}\right)^5 + 20\left(\frac{1}{2}\right)^6 + ...$		
	$= \frac{20(\frac{1}{2})^4}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= (1.25)(2)\} = 2.5$ o.e.	A1	1.1b
		(3)	
(i) Way 2	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = \sum_{r=1}^{\infty} 20\times\left(\frac{1}{2}\right)^r - \sum_{r=1}^{3} 20\times\left(\frac{1}{2}\right)^r$		
	$= \frac{10}{1-\frac{1}{2}} - (10+5+2.5)$ or $= \frac{10}{1-\frac{1}{2}} - \frac{10(1-(\frac{1}{2})^3)}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= 20-17.5\} = 2.5$ o.e.	A1	1.1b
		(3)	
(i) Way 3	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = \sum_{r=0}^{\infty} 20\times\left(\frac{1}{2}\right)^r - \sum_{r=0}^{3} 20\times\left(\frac{1}{2}\right)^r$		
	$= \frac{20}{1-\frac{1}{2}} - (20+10+5+2.5)$ or $= \frac{20}{1-\frac{1}{2}} - \frac{20(1-(\frac{1}{2})^4)}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= 40-37.5\} = 2.5$ o.e.	A1	1.1b
		(3)	
(ii) Way 1	$\left\{\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) =\right\}$		
	$= \log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + ... + \log_5\left(\frac{50}{49}\right) = \log_5\left(\frac{3}{2}\times\frac{4}{3}\times...\times\frac{50}{49}\right)$	M1	1.1b
		M1	3.1a
	$= \log_5\left(\frac{50}{2}\right)$ or $\log_5(25) = 2$ *	A1*	2.1
		(3)	
(ii) Way 2	$\left\{\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) =\right\} \sum_{n=1}^{48} (\log_5(n+2) - \log_5(n+1))$	M1	1.1b
	$= (\log_5 3 + \log_5 4 + ... + \log_5 50) - (\log_5 2 + \log_5 3 + ... + \log_5 49)$	M1	3.1a
	$= \log_5 50 - \log_5 2$ or $\log_5\left(\frac{50}{2}\right)$ or $\log_5(25) = 2$*	A1*	2.1
		(3)	
			(6 marks)

Notes for Question 8	
(i)	**Way 1**
M1:	Applies $\frac{a}{1-r}$ for their r (where $-1 <$ their $r < 1$) and their value for a
M1:	Finds the infinite sum by using a complete strategy of applying $\frac{20(\frac{1}{2})^4}{1-\frac{1}{2}}$
A1:	2.5 o.e.
(i)	**Way 2**
M1:	Applies $\frac{a}{1-r}$ for their r (where $-1 <$ their $r < 1$) and their value for a
M1:	Finds the infinite sum by using a completely correct strategy of applying $\frac{10}{1-\frac{1}{2}} - (10 + 5 + 2.5)$ or $\frac{10}{1-\frac{1}{2}} - \frac{10(1-(\frac{1}{2})^3)}{1-\frac{1}{2}}$
A1:	2.5 o.e.
(i)	**Way 3**
M1:	Applies $\frac{a}{1-r}$ for their r (where $-1 <$ their $r < 1$) and their value for a
M1:	Finds the infinite sum by using a completely correct strategy of applying $\frac{20}{1-\frac{1}{2}} - (20+10 + 5 + 2.5)$ or $\frac{20}{1-\frac{1}{2}} - \frac{20(1-(\frac{1}{2})^4)}{1-\frac{1}{2}}$
A1:	2.5 o.e.
Note:	Give M1 M1 A1 for a correct answer of 2.5 from no working in (i)
(ii)	**Way 1**
M1:	Some evidence of applying the addition law of logarithms as part of a valid proof
M1:	Begins to solve the problem by just writing (or by combining) at least three terms including • **either** the first two terms and the last term • **or** the first term and the last two terms
Note:	The 2nd mark can be gained by writing any of • listing $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{4}{3}\right), \log_5\left(\frac{50}{49}\right)$ or $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{49}{48}\right), \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + ... + \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2}\right) + ... + \log_5\left(\frac{49}{48}\right) + \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2} \times \frac{4}{3} \times ... \times \frac{50}{49}\right)$ ***{this will also gain the 1st M1 mark}*** • $\log_5\left(\frac{3}{2} \times ... \times \frac{49}{48} \times \frac{50}{49}\right)$ ***{this will also gain the 1st M1 mark}***
A1*:	Correct proof leading to a correct answer of 2
Note:	Do not allow the 2nd M1 if $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{4}{3}\right)$ are listed and $\log_5\left(\frac{50}{49}\right)$ is used for the first time in their applying the formula $S_{48} = \frac{48}{2}\left(\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{50}{49}\right)\right)$
Note:	**<u>Listing all 48 terms</u>** Give M0 M1 A0 for $\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + \log_5\left(\frac{5}{4}\right) + ... + \log_5\left(\frac{50}{49}\right) = 2$ {lists all terms} Give M0 M0 A0 for $0.2519... + 0.1787... + 0.1386... + ... + 0.0125... = 2$ {all terms in decimals}

Notes for Question 8	
(ii)	**Way 2**
M1:	Uses the subtraction law of logarithms to give $\log_5\left(\frac{n+2}{n+1}\right)\rightarrow\log_5(n+2)-\log_5(n+1)$
M1:	Begins to solve the problem by writing at least three terms for each of $\log_5(n+2)$ and $\log_5(n+1)$ including • **either** the first two terms and the last term for both $\log_5(n+2)$ and $\log_5(n+1)$ • **or** the first term and the last two terms for both $\log_5(n+2)$ and $\log_5(n+1)$
Note:	This mark can be gained by writing any of • $(\log_5 3+\log_5 4+.....+\log_5 50)-(\log_5 2+\log_5 3+.....+\log_5 49)$ • $(\log_5 3+.....+\log_5 49+\log_5 50)-(\log_5 2+.....+\log_5 48+\log_5 49)$ • $(\log_5 3+\log_5 4+.....+\log_5 50)-(\log_5 2+\log_5 3+.....+\log_5 49)$ • $(\log_5 3-\log_5 2)+(\log_5 4-\log_5 3)+.....+(\log_5 50-\log_5 49)$ • $\log_5 3-\log_5 2,\ .....,\log_5 49-\log_5 48,\ \log_5 50-\log_5 49$
A1*:	Correct proof leading to a correct answer of 2
Note:	The base of 5 can be omitted for the M marks in part (ii), but the base of 5 must be included in the final line (as shown on the mark scheme) of their solution.
Note:	If a student uses a mixture of a Way 1 or Way 2 method, then award the best Way 1 mark only or the best Way 2 mark only.
Note:	Give M1 M0 A0 (1st M for implied use of subtraction law of logarithms) for $\sum_{n=1}^{48}\log_5\left(\frac{n+2}{n+1}\right)=91.8237...-89.8237...=2$
Note:	Give M1 M1 A1 for $\sum_{n=1}^{48}\log_5\left(\frac{n+2}{n+1}\right)=\sum_{n=1}^{48}(\log_5(n+2)-\log_5(n+1))$ $=\log_5(3\times4\times.....\times50)-\log_5(2\times3\times.....\times49)$ $=\log_5\left(\frac{50!}{2}\right)-\log_5(49!)$ or $=\log_5(25\times49!)-\log_5(49!)$ $=\log_5 25=2$

Question	Scheme	Marks	AOs
	$\{d = kV^n \Rightarrow\} \log_{10} d = \log_{10} k + n \log_{10} V$		

9 (a) Way 1	$\{d = kV^n \Rightarrow\}\ \log_{10} d = \log_{10} k + n\log_{10} V$ or $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ seen or used as part of their argument	M1	2.1
	Alludes to $d = kV^n$ and gives a full explanation by comparing their result with a linear model e.g. $Y = MX + C$	A1	2.4
	$\{k =\}\ 10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
9 (a) Way 2	$\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ or $\log_{10} d = \log_{10} k + n\log_{10} V$ seen or used as part of their argument	M1	2.1
	$\{d = kV^n \Rightarrow\} \log_{10} d = \log_{10}(kV^n)$ $\Rightarrow \log_{10} d = \log_{10} k + \log_{10} V^n \Rightarrow \log_{10} d = \log_{10} k + n\log_{10} V$	A1	2.4
	$\{k =\}\ 10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
(a) Way 3	Starts from $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$	M1	2.1
	$\log_{10} d = m\log_{10} V + c \Rightarrow d = 10^{m\log_{10} V + c} \Rightarrow d = 10^c V^m \Rightarrow d = kV^n$ **or** $\log_{10} d = m\log_{10} V - 1.77 \Rightarrow d = 10^{m\log_{10} V - 1.77}$ $\Rightarrow d = 10^{-1.77} V^m \Rightarrow d = kV^n$	A1	2.4
	$\{k =\}\ 10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
(b)	$\{d = 20, V = 30 \Rightarrow\}\quad 20 = k(30)^n$ or $\log_{10} 20 = \log_{10} k + n\log_{10} 30$	M1	3.4
	$20 = k(30)^n \Rightarrow \log 20 = \log k + n\log 30 \Rightarrow n = \dfrac{\log 20 - \log k}{\log 30} \Rightarrow n = ...$	M1	1.1b
	$\log_{10} 20 = \log_{10} k + n\log_{10} 30 \Rightarrow n = \dfrac{\log_{10} 20 - \log_{10} k}{\log_{10} 30} \Rightarrow n = ...$		
	$\{n = $ awrt $2.08 \Rightarrow\}\ d = (0.017)V^{2.08}$ **or** $\log_{10} d = -1.77 + 2.08\log_{10} V$	A1	1.1b
	Note: You can recover the A1 mark for a correct model equation given in part (c)	**(3)**	
(c)	$d = (0.017)(60)^{2.08}$	M1	3.4
	• $13.333... + 84.918... = 98.251... \Rightarrow$ Sean stops in time	M1	3.1b
	• $100 - 13.333... = 86.666...$ & $d = 84.918 \Rightarrow$ Sean stops in time	A1ft	3.2a
		(3)	
			(9 marks)
ADVICE: Ignore labelling (a), (b), (c) when marking this question **Note:** Give B0 in (a) for $10^{-1.77} = 0.01698...$ without reference to 0.017 in the same part			

Notes for Question 9	
Note:	In their solution to (a) and/or (b) condone writing log in place of $\log_{10}$
(a)	**Way 1**
M1:	See scheme
A1:	See scheme
B1*:	See scheme
(a)	**Way 2**
M1:	See scheme
A1:	Starts from $d = kV^n$ (which they do not have to state) and progresses to $\log_{10} d = \log_{10} k + n\log_{10} V$ with an intermediate step in their working.
B1*:	See scheme
(a)	**Way 3**
M1:	Starts their argument from $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$
A1:	Mathematical explanation is seen by showing any of either • $\log_{10} d = m\log_{10} V + c \rightarrow d = 10^c V^m$ or $d = kV^n$ • $\log_{10} d = m\log_{10} V - 1.77 \rightarrow d = 10^{-1.77} V^m$ or $d = kV^n$ with no errors seen in their working
B1*:	See scheme
Note:	Allow B1 for $\log_{10} 0.017 = -1.77$ or $\log 0.017 = -1.77$
(b)	
M1:	Applies $V = 30$ and $d = 20$ to their model **(correct way round)**
M1:	Applies $(V, d) = (30, 20)$ or $(20, 30)$ and applies logarithms correctly leading to $n = ...$
A1:	$d = (0.017)V^{2.08}$ or $\log_{10} d = -1.77 + 2.08\log_{10} V$ or $\log_{10} d = \log_{10}(0.017) + 2.08\log_{10} V$
Note:	Allow $k =$ awrt 0.017 and/or $n =$ awrt 2.08 in their final model equation
Note:	M0 M1 A0 is a possible score for (b)
(c)	
M1:	Applies $V = 60$ to their exponential model or their logarithmic model
M1:	Uses their model in a correct problem-solving process of either • adding a "thinking distance" to their value of their *d* to find an overall stopping distance • applying 100 – "thinking distance" and finds their value of *d*
Note:	$\frac{1}{75}$ or 48 are examples of acceptable thinking distances
A1ft:	**Either** adds 13.3… to their *d* to find a total stopping distance and gives a correct ft conclusion **or** finds their *d* and a comparative 86.666…(m) or awrt 87 (m) and gives a correct ft conclusion
Note:	The thinking distance must be dimensionally correct for the M1 mark. i.e. $0.8\times$ their velocity
Note:	A thinking distance of awrt 13 and a value of *d* in the range [81.5, 88.5] are required for A1ft
Note:	Allow "Sean stops in time" or "Yes he stops in time" or "he misses the puddle" as relevant conclusions.
Note:	A mark of M0 M1 A0 is possible in (c)

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
4 (a)	Attempts $f(3) =$ and $f(4) =$ where $f(x) = \pm(2\ln(8-x)-x)$	M1	2.1
	$f(3) = (2\ln(5)-x) = (+)0.22$ and $f(4) = (2\ln(4)-4) = -1.23$ Change of sign and function continuous in interval $[3,4] \Rightarrow$ Root *	A1*	2.4
		(2)	
(b)	For annotating the graph by drawing a cobweb diagram starting at $x_1 = 4$ It should have at least two spirals	M1	2.4
	Deduces that the iteration formula **can be used** to find an approximation for α because **the cobweb spirals inwards** for the cobweb diagram	A1	2.2a
		(2)	
			(4 marks)

Notes:

(a)

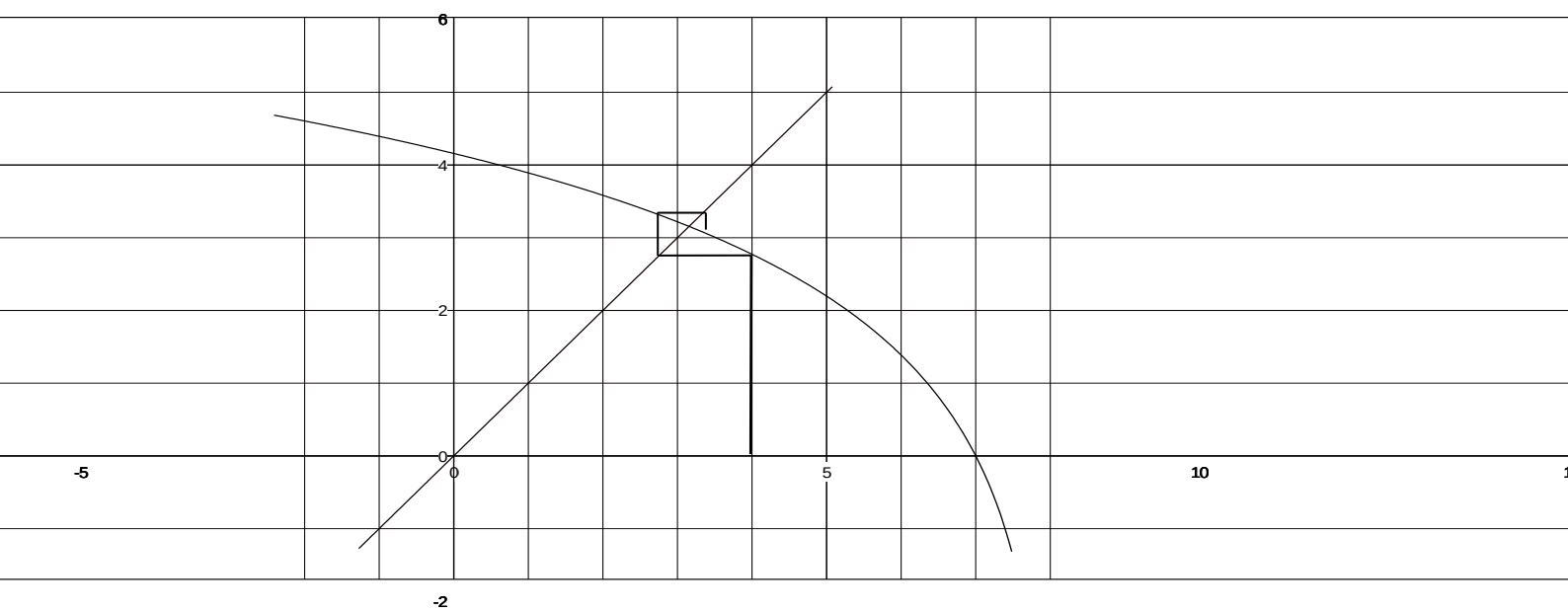
M1: Attempts $f(3) =$ and $f(4) =$ where $f(x) = \pm(2\ln(8-x)-x)$ or alternatively **compares** $2\ln 5$ to 3 and $2\ln 4$ to 4. This is not routine and cannot be scored by substituting 3 and 4 in both functions

A1: Both values (calculations) correct to at least 1 sf with correct explanation and conclusion. (See underlined statements)
When comparing terms, allow reasons to be $2ln8 = 3.21 > 3$, $2\ln 4 = 2.77 < 4$ or similar

(b)

M1: For an attempt at using a cobweb diagram. Look for 5 or more correct straight lines. It may not start at 4 but it must show an understanding of the method. **If there is no graph then it is M0 A0**

A1: For a correct attempt starting at 4 and deducing that the iteration **can be used** as the iterations **converge to the root**. You must statement that it can be used with a suitable reason. Suitable reasons could be " it spirals inwards", it gets closer to the root", it converges "



Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
12 (a)	(i) Method to find p Eg. Divides $32000 = Ap^4$ by $50000 = Ap^{11}$ $p^7 = \frac{50000}{32000} \Rightarrow p = \sqrt[7]{\frac{50000}{32000}} = ...$	M1	3.1a
	$p = 1.0658$	A1	1.1b
	(ii) Substitutes their $p = 1.0658$ into either equation and finds A $A = \frac{32000}{'1.0658'^4}$ or $A = \frac{50000}{'1.0658'^{11}}$	M1	1.1b
	$A = 24795 \rightarrow 24805 \simeq 24\,800$ *	A1*	1.1b
		(4)	
(b)	A / $(£)24\,800$ is the value of the car on 1st January 2001	B1	3.4
	p / 1.0658 is the factor by which the value rises each year. Accept that the value rises by 6.6 % a year (ft on their p)	B1	3.4
		(2)	
(c)	Attempts $100000 = '24800' \times '1.0658'^t$		
	$'1.0658'^t = \frac{100000}{24800}$	M1	3.4
	$t = \log_{1.0658}\left(\frac{100000}{24800}\right)$	dM1	1.1b
	$t = 21.8$ or 21.9	A1	1.1b
	cso 2022	A1	3.2a
		(4)	
			(10 marks)

(a) (i)

M1: Attempts to use both pieces of information within $V = Ap^t$, eliminates A correctly and solves an equation of the form $p^n = k$ to reach a value for p.

Allow for slips on the 32 000 and 50 000 and the values of t.

A1: $p =$ awrt 1.0658

Both marks can be awarded from incorrect but consistent interpretations of t. Eg.

$32000 = Ap^5$, $50000 = Ap^{12}$

(a)(ii)

M1: Substitutes their $p = 1.0658$ into either of their equations and finds A

Eg $A = \frac{32000}{1.0658^4}$ or $A = \frac{50000}{1.0658^7}$ but you may follow through on incorrect equations from part (i)

A1*: Shows that A is between 24 795 and 24 805 before you see ' =24 800' or ' ≈ 24800'. Accept with or without units.

An alternative to (ii) is to start with the given answer.

M1: Attempts $24800 \times '1.0658'^4 = (32000.34)$

$\approx 24\,800$

(b)

B1: States that A is the value of the car on 1st January 2001.
The statement must reference **the car**, its **cost/value**, and **"0" time**
Allow 'it is the initial value of the car" "it is the cost of the car at $t = 0$" "it is the cars starting value"

B1: States that p is the rate at which the value of the car rises each year.
The statement must reference **a yearly rate** and **an increase in value or multiplier**.
They could reference the 1.0658 Eg "The cars value rises by 6.5 % each year."
Allow "p is the rate the cars value is rising each year" "it is the proportional increase in value of the car each year" "the factor by which the value of the car is rising each year" 'its value appreciates by 6.5% per year' Allow ' the value of the car multiplies by p each year'
Do not allow "by how much the value of the car rises each year " or "it is the rate of inflation"

(c)

M1: Uses the model $100000 = \text{'}24800\text{'} \times \text{'}1.0658\text{'}^t$ and proceeds to their $\text{'}1.0658\text{'}^t = k$
Allow use of any inequality here.

dM1: For the complete method of (i) using the information given with their equation of the model **and** (ii) translating the situation into a correct method to find 't'

A1: $(t) =$ awrt 21.8 or 21.9 or $\log_{1.0658}\left(\frac{100000}{24800}\right)$ oe

A1: States in the year 2022. A candidate using a GP formula can be awarded full marks

Allow different methods in part (c).

Eg Via GP a formula

M1: $24800 \times \text{'}1.0658\text{'}^{n-1} = 100000 \Rightarrow \text{'}1.0658\text{'}^{n-1} = K$

dM1: Uses a correct method to find n.

A2: 2022

Via (trial and improvement)

M1: Uses the model by substituting integer values of t into their $V = Ap^t$ so that for $t = n, V < 100\,000$ or $t = n+1, V > 100\,000$

(So for the correct A and p this would be scored for $t = 21, V \approx £95000$ or $t = 21, V \approx £101\,000$

dM1: For a complete method showing that this is the least value. So both of the above values

A1: Allow for 22 following correct and accurate results (awrt nearest £1000 is sufficient accuracy)

A1: As before

13	$C: y = x\ln x$; l is a normal to C at $P(\mathrm{e}, \mathrm{e})$ Let x_A be the x-coordinate of where l cuts the x-axis		
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \ln x + x\left(\dfrac{1}{x}\right) \quad \{= 1 + \ln x\}$	M1 A1	2.1 1.1b
	$x = \mathrm{e},\ m_T = 2 \Rightarrow m_N = -\dfrac{1}{2} \Rightarrow y - \mathrm{e} = -\dfrac{1}{2}(x - \mathrm{e})$ $y = 0 \Rightarrow -\mathrm{e} = -\dfrac{1}{2}(x - \mathrm{e}) \Rightarrow x = \ldots$	M1	3.1a
	l meets x-axis at $x = 3\mathrm{e}$ (allow $x = 2\mathrm{e} + \mathrm{e}\ln\mathrm{e}$)	A1	1.1b
	{Areas:} **either** $\displaystyle\int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = \left[\ \ldots\ \right]_1^{\mathrm{e}} = \ldots$ **or** $\dfrac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$	M1	2.1
	$\left\{\displaystyle\int x\ln x\,\mathrm{d}x = \right\}\ \dfrac{1}{2}x^2\ln x - \int \dfrac{1}{x}.\left(\dfrac{x^2}{2}\right)\{\mathrm{d}x\}$	M1	2.1
	$\left\{= \dfrac{1}{2}x^2\ln x - \displaystyle\int \dfrac{1}{2}x\,\{\mathrm{d}x\}\right\} = \dfrac{1}{2}x^2\ln x - \dfrac{1}{4}x^2$	dM1 A1	1.1b 1.1b
	$\text{Area}(R_1) = \displaystyle\int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = \left[\ \ldots\ \right]_1^{\mathrm{e}} = \ldots$; $\text{Area}(R_2) = \dfrac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$ and so, $\text{Area}(R) = \text{Area}(R_1) + \text{Area}(R_2) \quad \{= \frac{1}{4}\mathrm{e}^2 + \frac{1}{4} + \mathrm{e}^2\}$	M1	3.1a
	$\text{Area}(R) = \frac{5}{4}\mathrm{e}^2 + \frac{1}{4}$	A1	1.1b
		(10)	

Notes for Question 13

M1:	Differentiates by using the product rule to give $\ln x + x(\text{their } \mathrm{g}'(x))$, where $\mathrm{g}(x) = \ln x$
A1:	Correct differentiation of $y = x\ln x$, which can be un-simplified or simplified
M1:	Complete strategy to find the x coordinate where their normal to C at $P(\mathrm{e}, \mathrm{e})$ meets the x-axis i.e. Sets $y = 0$ in $y - \mathrm{e} = m_N(x - \mathrm{e})$ to find $x = \ldots$
Note:	m_T is found by using calculus and $m_N \neq m_T$
A1:	l meets x-axis at $x = 3\mathrm{e}$, allowing un-simplified values for x such as $x = 2\mathrm{e} + \mathrm{e}\ln\mathrm{e}$
Note:	Allow $x =$ awrt 8.15
M1:	Scored for either • Area under curve $= \displaystyle\int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = \left[\ \ldots\ \right]_1^{\mathrm{e}} = \ldots$, with limits of e and 1 and some attempt to substitute these and subtract • **or** Area under line $= \dfrac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$, with a valid attempt to find x_A
M1:	Integration by parts the correct way around to give $Ax^2\ln x - \displaystyle\int B\left(\dfrac{x^2}{x}\right)\{\mathrm{d}x\}$; $A \neq 0,\ B > 0$
dM1:	**dependent on the previous M mark** Integrates the second term to give $\pm\lambda x^2$; $\lambda \neq 0$
A1:	$\dfrac{1}{2}x^2\ln x - \dfrac{1}{4}x^2$
M1:	Complete strategy of finding the area of R by finding the sum of two key areas. See scheme.
A1:	$\frac{5}{4}\mathrm{e}^2 + \frac{1}{4}$

Note:	$\text{Area}(R_2)$ can also be found by integrating the line *l* between limits of e and their x_A i.e. $\text{Area}(R_2) = \int_{\text{e}}^{\text{their } x_A} \left(-\frac{1}{2}x + \frac{3}{2}\text{e}\right) \text{d}x = \left[\ ... \ \right]_{\text{e}}^{\text{their } x_A} = ...$
Note:	**<u>Calculator approach with no algebra, differentiation or integration seen:</u>** • Finding *l* cuts through the *x*-axis at awrt 8.15 is 2nd M1 2nd A1 • Finding area between curve and the *x*-axis between $x = 1$ and $x = \text{e}$ to give awrt 2.10 is 3rd M1 • Using the above information (must be seen) to apply $\text{Area}(R) = 2.0972... + 7.3890... = 9.4862...$ is final M1 Therefore, a maximum of 4 marks out of the 10 available.

Question	Scheme	Marks	AOs
	$\int \frac{900}{N(300-N)}\,\mathrm{d}N = \int \frac{\mathrm{d}N}{N(300-N)}$		

14	$N=\dfrac{900}{3+7\mathrm{e}^{-0.25t}}=900(3+7\mathrm{e}^{-0.25t})^{-1},\ t\in\mathbb{R},\ t\geqslant 0\ ;\quad \dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{N(300-N)}{1200}$		
(a)	90	B1	3.4
		(1)	
(b) Way 1	$\dfrac{\mathrm{d}N}{\mathrm{d}t}=-900(3+7\mathrm{e}^{-0.25t})^{-2}\left(7(-0.25)\mathrm{e}^{-0.25t}\right)\quad\left\{=\dfrac{900(0.25)(7)\mathrm{e}^{-0.25t}}{(3+7\mathrm{e}^{-0.25t})^2}\right\}$	M1	2.1
		A1	1.1b
	$\Rightarrow \dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{900(0.25)\left(\left(\dfrac{900}{N}-3\right)\right)}{\left(\dfrac{900}{N}\right)^2}$	dM1	2.1
	correct algebra leading to $\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{N(300-N)}{1200}$ *	A1*	1.1b
		(4)	
(b) Way 2	$\dfrac{\mathrm{d}N}{\mathrm{d}t}=-900(3+7\mathrm{e}^{-0.25t})^{-2}\left(7(-0.25)\mathrm{e}^{-0.25t}\right)\quad\left\{=\dfrac{900(0.25)(7)\mathrm{e}^{-0.25t}}{(3+7\mathrm{e}^{-0.25t})^2}\right\}$	M1	2.1
		A1	1.1b
	$\dfrac{N(300-N)}{1200}=\dfrac{\left(\dfrac{900}{3+7\mathrm{e}^{-0.25t}}\right)\left(300-\dfrac{900}{3+7\mathrm{e}^{-0.25t}}\right)}{1200}$	dM1	2.1
	$\text{LHS}=\dfrac{1575\mathrm{e}^{-0..25t}}{(3+7\mathrm{e}^{-0.25t})^2}$ o.e., $\text{RHS}=\dfrac{900(300(3+7\mathrm{e}^{-0.25t})-900)}{1200(3+7\mathrm{e}^{-0.25t})^2}=\dfrac{1575\mathrm{e}^{-0..25t}}{(3+7\mathrm{e}^{-0.25t})^2}$ o.e. and states hence $\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{N(300-N)}{1200}$ (or LHS = RHS) *	A1*	1.1b
		(4)	
(c)	Deduces $N=150$ (can be implied)	B1	2.2a
	so $150=\dfrac{900}{3+7\mathrm{e}^{-0.25T}}\Rightarrow \mathrm{e}^{-0.25T}=\dfrac{3}{7}$	M1	3.4
	$T=-4\ln\left(\dfrac{3}{7}\right)$ or $T=$ awrt 3.4 (months)	dM1	1.1b
		A1	1.1b
		(4)	
(d)	either one of 299 or 300	B1	3.4
		(1)	
			(10 marks)

Notes for Question 14	
14 (b)	
M1:	Attempts to differentiate using • the chain rule to give $\frac{\mathrm{d}N}{\mathrm{d}t} = \pm A\mathrm{e}^{-0.25t}(3+7\mathrm{e}^{-0.25t})^{-2}$ or $\frac{\pm A\mathrm{e}^{-0.25t}}{(3+7\mathrm{e}^{-0.25t})^2}$ o.e. • the quotient rule to give $\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{(3+7\mathrm{e}^{-0.25t})(0)\pm A\mathrm{e}^{-0.25t}}{(3+7\mathrm{e}^{-0.25t})^2}$ • implicit differentiation to give $N(3+7\mathrm{e}^{-0.25t}) = 900 \Rightarrow (3+7\mathrm{e}^{-0.25t})\frac{\mathrm{d}N}{\mathrm{d}t} \pm AN\mathrm{e}^{-0.25t} = 0$, o.e. where $A \neq 0$
Note:	Condone a slip in copying $(3+7\mathrm{e}^{-0.25t})$ for the M mark
A1:	A correct differentiation statement
Note:	Implicit differentiation gives $(3+7\mathrm{e}^{-0.25t})\frac{\mathrm{d}N}{\mathrm{d}t} - 1.75N\mathrm{e}^{-0.25t} = 0$
dM1:	**Way 1:** Complete attempt, by eliminating *t*, to form an equation linking $\frac{\mathrm{d}N}{\mathrm{d}t}$ and N only **Way 2:** Complete substitution of $N = \frac{900}{3+7\mathrm{e}^{-0.25t}}$ into $\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{N(300-N)}{1200}$
Note:	**Way 1:** e.g. substitutes $3+7\mathrm{e}^{-0.25t} = \frac{900}{N}$ and $\mathrm{e}^{-0.25t} = \frac{900}{N}$ or substitutes $\mathrm{e}^{-0.25t} = \frac{\frac{900}{N}-3}{7}$ into their $\frac{\mathrm{d}N}{\mathrm{d}t} = ...$ to form an equation linking $\frac{\mathrm{d}N}{\mathrm{d}t}$ and N
A1*:	**Way 1:** Correct algebra leading to $\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{N(300-N)}{1200}$ * **Way 2:** See scheme
(c)	
B1:	Deduces or shows that $\frac{\mathrm{d}N}{\mathrm{d}t}$ is maximised when $N = 150$
M1:	Uses the model $N = \frac{900}{3+7\mathrm{e}^{-0.25t}}$ with their $N = 150$ and proceeds as far as $\mathrm{e}^{-0.25T} = k$, $k > 0$ or $\mathrm{e}^{0.25T} = k$, $k > 0$. Condone $t \equiv T$
dM1:	Correct method of using logarithms to find a value for T. Condone $t \equiv T$
A1:	see scheme
Note:	$\frac{\mathrm{d}^2N}{\mathrm{d}t^2} = \frac{\mathrm{d}N}{\mathrm{d}t}\left(\frac{300}{1200} - \frac{2N}{1200}\right) = 0 \Rightarrow N = 150$ is acceptable for B1
Note:	Ignore units for T
Note:	Applying $300 = \frac{900}{3+7\mathrm{e}^{-0.25t}} \Rightarrow t = ...$ or $0 = \frac{900}{3+7\mathrm{e}^{-0.25t}} \Rightarrow t = ...$ is M0 dM0 A0
Note:	M1 dM1 can only be gained in (c) by using an N value in the range $90 < N < 300$
(d)	
B1:	300 (or accept 299)

Question	Scheme	Marks	AOs
14	$N=\dfrac{900}{3+7\mathrm{e}^{-0.25t}}=900(3+7\mathrm{e}^{-0.25t})^{-1},\ t\in\mathbb{R},\ t\geqslant 0\ ;\quad \dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{N(300-N)}{1200}$		
(b) Way 3	$\displaystyle\int\frac{1}{N(300-N)}\,\mathrm{d}N=\int\frac{1}{1200}\,\mathrm{d}t$ $\displaystyle\int\frac{1}{300}\left(\frac{1}{N}+\frac{1}{300-N}\right)\mathrm{d}N=\int\frac{1}{1200}\,\mathrm{d}t$ $\dfrac{1}{300}\ln N-\dfrac{1}{300}\ln(300-N)=\dfrac{1}{1200}t\ \{+c\}$	M1 A1	2.1 1.1b
	$\{t=0,\ N=90\Rightarrow\}\ \ c=\dfrac{1}{300}\ln(90)-\dfrac{1}{300}\ln(210)\ \Rightarrow c=\dfrac{1}{300}\ln\left(\dfrac{3}{7}\right)$ $\dfrac{1}{300}\ln N-\dfrac{1}{300}\ln(300-N)=\dfrac{1}{1200}t+\dfrac{1}{300}\ln\left(\dfrac{3}{7}\right)$ $\ln N-\ln(300-N)=\dfrac{1}{4}t+\ln\left(\dfrac{3}{7}\right)$ $\ln\left(\dfrac{N}{300-N}\right)=\dfrac{1}{4}t+\ln\left(\dfrac{3}{7}\right)\ \Rightarrow\ \dfrac{N}{300-N}=\dfrac{3}{7}\mathrm{e}^{\frac{1}{4}t}$	dM1	2.1
	$7N=3\mathrm{e}^{\frac{1}{4}t}(300-N)\ \Rightarrow 7N+3N\mathrm{e}^{\frac{1}{4}t}=900\mathrm{e}^{\frac{1}{4}t}$ $N(7+3\mathrm{e}^{\frac{1}{4}t})=900\mathrm{e}^{\frac{1}{4}t}\ \Rightarrow\ N=\dfrac{900\mathrm{e}^{\frac{1}{4}t}}{7+3\mathrm{e}^{\frac{1}{4}t}}\ \Rightarrow\ N=\dfrac{900}{3+7\mathrm{e}^{-0.25t}}$ *	A1*	1.1b
		(4)	
(b) Way 4	$N(3+7\mathrm{e}^{-0.25t})=900\ \Rightarrow\mathrm{e}^{-0.25t}=\dfrac{1}{7}\left(\dfrac{900}{N}-3\right)\Rightarrow\mathrm{e}^{-0.25t}=\dfrac{900-3N}{7N}$ $\Rightarrow t=-4\big(\ln(900-3N)-\ln(7N)\big)$ $\Rightarrow\dfrac{\mathrm{d}t}{\mathrm{d}N}=-4\left(\dfrac{-3}{900-3N}-\dfrac{7}{7N}\right)$	M1 A1	2.1 1.1b
	$\dfrac{\mathrm{d}t}{\mathrm{d}N}=4\left(\dfrac{1}{300-N}+\dfrac{1}{N}\right)\Rightarrow\dfrac{\mathrm{d}t}{\mathrm{d}N}=4\left(\dfrac{N+300-N}{N(300-N)}\right)$	dM1	2.1
	$\dfrac{\mathrm{d}t}{\mathrm{d}N}=\left(\dfrac{1200}{N(300-N)}\right)\Rightarrow\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{N(300-N)}{1200}$ *	A1*	1.1b
		(4)	

(b) Way 3	
M1:	Separates the variables, an attempt to form and apply partial fractions and integrates to give ln terms $= kt$ $\{+c\}$, $k \neq 0$, with or without a constant of integration c
A1:	$\frac{1}{300}\ln N - \frac{1}{300}\ln(300-N) = \frac{1}{1200}t$ $\{+c\}$ or equivalent with or without a constant of integration c
dM1:	Uses $t=0$, $N=90$ to find their constant of integration and obtains an expression of the form $\lambda e^{\frac{1}{4}t} = f(N); \lambda \neq 0$ or $\lambda e^{-\frac{1}{4}t} = f(N); \lambda \neq 0$
A1*:	Correct manipulation leading to $N = \frac{900}{3+7e^{-0.25t}}$ *
(b) Way 4	
M1:	Valid attempt to make t the subject, followed by an attempt to find two ln derivatives, condoning sign errors and constant errors.
A1:	$\frac{dt}{dN} = -4\left(\frac{-3}{900-3N} - \frac{7}{7N}\right)$ or equivalent
dM1:	Forms a common denominator to combine their fractions
A1*:	Correct algebra leading to $\frac{dN}{dt} = \frac{N(300-N)}{1200}$ *

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