adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Differentiation from first principles

Mark scheme

3 questions

Exam questions and marking instructions remain credited to their board.

Contents

1 June 2024 - Paper 1 - Q4

2 June 2023 - Paper 1 - Q12

3 June 2022 - Paper 2 - Q4

Question	Scheme	Marks	AOs
4	$\dfrac{(x+h)^2-x^2}{h}=\dfrac{x^2+2xh+h^2-x^2}{h}$	M1	2.1
	$=\dfrac{2xh+h^2}{h}$	A1	1.1b
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\lim_{h\to 0}\dfrac{2xh+h^2}{h}=\lim_{h\to 0}(2x+h)=2x$*	A1*	2.5
		(3)	
			(3 marks)

Notes

Note: Throughout the question allow use of δx for h or any other letter e.g. α if used consistently. If δx is used then you can condone e.g. $\delta^2 x$ for δx^2 as well as condoning e.g. poorly formed δ's

M1: Begins the process by writing down the gradient of the chord and attempts to expand the correct squared bracket – you can condone "poor" squaring e.g. $(x+h)^2=x^2+h^2$ but the $-x^2$ must be present.

A1: Reaches a correct fraction o.e. with the x^2 terms cancelled out and with no algebraic errors, e.g. $\dfrac{x^2+2xh+h^2-x^2}{h}$, $2x+h$ is correct.

A1*: Completes the process by applying a limiting argument and deduces that $\dfrac{\mathrm{d}y}{\mathrm{d}x}=2x$ with no errors seen. They must have $= 2x$ and not just $\lim_{h\to 0}2x$ to complete the proof.

$\dfrac{\mathrm{d}y}{\mathrm{d}x}=$ or an equivalent e.g. $\mathrm{f}'(x)=$ or "Gradient =" must be evident somewhere in their working or final line. If $\mathrm{f}'(x)$ is used then there is no requirement to see f (x) defined first. Condone e.g. $\dfrac{\mathrm{d}y}{\mathrm{d}x}\to 2x$ or $\mathrm{f}'(x)\to 2x$.

Condone missing brackets to allow e.g. $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\lim_{h\to 0}\dfrac{2xh+h^2}{h}=\lim_{h\to 0}2x+h=2x$

Do not allow $h = 0$ if there is never a reference to $h\to 0$.

e.g. $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\lim_{h\to 0}\dfrac{2xh+h^2}{h}=\lim_{h\to 0}2x+0=2x$ is acceptable

but e.g. $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2xh+h^2}{h}=2x+0=2x$ is not unless $h\to 0$ is seen.

The $h\to 0$ does not need to be present throughout the proof e.g. appear on every line but must appear at least once.

They must reach $2x+h$ at the end and not $\dfrac{2xh+h^2}{h}$ (without the h's cancelled) to complete the limiting argument.

Question	Scheme	Marks	AOs

12	$\dfrac{\sin(x+h)-\sin x}{h}$	B1	2.1
	$\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}$	M1	1.1b
		A1	1.1b
	(As $h\to 0$), $\quad \sin x\left(\dfrac{\cos h-1}{h}\right)+\cos x\left(\dfrac{\sin h}{h}\right)\to 0\times\sin x+1\times\cos x$	dM1	2.1
	so $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\cos x \quad *$	A1*	2.5
			(5 marks)

Notes

Throughout the question allow the use of $h=\delta x$ if used consistently

There is no requirement to see "gradient of chord" written down.

B1: Gives the correct fraction such as $\dfrac{\sin(x+h)-\sin x}{x+h-x}$ or $\dfrac{\sin x-\sin(x+h)}{-h}$ or $\dfrac{\sin(x+h)-\sin(x-h)}{2h}$ or $\dfrac{\sin(x-h)-\sin x}{x-h-x}$. Condone invisible brackets. May be implied by $\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}$

M1: Uses the compound angle formula for $\sin(x\pm h)$ to give $\sin x\cos h\pm\cos x\sin h$

A1: Achieves $\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}$ or equivalent (may be implied by further work).

Allow invisible brackets to be recovered.

dM1: **It is dependent on both the B and the M marks being awarded.**

Complete attempt to apply the given limits to the gradient of their chord. They must isolate $\left(\dfrac{\cos h-1}{h}\right)$ and replace with 0 and isolate $\left(\dfrac{\sin h}{h}\right)$ and replace with 1.

e.g. $\sin x\left(\dfrac{\cos h-1}{h}\right)+\cos x\left(\dfrac{\sin h}{h}\right)=\sin x\times 0+\cos x\times 1$

Accept as a minimum $\sin x\left(\dfrac{\cos h-1}{h}\right)+\cos x\left(\dfrac{\sin h}{h}\right)=\cos x$ (implying the application of the limits)

If they do not fully show $\left(\dfrac{\cos h-1}{h}\right)$ and $\left(\dfrac{\sin h}{h}\right)$ being isolated but proceed from

e.g. $\dfrac{\sin x(\cos h-1)+\cos x\sin h}{h}$ to $0\times\sin x+\cos x$ (or e.g. $0+\cos x$) then this can be implied and score dM1

$\dfrac{\sin x(\cos h-1)+\cos x\sin h}{h}=\cos x$ is dM0

Condone if limit notation remains within their expression after the limits have been applied.

e.g. $\lim\limits_{h\to 0}(\sin x\times 0+\cos x\times 1)$

Alternatively, condone use of the small angle approximations such that

$\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}\to\dfrac{-\frac{h^2}{2}\sin x+h\cos x}{h}=-\dfrac{h}{2}\sin x+\cos x$ and replaces $\dfrac{h}{2}$ with 0

A1*: Uses correct mathematical language of limiting arguments to show that $\frac{dy}{dx} = \cos x$ with no errors seen. (cso)

We need to see $h \to 0$ at some point in their solution and linking $\frac{dy}{dx}$ with $\cos x$ e.g.

- $\frac{dy}{dx} = ... = \lim_{h\to 0}\left(\sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right)\right) = \cos x$
- $\frac{dy}{dx} = ... = \lim_{h\to 0}\left(-\frac{h}{2}\sin x + \cos x\right) = 0 \times \sin x + \cos x = \cos x$ (using small angle approximations)
- $\frac{dy}{dx} = ... = \frac{\sin x(\cos h - 1) + \cos x \sin h}{h} = \sin x \times 0 + 1 \times \cos x = \cos x$ as $h \to 0$

Condone $f'(x)$ or y' in place of $\frac{dy}{dx}$

Give final A0 for no evidence of limiting arguments**:**

e.g. when $h = 0$ $\frac{dy}{dx} = ... = \sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right) = \sin x \times 0 + \cos x \times 1 = \cos x$ is A0

Do not allow the final A1 for just stating $\frac{\sin h}{h} = 1$ and $\frac{\cos h - 1}{h} = 0$ and attempting to apply these (without seeing e.g. $h \to 0$ at some point in their solution)

If they work in another variable (e.g. θ) then withhold the final mark. If they have mixed variables within some of their statements, then allow recovery but withhold the final mark.

Withhold this mark if there has been incorrect bracketing or invisible brackets when isolating $\sin x(\cos h - 1)$ e.g. $\frac{\sin x \cos h - 1 + \cos x \sin h}{h}$ but accept terms written as e.g. $\sin x \frac{\cos h - 1}{h}$ which do not require brackets. Condone a missing trailing bracket if the intention is clear.

Question	Scheme	Marks	AOs

4	$\frac{2(x+h)^2-2x^2}{h}=...$	M1	2.1
	$\frac{2(x+h)^2-2x^2}{h}=\frac{4xh+2h^2}{h}$	A1	1.1b
	$\frac{dy}{dx}=\lim_{h\to 0}\frac{4xh+2h^2}{h}=\lim_{h\to 0}(4x+2h)=4x*$	A1*	2.5
		(3)	
			(3 marks)

Notes:

Throughout the question allow the use of δx for *h* or any other letter e.g. α if used consistently. If δx is used then you can condone e.g. $\delta^2 x$ for δx^2 as well as condoning e.g. poorly formed δ's

M1: Begins the process by writing down the gradient of the chord and attempts to expand the correct bracket – you can condone "poor" squaring e.g. $(x+h)^2=x^2+h^2$.

Note that $\frac{2(x-h)^2-2x^2}{-h}=...$ is also a possible approach.

A1: Reaches a correct fraction oe with the x^2 terms cancelled out.

E.g. $\frac{4xh+2h^2}{h}$, $\frac{\cancel{2x^2}+4xh+2h^2-\cancel{2x^2}}{h}$, $4x+2h$

A1*: Completes the process by applying a limiting argument and deduces that $\frac{dy}{dx}=4x$ with no errors seen. The "$\frac{dy}{dx}=$" doesn't have to appear but there must be something equivalent e.g. "$f'(x)=$" or "Gradient =" which can appear anywhere in their working. If $f'(x)$ is used then there is no requirement to see f (*x*) defined first. Condone e.g. $\frac{dy}{dx}\to 4x$ or $f'(x)\to 4x$.

Condone missing brackets so allow e.g. $\frac{dy}{dx}=\lim_{h\to 0}\frac{4xh+2h^2}{h}=\lim_{h\to 0}4x+2h=4x$

Do not allow *h* = 0 if there is never a reference to h→0

e.g. $\frac{dy}{dx}=\lim_{h\to 0}\frac{4xh+2h^2}{h}=\lim_{h\to 0}4x+2(0)=4x$ is acceptable

but e.g. $\frac{dy}{dx}=\frac{4xh+2h^2}{h}=4x+2h=4x+2(0)=4x$ is not if there is no h→0 seen.

The h→0 does not need to be present throughout the proof e.g. on every line.

They must reach $4x+2h$ at the end and not $\frac{4xh+2h^2}{h}$ (without the *h*'s cancelled) to complete the limiting argument.