adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Forces and Newtonâ€™s laws

Mark scheme

20 questions

Exam questions and marking instructions remain credited to their board.

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1(a)	$v = 15 + (2 \times 4)$	M1	3.1b
	$= 23\ (\text{m s}^{-1})$	A1	1.1b
		(2)	
1(b)	$D - 400 = 800 \times 2$	M1	3.1b
	$(D =)\ 2000$	A1	1.1b
		(2)	
			(4 marks)

Notes:

1(a)	M1	Use of $v = u + at$ or any other **complete** method to form an equation in v only. Condone sign errors. e.g. May find s first: $s = (15 \times 4) + \frac{1}{2} \times 2 \times 4^2 = 76$ then either: $76 = 4v - \frac{1}{2} \times 2 \times 4^2$ or: $76 = \frac{(15 + v)}{2} \times 4$ or: $v^2 = 15^2 + 2 \times 2 \times 76$ **N.B.** If they use $v = u + at$ and have $15 = u + (2 \times 4)$, M1A0 M0 if they clearly use an incorrect *suvat* formula or $u = 0$
	A1	cao. Must be positive. A0 if they get 23 but go on and do something with it. i.e. not ISW **N.B.** 23 with no working can score both marks.
1(b)	M1	Use of $F = ma$ to form an equation in D or a complete expression for D, with correct number of terms, condone sign errors and g in the ma term.
	A1	cao **N.B.** 2000 with no working can score both marks.

Question	Scheme	Marks	AOs
N.B. Penalise the use of $g = 9.81$ ONCE (the first time it is used) for the whole question.			

N.B. Penalise the use of $g = 9.8$ ONCE (the first time it is used) for the whole question.

2(a)	Resolve vertically	M1	3.4
	$(\uparrow) 5\sin\alpha + R = 2g$ oe	A1	1.1b
	$(R =)$ $19.6 - 3 = 16.6$ (N)*	A1*	1.1b
		(3)	
2(b)	The box decelerates, slows down, loses speed or velocity, will come to a stop or rest, loses momentum	B1	2.4
		(1)	
2(c)	$(S =)$ 19.6 or 20 (N)	B1	3.3
		(1)	
2(d)	Equation of motion along the plane: $-F = 2a$ or $F = 2a$	M1	3.4
	$F = 0.2\times$ their S	B1	1.1b
	$0^2 = 4^2 - 2\times 0.2g \times d$ or $0^2 = 4^2 + 2\times 0.2g \times d$ or $4^2 = 0^2 + 2\times 0.2g \times d$	M1	3.1b
	$(OX =)$ 4.1 (m) or 4.08 (m)	A1	1.1b
	OR		
	Work done against friction = Fd	M1	3.4
	$F = 0.2\times$ their S	B1	1.1b
	$0.2\times 2g \times d = \frac{1}{2}\times 2\times 4^2$	M1	3.1b
	($d = OX =$) 4.1 (m) or 4.08 (m)	A1	1.1b
		(4)	
2(e)	the box (it) has been modelled as a particle the box (it) will have size or shape or dimensions the coefficient of friction has been modelled as being constant the coefficient of friction may not be exactly 0.2 or may vary the friction may vary B0 : the ground may not be horizontal, an inaccurate value of g has been used, has not considered wind, the angle may not be accurate, any reference to the particle having mass (or not having mass). Ignore any reference to air resistance.	B1	3.5b
		(1)	

		(10 marks)
Notes:		
2(a)	M1	Form an equation in R and α only, with correct number of terms, condone sign errors and sin/cos confusion, $\sin\alpha$ may or may not be substituted. **N.B**. $3 + R = 2g$ oe , with no evidence of resolving the 5 N, e.g. not using $\sin\alpha$, is M0.
	A1	Correct equation, $\sin\alpha$ may or may not be be substituted.
	A1*	Given answer correctly obtained, with at least one line of working.
2(b)	B1	Any equivalent statement. B0 if any incorrect extras e.g. box is stationary or at rest, doesn't move, will stop moving.
2(c)	B1	Accept $2g$ if g is not substituted for. B0 if they use $g = 9.81$ unless they have already been penalised in part (a).
2(d)	M1	Correct number of terms. Condone an extra g in ma term. M0 if 2 is missing. **N.B**. M0 if they use a vertical force for F e.g. $2g$ or 16.6
	B1	Seen (e.g. on a diagram) or implied.
	M1	Complete method to form an equation in d (= OX) only, condone sign errors, using their calculated acceleration from an attempt at using $F = ma$ M0 if clearly using $\mu(0.2)$ or $-\mu$ for a without any calculation.. e.g. may find t first: $0 = 4 - 0.2gt$ => $t = \frac{20}{g}$ $(= \frac{100}{49})$ then $d = 4 \times \frac{20}{g} - \frac{1}{2} \times 0.2g \times \left(\frac{20}{g}\right)^2$ or $d = 0 - \frac{1}{2} \times (-0.2g) \times \left(\frac{20}{g}\right)^2$ or $0^2 = 4^2 + 2 \times (-0.2g) \times d$
	A1	Either answer. A0 for $\frac{40}{g}$
2(e)	B1	Any equivalent statement which refers to the model. B0 if incorrect extras.

Question	Scheme	Marks	AOs

3(a)	$(\mathbf{F} =)\ (\mathbf{i} - 1.2\mathbf{j})$	B1	3.4
		(1)	
3(b)	$\mathbf{v} = (-7\mathbf{i} + 7.8\mathbf{j}) + 2(2\mathbf{i} - 2.4\mathbf{j})$	M1	2.1
	$= (-3\mathbf{i} + 3\mathbf{j})\ (\text{m s}^{-1})$	A1	1.1b
		(2)	
3(c)	Use trig to find an equation in a relevant angle	M1	2.1
	e.g. $\tan\alpha = 1$ or -1 or $\sin\alpha = \frac{3}{\sqrt{3^2+3^2}}$ or $\frac{-3}{\sqrt{3^2+3^2}}$ or $\cos\alpha = \frac{3}{\sqrt{3^2+3^2}}$ or $\frac{-3}{\sqrt{3^2+3^2}}$	A1	1.1b
	315^o	A1	2.2a
		(3)	
3(d)	$\overrightarrow{OA} = 5(-7\mathbf{i} + 7.8\mathbf{j}) + \frac{1}{2}(2\mathbf{i} - 2.4\mathbf{j}) \times 5^2$	M1	3.1a
	$= (-10\mathbf{i} + 9\mathbf{j})\ (\text{m})$	A1	1.1b
		(2)	
			(8 marks)

Notes:

Accept column vectors throughout apart from in the final answer for (a) and (d) which must be in terms of i and j, but only penalise ONCE for the whole question.

N.B. For final answers to (a), (b) and (d) , penalise incorrect vector notation.

3(a)	B1	Must be in terms of **i** and **j** Do not accept $0.5 \times (2\mathbf{i} - 2.4\mathbf{j})$ Do not need **F** =
3(b)	M1	Use of **v** = **u** + **a***t* or any other complete method, e.g. integration, to find an unsimplified expression for **v** at $t = 2$, condone sign errors. **N.B.** If using integration, must have used $t = 0$, $\mathbf{v} = (-7\mathbf{i} + 7.8\mathbf{j})$ to find the constant of integration and included it in their expression for **v** at $t = 2$.
	A1	cao. **N.B.** Ignore if they go on and find the speed.
3(c)	M1	Use trig to find an equation in a relevant angle for their **v**
	A1	Correct equation.
	A1	Cao (with or without the degree sign), must come from a correct **v** **N.B.** $\pm 45^o, \pm 135^o, \pm 225^o, -315^o$ with no working scores M1A1 315^o with no working scores all 3 marks.

3(d)	M1	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with $t = 5$ or any other complete method, e.g. integration of their $\mathbf{v} = f(t)$ and use of $t = 5$, to find an unsimplified expression for $\overrightarrow{OA}$ (No need to show that $\mathbf{C} = \mathbf{0}$) M0 if they use $\mathbf{u} = \mathbf{0}$
	A1	Must be in terms of **i** and **j** **N.B.** Ignore if they go on and find the length *OA*

Question	Scheme	Marks	AOs

4(a)	$\mathbf{r} = (32\mathbf{i} - 16\mathbf{j})$	B1	1.1b
		(1)	
4(b)	Use Pythagoras: $\sqrt{32^2 + 16^2}$ oe for their **r**	M1	3.1a
	$16\sqrt{5}$ (m)	A1	1.1b
		(2)	
4(c)	Differentiate **r** wrt *t* to obtain **v**	M1	3.4
	$6t^{\frac{1}{2}}\mathbf{i} - 2t\mathbf{j}$ oe (m s^{-1})	A1	1.1b
		(2)	
4(d)	Differentiate their **v** wrt *t* to obtain **a**	M1	3.4
	$3t^{-\frac{1}{2}}\mathbf{i} - 2\mathbf{j}$	A1	1.1b
	$\frac{3T^{-\frac{1}{2}}}{-2} = -\frac{1}{3}$ must see $\frac{\mathbf{i} \text{ component of their } \mathbf{a}}{\mathbf{j} \text{ component of their } \mathbf{a}} = -\frac{1}{3}$ oe	M1	2.1
	$(T =) \frac{81}{4}$ oe	A1	1.1b
		(4)	
			(9 marks)

Notes: N.B. Accept column vectors throughout apart from in the answer to (c).

4(a)	B1	cao. **N.B.** Must be using correct vector notation, including brackets if using column vectors.
4(b)	M1	For an unsimplified expression, using their **r**, with the square root
	A1	Accept any surd equivalent isw **N.B**. Must come from $\mathbf{r} = (32\mathbf{i} - 16\mathbf{j})$
4(c)	M1	Both powers of *t* decreasing by 1 (but not just division by *t*) **N.B.** M0 if **i** and/or **j** are missing and never reappear.
	A1	Must be in terms of *t*, **i** and **j**
4(d)	M1	Both powers of *t* decreasing by 1 (but not just division by *t*) **If no i's and/or j's,** can score M1 if **EITHER** they have 2 separate components, provided they are clearly treated as such in the subsequent working **OR** the **i's** and **j's** reappear **otherwise** M0.
	A1	Correct vector **or** 2 correct separate components, provided they are clearly treated as such in the subsequent working

	M1	Complete method to form a correct equation in T (t) only, for their **a,** using the 2 components from their **a**
	A1	Accept 20.3 or 20.25 or any equivalent fraction (allow t instead of T)

Question	Scheme	Marks	AOs

5(a)	Using horizontal motion	M1	3.3
	$14\cos\theta \times t = 40$	A1	1.1b
	Using vertical motion	M1	3.4
	$(s =)14\sin\theta \times t - \frac{1}{2}gt^2$ oe with down as positive.	A1	1.1b
	OR		
	Using the equation of the path		
	$(s =)40\tan\theta - \frac{g \times 40^2}{2 \times 14^2 \cos^2\theta}$ oe	M2 A2	
	$(H =)$ 30 or 30.0	A1	1.1b
		(5)	
5(b)	Using vertical motion **OR** Using cons. of energy	M1	2.1
	$0^2 = (14\sin\theta)^2 - 2gh$ $\quad$ $\frac{1}{2}m(14^2 - (14\cos\theta)^2) = mgh$	A1	1.1b
	32 (m)	A1**ft**	1.1b
		(3)	
			(8 marks)

Notes:

N.B. Penalise the use of g = 9.81 **once** (the first time it is used) for the whole question.
All marks available if they use a different letter for θ

5(a)	M1	Complete method, using the horizontal motion, to give an equation in t and θ only, condone sin/cos confusion and sign errors (θ may or may not be substituted for)
	A1	Correct equation (t and/or θ may or may not be substituted for)
	M1	Complete method, using the vertical motion, to give an equation in t and θ only, condone sin/cos confusion and sign errors (t and/or θ may or may not be substituted for)
	A1	Correct equation (t and/or θ may or may not be substituted for) **N.B.** Just mark the RHS of the equation for this A mark.
	OR	
	M2 A2	Quoting and using correctly the equation of the path. **N.B.** No marks if they misquote the equation and no marks if they put in 14 and/or 40 incorrectly. These 4 marks are indivisible i.e. all 4 or 0.
	A1	H = 30 or 30.0 to 2 or 3 sf, after use of g = 9.8, **obtained correctly**.

5(b)	M1	Complete method to give equation in h and θ only, condone sin/cos confusion and sign errors They may find t first: $0 = 14\sin\theta - gt \Rightarrow t = \dfrac{14\sin\theta}{g}$ then $h = (14\sin\theta)\dfrac{14\sin\theta}{g} - \dfrac{1}{2}g\left(\dfrac{14\sin\theta}{g}\right)^2$ or $h = -\dfrac{1}{2}(-g)\left(\dfrac{14\sin\theta}{g}\right)^2$ or $h = \dfrac{(14\sin\theta + 0)}{2}\left(\dfrac{14\sin\theta}{g}\right)$
	A1	Correct equation
	A1**ft**	$\lvert$their $H\rvert + 2$, to 2sf or 3sf

Question	Scheme	Marks	AOs

1(a)	$0.5g$, $\frac{1}{2}g$ or 4.9 (N) seen	B1	3.4
		(1)	
1(b)	$\frac{2}{7}\times 4.9$ oe seen	M1	3.1a
	1.4, 1.40 or $\frac{1}{7}g$	A1	1.1b
		(2)	
			(3 marks)

Notes: Ignore units in this question.

N.B. Use of $g = 9.81$ should only be penalised **once** for the whole question as should two fractional answers (a) 49/10 (b) 7/5.

Penalise the use of $g = 9.81$ the first time you see it.

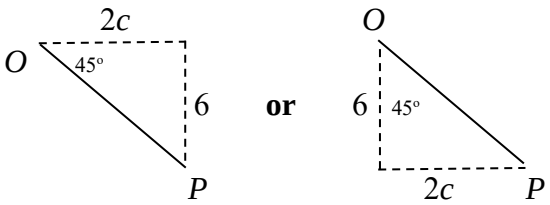
N.B.

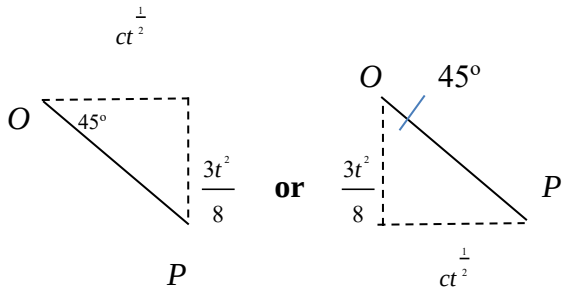
If $g = 9.81$ is used in (a), B0. If it is then used again in part (b), and they give the answer as 1.4 or 1.40, they can score M1A1 in part (b).

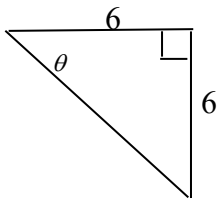
If $g = 9.81$ is only used in (b), they can score max M1A0 for (b).

1a	B1	cao. (must be positive) B0 for a fraction ($\frac{49}{10}$) B0 if they have $0.5g$ and then clearly use $g = 9.81$ i.e. **NOT** isw This answer must appear in (a) to earn this mark. If no labelling, give BOD and award as many marks as possible.
1b	M1	$\frac{2}{7}\times$ their (a) (must be a numerical value) If no answer for (a) or if (a) is incorrect and they don't use it, allow a correct restart i.e. $\frac{2}{7}\times 4.9$ or $\frac{2}{7}\times 0.5g$ or $\frac{2}{7}\times 0.5$ (missing g is not an M error)
	A1	A0 for a fraction **N.B.** $X =$ is not needed **BUT** A0 for F (or P or horizontal force) = a correct answer if they don't go on to state $X =$... If they obtain the correct answer for F and they have said $F = X$ they can score A1.
		N.B. 1.4, 1.40 or $\frac{1}{7}g$ **with no working,** scores M1A1

Question	Scheme	Marks	AOs

4(a)	**ALTERNATIVES** when $t = 4$ is substituted at the beginning.		
	$2c\mathbf{i} - 6\mathbf{j}$ or as a column vector, seen or implied.	B1	1.1b
	ALT 1  **AND** **either** $\tan 45^o = \frac{2c}{6} \Rightarrow 2c = 6$ **or** **states** isosceles triangle so $2c = 6$ **N.B.** In both of the above,we must see the justification for the equation. **ALT 2** $\tan 135^o = \frac{2c}{-6} \Rightarrow 2c = 6$ **N.B.** M0 if they are using the wrong bearing.	M1	3.1a
	$c = 3$ *	A1*	2.2a
	SC 1 M1A0: no right-angled triangle $2c\mathbf{i} - 6\mathbf{j} = k(\mathbf{i} - \mathbf{j}) \Rightarrow 2c = 6$ **or** **i**-cpt = – **j**-cpt $\Rightarrow 2c = 6$ **N.B.** In both of the above,we must see the justification for the equation. **SC 2 M1A0:** no right-angled triangle $\tan 45^o = \frac{2c}{6}$ or $\frac{6}{2c} \Rightarrow 2c = 6$ **N.B.** In the above,we must see the justification for the equation.		

	ALTERNATIVES when $t = 4$ is substituted at the end:		
	$ct^{\frac{1}{2}} = 2c$ **and** $(-)\frac{3t^2}{8} = (-)6$ when $t = 4$, seen or implied	B1	
	ALT 3  **AND**	M1	
	either $\tan 45^o = \frac{\frac{3t^2}{8}}{ct^{\frac{1}{2}}}$ => $2c = 6$ when $t = 4$ **or** **states** isosceles triangle, so $ct^{\frac{1}{2}} = \frac{3t^2}{8}$ => $2c = 6$ when $t = 4$ **N.B.** In both of the above,we must see the justification for the equation. **N.B.** M0 if they are using the wrong bearing.		
	$c = 3$	A1*	
	SC 3 M1A0: no right-angled triangle $(ct^{\frac{1}{2}}\mathbf{i} - \frac{3t^2}{8}\mathbf{j}) = k(\mathbf{i} - \mathbf{j})$ => $2c = 6$ when $t = 4$ **or** **i**-cpt = – **j**-cpt => $ct^{\frac{1}{2}} = \frac{3t^2}{8}$ => $2c = 6$ when $t = 4$ **N.B.** In both of the above,we must see the justification for the equation. **SC 4 M1A0:** no right-angled triangle $\tan 45^o = \frac{\frac{3t^2}{8}}{ct^{\frac{1}{2}}}$ => $2c = 6$ when $t = 4$.		

	N.B. Allow a **verification**: i.e. use $c = 3$ and $t = 4$ to show that P is on a bearing of 135° from O. $6\mathbf{i} - 6\mathbf{j}$ then a diagram:  **AND** $\tan\theta = \frac{6}{6}$ or isosceles triangle => $\theta = 45°$ **N.B.** In the above,we must see the justification for the equation.	B1 M1	
	bearing = 45° + 90° = 135°	A1*	
		(3)	
4(b)	Differentiate **r** wrt t to obtain **v**	M1	2.1
	$\mathbf{v} = 3\times\frac{1}{2}t^{-\frac{1}{2}}\mathbf{i} - \frac{3}{8}\times 2t\mathbf{j} = \frac{3}{2}t^{-\frac{1}{2}}\mathbf{i} - \frac{3}{4}t\mathbf{j}$ oe	A1	1.1b
	Put $t = 4$ into **both** components and use Pythagoras: $\sqrt{\left(\frac{3}{4}\right)^2 + (-3)^2}$	M1	3.1a
	$\sqrt{\frac{153}{16}}$ or $\frac{\sqrt{153}}{4}$ or $\frac{3\sqrt{17}}{4}$ or $3\sqrt{\frac{17}{16}} = 3.0923..$ (m s^{-1})	A1	1.1b
		(4)	
4(c)	Differentiate their **v** wrt t to obtain **a**	M1	3.4
	$\mathbf{a} = -\frac{3}{4}t^{-\frac{3}{2}}\mathbf{i} - \frac{3}{4}\mathbf{j}$	A1	1.1b
	$\frac{-\frac{3}{4}T^{-\frac{3}{2}}}{-\frac{3}{4}} = \frac{-1}{-27}$ oe	M1	2.1
	$(T =)$ 9	A1	1.1b
		(4)	

		(11 marks)
Notes: Accept column vectors throughout		
4a	B1	$2c\mathbf{i} - 6\mathbf{j}$ seen or implied. B0 for r = $2c - 6$ if no evidence of components.
	M1	**ALT 1:** Use the bearing to obtain a CORRECT diagram showing a right-angled triangle with at least one 45° angle marked or clearly explained (i.e. 135° marked on the diagram and either $135^o - 90^o = 45^o$ or $180^o - 135^o = 45^o$), and $2c$ and ± 6 marked **AND** use of isosceles triangle or tan or (sin/cos and Pythag) to obtain $2c = 6$ **ALT 2: No diagram required** Use $\tan 135^\circ = \frac{2c}{-6} \Rightarrow 2c = 6$
	A1*	Given answer correctly obtained
		ALTERNATIVE when $t = 4$ is substituted at the end:
	B1	$ct^{\frac{1}{2}} = 2c$ **and** $(-)\frac{3t^2}{8} = (-)6$ when $t = 4$, seen or implied
	M1	**ALT 3:** Use the bearing to obtain a CORRECT diagram showing a right-angled triangle with at least one 45° angle marked or clearly explained (i.e. 135° marked on the diagram and either $135^o - 90^o = 45^o$ or $180^o - 135^o = 45^o$), and $ct^{\frac{1}{2}}$ and $\pm \frac{3t^2}{8}$ marked **AND** use of isosceles triangle or tan or (sin/cos and Pythag) to obtain $2c = 6$ when $t = 4$
	A1*	Given answer correctly obtained
4b	M1	Both powers of t decreasing by 1 (M0 if **i** or **j** is missing but allow recovery or working with components only) **N.B.** This mark is available if c has not been substituted for.
	A1	Correct unsimplified derivative or two correct components
	M1	Put $t = 4$ in their **v** (must be using an attempted derivative of **r**) and then use Pythagoras with the root, allow a missing $-$ sign **N.B.** If they **state** $t = 4$, allow a slip when they substitute in, for this M mark. This mark is available if c has not been substituted for.
	A1	Accept 3.1 or better
4c	M1	Both powers of t decreasing by 1 **N.B.** This mark is available if c has not been substituted for (M0 if **i** or **j** is missing but allow recovery or working with components only).
	A1	Correct unsimplified derivative
	M1	Use of an appropriate **ratio** (must be using an attempted derivative of **their v)**, condone sign error and the **reciprocal,** to obtain an equation in t or ***T*** **only**. **N.B.** If they state that $-\frac{3}{4}T^{-\frac{3}{2}}\mathbf{i} - \frac{3}{4}\mathbf{j} = k(-\mathbf{i} - 27\mathbf{j})$ and then equate coefficients to give two simultaneous equations in k and T, these need to be used to produce an equation in ***T*** **only**, before the M mark is earned.
	A1	cao (allow t instead of T)

Question	Scheme	Marks	AOs

2(a)	Resolve vertically, $R = 5g = 49$ (N)	B1	1.1b
		(1)	
2(b)	Equation of motion: $28 - F = 5 \times 1.4$	M1	3.1a
	$F = 21$	A1	1.1b
		(2)	
2(c)	$\mu = 0.43$ (2sf required)	B1 **ft**	3.4
		(1)	
			(4 marks)

Notes:

2a	B1	Allow either $5g$ or 49. No penalty for using $g = 9.81$ or 10. Ignore any working. Must be a positive number. B0 if m is involved. **N.B**. Could be seen on a diagram, provided it's clearly the reaction.
2b	M1	Equation with correct terms, dimensionally correct, condone sign errors.
	A1	cao but allow $\frac{15g}{7}$. Ignore units.
2c	B1**ft**	$\mu = \frac{\text{their (b)}}{\text{their (a)}}$. **Answer must be a positive number given to 2sf**. **N.B**. B0 if they use $g = 9.81$ or 10 in this part of the question. Do not allow restarts. Allow $\mu > 1$.

Question	Scheme	Marks	AOs

4(a)	$\mathbf{v}_B = (-16\mathbf{i} - 3\mathbf{j}) + 5(2.4\mathbf{i} + \mathbf{j})$	M1	3.4
	$\mathbf{v}_B = (-4\mathbf{i} + 2\mathbf{j})$	A1	1.1b
	$\sqrt{(-4)^2 + 2^2}$	M1	3.1a
	$\sqrt{20}$ = $2\sqrt{5}$, 4.5 or better ($\mathrm{m\,s^{-1}}$)	A1	1.1b
		(4)	
4(b)	Using *A* as the initial position: $\mathbf{r}_C = \mathbf{v}_A t + \frac{1}{2}\mathbf{a}t^2 + \mathbf{r}_A$ where $t = T$ $(4\mathbf{i} + c\mathbf{j}) = (-16\mathbf{i} - 3\mathbf{j})T + \frac{1}{2}(2.4\mathbf{i} + \mathbf{j})T^2 + (44\mathbf{i} - 10\mathbf{j})$ **OR** $\begin{pmatrix}4\\c\end{pmatrix} = \begin{pmatrix}-16\\-3\end{pmatrix}T + \frac{1}{2}\begin{pmatrix}2.4\\1\end{pmatrix}T^2 + \begin{pmatrix}44\\-10\end{pmatrix}$ Equating **i**-components, to give a **quadratic** equation in *T* only. Allow *t* instead of *T*. **N.B.** Allow omission of 44 for this M mark. Also allow ± 4 but M0 if 4 is not used at all i.e. $4 = -16T + \frac{1}{2} \times 2.4T^2$ scores M1A0A0	M1	3.1a
	$4 = -16T + \frac{1}{2} \times 2.4T^2 + 44$	A1	1.1b
	(*T* =) 10	A1	1.1b
	ALTERNATIVE using *B* as the initial position: (The position vector of *B*, $\mathbf{r}_B$, should be $-6\mathbf{i} - 12.5\mathbf{j}$ but no credit for finding this) $\mathbf{r}_C = \mathbf{v}_B t + \frac{1}{2}\mathbf{a}t^2 + \mathbf{r}_B$ using **their** $\mathbf{v}_B$ from (a) and **their** $\mathbf{r}_B$ $(4\mathbf{i} + c\mathbf{j}) = (-4\mathbf{i} + 2\mathbf{j})t + \frac{1}{2}(2.4\mathbf{i} + \mathbf{j})t^2 + (-6\mathbf{i} - 12.5\mathbf{j})$ $\begin{pmatrix}4\\c\end{pmatrix} = \begin{pmatrix}-4\\2\end{pmatrix}t + \frac{1}{2}\begin{pmatrix}2.4\\1\end{pmatrix}t^2 + \begin{pmatrix}-6\\-12.5\end{pmatrix}$ Equating **i**-components, to give a **quadratic** equation in *t* only. Allow if they have *T* instead of *t*.	M1	3.1a

	N.B. Allow omission of their -6 or if they use 44 for this M mark. Also allow ± 4 but M0 if 4 is not used at all. e.g. $4 = -4t + \frac{1}{2} \times 2.4t^2$ scores M1A0A0		
	$4 = -4t + \frac{1}{2} \times 2.4t^2 - 6$	A1	1.1b
	t = 5 so (T =) 10	A1	1.1b
		(3)	
4(c)	Equating **j**-components, with <u>their value of T or t substituted</u>, to give an equation, which must have a square term, in c only. **N.B.** Allow $\pm c$ in their equation. **(N.B.** Allow omission of -10 or their -12.5 for this M mark i.e. if using A as initial position $c = (-3 \times 10) + \frac{1}{2} \times 1 \times 10^2$ scores M1M0A0 **OR** if using B as initial position $c = (2 \times 5) + \frac{1}{2} \times 1 \times 5^2$ scores M1M0A0)	M1	2.1
	if using A as initial position $c = (-3 \times 10) + \frac{1}{2} \times 1 \times 10^2 + (-10)$ **N.B.** Allow $\pm c$ and/or $\pm(-10)$ in their equation **OR** if using B as initial position $c = (2 \times 5) + \frac{1}{2} \times 1 \times 5^2 + (-12.5)$ **N.B.** Allow $\pm c$ and/or $\pm(-12.5)$ in their equation	M1	1.1b
	c = 10	A1	1.1b
		(3)	
			(10 marks)

Notes: Accept column vectors throughout		
4a	M1	Use of **v** = **u** + **a***t* with t = 5 to give an unsimplified $\mathbf{v}_B$ M0 if **u** = **0**

		N.B. If using integration, they must get to the same stage i.e. have found the constant and put $t = 5$ M0 if they omit the constant altogether
	A1	Correct $\mathbf{v}_B$ with **i**'s and **j**'s collected
	M1	Use of Pythagoras on *their* $\mathbf{v}_B$ to give a magnitude (need the root)
	A1	Must be positive
4b	M1	Equating components of **i** to give an equation in T or t only. **N.B.** (they could use integration to get to the same stage) for this M mark, they **only** need to be equating the **i**-components, and receive no credit until they do so. M0 if **u** = **0**
	A1	A correct equation in T or t only (could be in $(T - 5)$ if using B as initial position)
	A1	$T = 10$
4c	M1	Equating components of **j** to give an equation in c only but allow omission of their initial position
	M1	With their value of T or t and **must include t = 0 position** (should be -10 if using A **OR** their -12.5 if using B)
	A1	cao

Question	Scheme	Marks	AOs
	N.B. In this question, allow misread of α for a.		

5(a)	Use horizontal motion to give an equation in *T* and α only: $28\cos\alpha \times T = 40$	M1	3.4
	$T = \dfrac{10}{7\cos\alpha}$ *	A1*	1.1b
		(2)	
5(b)	Use vertical motion to give an equation in *T* and α only	M1	3.3
	$20 = (28\sin\alpha)T - \dfrac{1}{2}gT^2$	A1	1.1b
	Eliminate *T* to give an unsimplified equation in α only: $20 = (28\sin\alpha) \times \dfrac{10}{7\cos\alpha} - \dfrac{1}{2}g\left(\dfrac{10}{7\cos\alpha}\right)^2$	M1	1.1b
	Use $\sec^2\alpha = 1 + \tan^2\alpha$ oe to give an unsimplified equation in **tan α only** : $20 = 40\tan\alpha - \dfrac{1}{2}g \times \dfrac{100}{49}(1 + \tan^2\alpha)$	M1	3.1b
	$\tan^2\alpha - 4\tan\alpha + 3 = 0$ * (allow $0 = \tan^2\alpha - 4\tan\alpha + 3$)	A1*	2.2a
		(5)	
5(c)	Solve and use of $\tan\alpha = 3$ or $\sin\alpha = \dfrac{3}{\sqrt{10}}$ or $\alpha = 71.565..^o$ to find an equation in *H* only.	M1	3.1b
	$0 = (28\sin\alpha)^2 - 2gH$ **where** $\tan\alpha = 3$ $(\alpha = 71.565..^o)$	M1	3.4
	H = 36 or 36.0 (m)	A1	1.1b
		(3)	
5(d)	e.g. spin of the ball, the wind, the dimensions or shape of the ball, ball is modelled as a particle, uses an inaccurate value of *g*, motion takes place in 3D not in 2D, *g* could be variable. B0 if mass or weight are mentioned. B0 for ground may not be horizontal.	B1	3.5b
		(1)	

		(11 marks)
Notes:		
5a	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors
	A1*	Correct printed answer correctly obtained. Allow $\frac{10}{7\cos\alpha} = T$ OR $T = \frac{40}{28\cos\alpha} = \frac{10}{7\cos\alpha}$ OR $\frac{40}{28\cos\alpha} = \frac{10}{7\cos\alpha} = T$ OR *t* instead of *T*
5b	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors
	A1	Correct equation
	M1	Eliminate *T*, using either the given answer in (a) or their own *T* expression, from their equation to give an unsimplified equation in α only
	M1	Use $\sec^2\alpha = 1 + \tan^2\alpha$ to produce an equation in $\tan\alpha$ only
	A1*	Given answer correctly obtained. **N.B.** Must be α (or *a*) in the final answer but allow a different angle in the working.
5c	M1	Solve given equation and select larger value of $\tan\alpha$ and use it to try to obtain an equation in *H* only.
	M1	Complete method to give an equation in ***H* only**, using **<u>larger</u>** value of α , correct no. of terms, dim correct, condone sin/cos confusion and sign errors.
	A1	cao. Must be positive, (allow a negative value, changed to a positive answer). **N.B.** This answer comes from use of *g* = 9.8, so must be rounded to 2 or 3 sf.
5d	B1	B0 if any incorrect extras

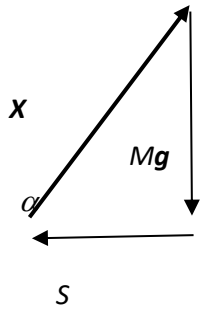
Question	Scheme	Marks	AOs

6(a)	The normal reaction at *B* is acting to the left so it must act to the right, right as it needs to balance (oppose, counter) the force at *B*, right as it prevents the rod from sliding (slipping, falling), right as the weight (mass) of the rod will mean the rod tends to slip left, mass or weight will be pushing the rod to the left so friction will oppose that. **N.B.** You may see an arrow on the diagram at *A*, instead of 'right'. B0 if they say the rod is moving oe Accept towards the wall instead of to the right.	B1	2.4
		(1)	
6(b)	Take moments about *A*	M1	3.4
	$S \times 2a\sin\theta = Mga\cos\theta$	A1	1.1b
	$S = \frac{1}{2}Mg\cot\theta$ *	A1*	2.2a
		(3)	
6(c)	Resolve vertically, $R = Mg$	B1	3.3
	Resolve horizontally, $F = S$	B1	3.3
	Other possible equations: Resolve along the rod, $F\cos\theta + R\sin\theta = S\cos\theta + Mg\sin\theta$ Resolve perp to the rod, $R\cos\theta + S\sin\theta = F\sin\theta + Mg\cos\theta$ M(*B*), $R \times 2a\cos\theta = F \times 2a\sin\theta + Mga\cos\theta$ M(*G*), $Ra\cos\theta = Fa\sin\theta + Sa\sin\theta$ **N.B.** When entering these two B marks on ePEN**,** **First** B1 is for a **vertical** resolution, **second** B1 is for a **horizontal** resolution, and if either is replaced by a different equation, enter appropriately. If both are replaced by other equations, enter in the order in which they appear in their working.		
	$F = \mu R$	B1	1.2
	$\frac{1}{2}Mg \times \frac{4}{3} = \mu Mg$	**dM**1	2.1
	$\mu = \frac{2}{3}$ oe Accept 0.67 or better	A1	2.2a
	S.C. For F ,, μR, B0 $\frac{1}{2}Mg \times \frac{4}{3}$,, μMg M1		

	$\frac{2}{3}$,, μ A0 **N.B**. If $\mu = \frac{2}{3}$ follows this, they could score all the marks.		
		(5)	
6(d)	$\sqrt{F^2 + R^2}$	M1	3.1a
	$\sqrt{\left(\frac{2}{3}Mg\right)^2 + (Mg)^2}$	M1	1.1b
	$\frac{1}{3}Mg\sqrt{13}$ or 1.2*Mg* or better	A1	2.2a
		(3)	
6(e)	New value of *S* would be **larger** as the **moment** of the **weight** about ***A*** would be larger	B1	3.5a
		(1)	
			(13 marks)

Notes:

6a	B1	Any equivalent appropriate statement.
6b	M1	Correct no. of terms, dimensionally correct, condone sin/cos confusion and sign errors. **N.B.** If *a*'s never appear, M0
	A1	Correct equation
	A1*	Correct given answer correctly obtained, **with no wrong working seen.** Allow $\frac{1}{2}Mg\cot\theta = S$ or $S = \frac{Mg\cot\theta}{2}$ or $\frac{Mg\cot\theta}{2} = S$ or $S = \frac{Mg}{2}\cot\theta$ or similar but NOT $S = \frac{1}{2}\cot\theta\ \mathrm{Mg}$ or similar **N.B.** Allow *m* instead of *M* Must be θ in final answer but allow a different angle in the working.
6c	B1	cao
	B1	cao
	B1	Seen anywhere, e.g. on the diagram
	dM1	Using $F = \mu R$, their two equations and substitute for trig (not necessarily correctly) to produce an equation in μ only. This mark is **dependent** on the 3 previous B marks.
	A1	Accept 0.67 or better

6d	M1	Use of Pythagoras with square root to find the required magnitude, but *F* and *R* do not need to be substituted
	M1	Substitute for their *F* and their *R* in terms of *Mg* and take square root to obtain magnitude in terms of *M* and *g* only. **N.B.** Must be using Pythagoras
		ALTERNATIVE: Using trig on triangle of forces  M1: $X = \frac{Mg}{\sin\alpha}$ or $\frac{S}{\cos\alpha}$ M1: substitute for $\sin\alpha$ or $\cos\alpha$ and *S*, where $\tan\alpha = \frac{Mg}{S}$ $(=\frac{3}{2})$, to obtain *X* in terms of *M* and *g* only.
	A1	Any equivalent surd form or 1.2*Mg* or better Must be in terms of *M* and *g*
6e	B1	Correct answer and any equivalent appropriate statement.

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3(a)	$(4\mathbf{i}-\mathbf{j})+(\lambda\mathbf{i}+\mu\mathbf{j})=(4+\lambda)\mathbf{i}+(-1+\mu)\mathbf{j}$	M1	3.4
	Use **ratios** to obtain an equation in λ and μ *only*	M1	2.1
	$\frac{(4+\lambda)}{(-1+\mu)}=\frac{3}{1}$ or $\frac{\frac{1}{4}(4+\lambda)}{\frac{1}{4}(-1+\mu)}=\frac{3}{1}$	A1	1.1b
	$\lambda-3\mu+7=0$ * Allow $0=\lambda-3\mu+7$ but nothing else.	A1*	1.1b
		(4)	
(b)	λ = 2 => μ = 3 ; Resultant force = $(6\mathbf{i}+2\mathbf{j})$ (N)	M1	3.1a
	$(6\mathbf{i}+2\mathbf{j})=4\mathbf{a}$ **OR** $\lvert(6\mathbf{i}+2\mathbf{j})\rvert=4a$	M1	1.1b
	Use of $\mathbf{r}=\mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$ with **u** = **0**, their **a** and t = 4 : **Or** they may integrate their **a** twice with **u** = **0** and put t = 4 : $\mathbf{r}=\frac{1}{2}\times\frac{(6\mathbf{i}+2\mathbf{j})}{4}4^2=(12\mathbf{i}+4\mathbf{j})$	DM1	2.1
	$\sqrt{12^2+4^2}$	M1	1.1b
	ALTERNATIVE 1 for last two M marks: Use of $s=ut+\frac{1}{2}at^2$, with u = 0, their a and t = 4 : DM1 $s=\frac{1}{2}\times\sqrt{1.5^2+0.5^2}\times4^2$ Use of Pythagoras to find mag of **a** : $a=\sqrt{1.5^2+0.5^2}$ M1		
	ALTERNATIVE 2 for last two M marks: Use of $s=ut+\frac{1}{2}at^2$, with u = 0, their a and t = 4 : DM1 $s=\frac{1}{2}\times\left(\frac{\sqrt{6^2+2^2}}{4}\right)\times4^2$ Use of Pythagoras to find $\lvert(6\mathbf{i}+2\mathbf{j})\rvert$: $=\sqrt{6^2+2^2}$ M1		
	$\sqrt{160}$, $2\sqrt{40}$, $4\sqrt{10}$ oe or 13 or better (m)	A1	1.1b
		(5)	
			(9 marks)

Notes: Accept column vectors throughout

3a	M1	Adding the two forces, **i**'s and **j**'s must be collected (or must be a **single** column vector) seen or implied
	M1	Must be using ratios; Ignore an equation e.g. $(4+\lambda)\mathbf{i}+(-1+\mu)\mathbf{j}=3\mathbf{i}+\mathbf{j}$ if they go on to use ratios

		$4+\lambda = 3(-1+\mu)$ with no use of a constant, it's M0 They may use the acceleration, with a factor of $\frac{1}{4}$ top and bottom, see alternative **Allow one side of the equation to be inverted**
	A1	Correct equation
	A1*	Given answer correctly obtained. Must see at least one line of working, with the LH fraction 'removed'**.**
3b	M1	Adding $\mathbf{F}_1$ and $\mathbf{F}_2$ to find the resultant force, λ and μ must be substituted **N.B.** M0 if they use $\mu = 2$ coming from $-1+\mu = 1$ in part (a).
	M1	Use of $\mathbf{F} = 4\mathbf{a}$ Or $\lvert\mathbf{F}\rvert = 4a$, where **F** is <u>their</u> resultant. (including $3\mathbf{i} + \mathbf{j}$) This is an independent mark, so could be earned, for example, if they have subtracted the forces to find the 'resultant' **N.B.** M0 if only using $\mathbf{F}_1$ or $\mathbf{F}_2$
	DM 1	Dependent on previous M mark for **Either**: use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with **u** = **0**, their **a** and $t = 4$ to produce a displacement vector **Or** : integrate twice, with **u** = **0**, their **a** and $t = 4$ to produce a displacement Vector **Or**: use of $s = ut + \frac{1}{2}at^2$ with $u = 0$, their a and $t = 4$ to produce a length
	M1	Use of Pythagoras, with square root, to find the magnitude of their displacement vector, **a** or **F (**M0 if only using $\mathbf{F}_1$ or $\mathbf{F}_2$) depending on which method they have used.
	A1	cao

Question	Scheme	Marks	AOs

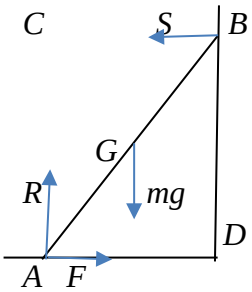
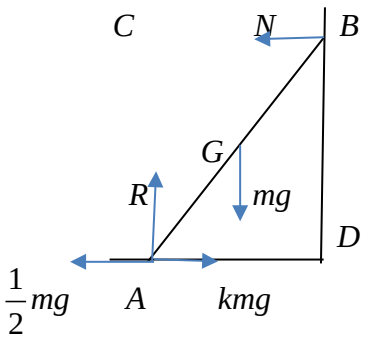
5(a)	Using horizontal motion	M1	3.3
	Whole Motion **Half way**		
	$U\cos\alpha \times t = 120$ $U\cos\alpha \times t = 60$	A1	1.1b
	Using vertical motion **OR**	M1	3.4
	$U\sin\alpha \times t - \frac{1}{2}gt^2 = 0$ $0 = U\sin\alpha - gt$	A1	1.1b
	Attempt to solve problem by eliminating t	DM1	3.1b
	$U^2\sin\alpha\cos\alpha = 588$ *	A1*	2.2a
		(6)	
	N.B. No credit given if they use the given answer from (b).		
5(b)	Using vertical motion **OR** conservation of energy	M1	3.4
	$0^2 = (U\sin\alpha)^2 - 2g\times 10$ $\frac{1}{2}mU^2 - \frac{1}{2}m(U\cos\alpha)^2 = mg\times 10$	A1	1.1b
	ALTERNATIVE 1: If t is time to top: use of $10 = \frac{1}{2}gt^2$ oe ($t = \frac{10}{7}$) to obtain an equation in U and α only M1 $U\sin\alpha = 14$ or $U\cos\alpha = 42$ A1 **ALTERNATIVE 2:** If t is time to top: use of : $10 = U\sin\alpha\ t - \frac{1}{2}gt^2$ with $t = \frac{60}{U\cos\alpha}$ substituted to obtain an equation in U and α only : M1 $10 = U\sin\alpha \times \frac{60}{U\cos\alpha} - \frac{1}{2}g\left(\frac{60}{U\cos\alpha}\right)^2$ A1		
	Attempt to solve problem by eliminating α : e.g. $U\sin\alpha = 14$ => $U\cos\alpha = 42$, from part (a) or from using $t = \frac{10}{7}$, then square and add to give result **OR:** $U^2\sin^2\alpha = 20g = 196$ and $U^2\sin\alpha\cos\alpha = 588$, divide to give $\tan\alpha = \frac{1}{3}$ then $\sin^2\alpha = \frac{1}{10}$, hence result **OR in ALTERNATIVE 2**: sub for U^2 using part (a), to give $\tan\alpha = \frac{1}{3}$ then $\sin^2\alpha = \frac{1}{10}$, hence result	DM1	3.1b

	N.B. Just stating that $\sin\alpha = \frac{4}{10}$, with no working is DM0A0.		
	$U^2 = 1960$ *	A1*	2.2a
	N.B. Verification (i.e. starting with $U^2 = 1960$ and trying to work backwards) is not an acceptable method for this question.		
		(4)	
5(c)	V, since air resistance has to be overcome, or just 'because of <u>air resistance' isw</u>	B1	3.5a
		(1)	
5(d)	e.g. wind effects, more accurate value of g, spin of ball, size of ball, shape of ball, dimensions of ball, not a particle, variable acceleration, surface area of ball, humidity. Allow wind resistance and rotational resistance (Ignore any mention of air resistance or drag)	B1	3.5c
		(1)	
			(12 marks)

Notes:

5a		**N.B**. Could score 2/6 for any one of the 4 given equations if there is no corresponding second equation or there is an attempt but it's incorrect.
	M1	Complete method to give equation in U, α and t only, condone sin/cos confusion and sign errors, each term that needs to be resolved must be resolved
	A1	Correct equation
	M1	Complete method to give equation in U, α and t only, condone sin/cos confusion and sign errors, each term that needs to be resolved must be resolved
	A1	Correct equation
	DM 1	Eliminate t, dependent on first and second M1's
	A1*	Given answer correctly obtained, <u>with no wrong working seen.</u> Allow $588 = U^2 \sin\alpha\cos\alpha$ but nothing else
5b	M1	Complete method to give equation in U and α only with correct no. of terms, condone sin/cos confusion and sign errors, each term that needs to be resolved must be resolved
	A1	Correct equation
	DM 1	Eliminate α and rearrange, dependent on first M1
	A1*	Given answer correctly obtained with <u>no wrong working seen</u> (**N.B**. If they use a value for α (18.43.°.) they lose the final A1*)
5c	B1	Clear statement isw
5d	B1	B0 if there is an incorrect extra e.g. mass or weight

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3(a)			
	Moments equation: (M1A0 for a moments inequality)	M1	3.3
	M(A), $mga\cos\theta = 2Sa\sin\theta$ M(B), $mga\cos\theta + 2Fa\sin\theta = 2Ra\cos\theta$ M(C), $F \times 2a\sin\theta = mga\cos\theta$ M(D), $2Ra\cos\theta = mga\cos\theta + 2Sa\sin\theta$ M(G), $Ra\cos\theta = Fa\sin\theta + Sa\sin\theta$.	A1	1.1b
	$(\updownarrow)$ $R = mg$ **OR** $(\leftrightarrow)$ $F = S$	B1	3.4
	Use their equations (they must have enough) and $F \le \mu R$ to give an inequality in μ and θ **only** (allow DM1 for use of $F = \mu R$ to give an *equation* in μ and θ only)	DM1	2.1
	$\mu \ge \frac{1}{2}\cot\theta$ *	A1*	2.2a
		(5)	
3(b)			
	Moments equation:	M1	3.4
	M(A), $mga\cos\theta = 2Na\sin\theta$ M(B), $mga\cos\theta + 2kmga\sin\theta = 2Ra\cos\theta + \frac{1}{2}mg2a\sin\theta$ M(D), $2Ra\cos\theta = mga\cos\theta + N2a\sin\theta$ M(G), $kmga\sin\theta + Na\sin\theta = \frac{1}{2}mga\sin\theta + Ra\cos\theta$	A1	1.1b

	S.C. M(C), $mga\cos\theta + \frac{1}{2}mg2a\sin\theta = kmg2a\sin\theta$ **M1A1B1** $1+\frac{5}{4}=\frac{5k}{2}$ **M1** $k=0.9$ **A1**		
	$N = kmg - F$ **OR** $R = mg$	B1	3.3
	Use their equations (they must have enough) to solve for k (numerical)	DM1	3.1b
	$k = 0.9$ oe	A1	1.1b
		(5)	
			(10 marks)

Notes:

3a	M1	Any moments equation with correct terms, condone sign errors and sin/cos confusion
	A1	Correct equation
	B1	Correct equation
	DM1	Dependent on M1, for using their equations (they must have enough) and $F \le \mu R$ to give an inequality in μ and θ only (allow M1 for use of $F = \mu R$ to give an equation in μ and θ only)
	A1*	Given answer correctly obtained with no wrong working seen (e.g. if they use $F = \mu R$ anywhere, A0)
3b	M1	Any moments equation with correct terms, condone sign errors
	A1	Correct equation
	B1	Correct equation
	DM1	Dependent on M1, for using their equations (they must have enough) with trig substituted, to solve for k, which must be numerical.
	A1	cao

Question	Scheme	Marks	AOs
	Note that $g = 10$; penalise once for whole question if $g = 9.8$		

4(a)	Take moments about A	M1	3.3
	$N \times \frac{4a}{\sin\alpha} = Mg \times 3a\cos\alpha$	A1	1.1b
	$\frac{9Mg}{25}$ *	A1*	1.1b
		(3)	
4(b)	Resolve horizontally	M1	3.4
	$(\rightarrow)\ F = \frac{9Mg}{25}\sin\alpha$	A1	1.1b
	Resolve vertically	M1	3.4
	$(\uparrow)\ R + \frac{9Mg}{25}\cos\alpha = Mg$	A1	1.1b
	Other possible equations: $(\nwarrow), R\cos\alpha + \frac{9Mg}{25} = Mg\cos\alpha + F\sin\alpha$ $(\nearrow), Mg\sin\alpha = F\cos\alpha + R\sin\alpha$ $\text{M}(C), Mg.2a\cos\alpha + F.5a\sin\alpha = R.5a\cos\alpha$ $\text{M}(G), \frac{9Mg}{25}.2a + F.3a\sin\alpha = R.3a\cos\alpha$ $\text{M}(B), Mg.3a\cos\alpha + F.6a\sin\alpha = R.6a\cos\alpha + \frac{9Mg}{25}a$ $(F = \frac{36Mg}{125}\ ,\ R = \frac{98Mg}{125})$		
	$F = \mu R$ used	M1	3.4
	Eliminate R and F and solve for μ	M1	3.1b
	Alternative equations if they have at A: X horizontally and Y perpendicular to the rod. $(\nwarrow), Y + \frac{9Mg}{25} = Mg\cos\alpha + X\sin\alpha$ $(\nearrow), Mg\sin\alpha = X\cos\alpha$ $(\uparrow), \frac{9Mg}{25}\cos\alpha + Y\cos\alpha = Mg$ $(\rightarrow), Y\sin\alpha + \frac{9Mg}{25}\sin\alpha = X$		

	M(C), $Mg.2a\cos\alpha + X.5a\sin\alpha = Y.5a$ M(G), $\frac{9Mg}{25}.2a + X.3a\sin\alpha = Y.3a$ M1A1 M1A1 M(B), $Mg.3a\cos\alpha + X.6a\sin\alpha = Y.6a + \frac{9Mg}{25}a$ $(X = \frac{4Mg}{3}, Y = \frac{98Mg}{75})$ Then $F = \mu R$ becomes: $X - Y\sin\alpha = \mu Y\cos\alpha$ M1 Eliminate X and Y and solve for μ M1		
	$\mu = \frac{18}{49}$ (0.3673…..accept 0.37 or better)	A1	2.2a
		(7)	
			(10 marks)

Notes:

4a	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors for an equation in N and Mg only. For perp distance allow any of : $\frac{4a}{\sin\alpha}, \frac{4a}{\cos\alpha}, 5a$ but use of any of : $6a, 5a\sin\alpha, 4a\cos\alpha, \ldots$ or anything involving $\tan\alpha$ is M0 Also M0 if no a's in their first equation.
	A1	Correct equation, trig does not need to be substituted
	A1*	Given answer correctly obtained.
4b	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors
	A1	Correct equation, trig does not need to be substituted but N does.
	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors
	A1	Correct equation, trig does not need to be substituted but N does.
		N.B. The above 4 marks are for any two equations, either resolutions or moments or one of each. Mark best two equations. Equations may appear in part (a) but must be used in (b) to earn marks.
	M1	Must be used, e.g. seen on the diagram. i.e. M0 if merely quoting it. (M0 if $F = \mu \times \frac{9Mg}{25}$ used)
	M1	Must have 3 equations (<u>and all 3 previous M marks</u>)
	A1	Accept 0.37 or better

Question	Scheme	Marks	AOs

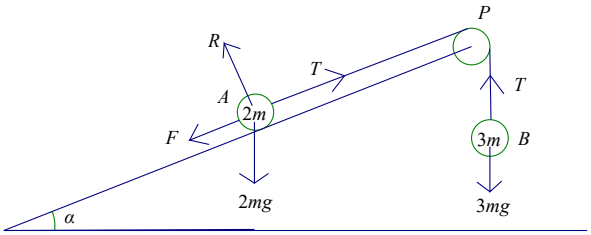
5(a)	Using horizontal motion	M1	3.3
	$U\cos 45^\circ t = 100$	A1	1.1b
	Using vertical motion	M1	3.4
	$U\sin 45^\circ t - \frac{1}{2}gt^2 = -25$	A1	1.1b
	Solve problem by eliminating t and solving for U	M1	3.1b
	$U = 28$*	A1*	1.1b
		(6)	
5(b)	Using vertical motion	M1	3.4
	$0^2 = (28\sin 45^\circ)^2 - 2gh$	A1	1.1b
	Greatest height = 45 m	A1	1.1b
		(3)	
5(c)	New value > 28	B1	3.5a
		(1)	
5(d)	e.g. wind effects, more accurate value of g, spin of ball, include size of the ball, not model as a particle, shape of ball	B1	3.5c
		(1)	
			(11 marks)

Notes:

5a	M1	Complete method to give equation in U and t only, condone sin/cos confusion and sign errors
	A1	Correct equation
	M1	Complete method to give equation in U and t only, condone sin/cos confusion and sign errors
	A1	Correct equation (g does not need to be substituted)
	M1	Must have earned the previous two M marks. Eliminate t and solve for U. **N.B**. They may solve for t first ($100 - \frac{1}{2}gt^2 = -25$) and then use it to find U.
	A1*	Exact given answer correctly obtained with no wrong working (e.g. g = 9.81 used) or approximation seen.
5b	M1	Complete method to give equation in h only (allow if U not substituted), condone sin/cos confusion and sign errors

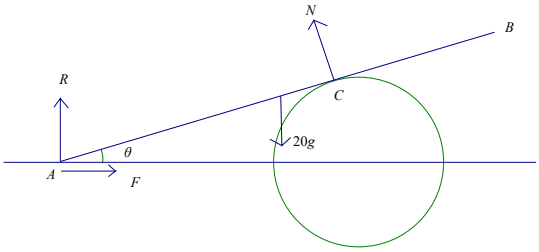
	A1	Correct equation (g does not need to be substituted) (A0 if U is used instead of 28)
	A1	cao
5c	B1	Clear statement
5d	B1	Penalise incorrect extras i.e. B0 if there are incorrect extras. The ground being horizontal, the cliff being vertical, .. are not part of the model so B0 Include weight/mass of the ball B0

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3(a)			
	$R = 2mg\cos\alpha$	B1	3.4
	$F = \frac{2}{3}R$	B1	1.2
	Equation of motion for A:	M1	3.3
	$T - F - 2mg\sin\alpha = 2ma$	A1	1.1b
	Equation of motion for B:	M1	3.3
	$3mg - T = 3ma$	A1	1.1b
	Complete strategy to find an equation in T, m and g only.	M1	3.1b
	$T = \frac{12mg}{5}$ *	A1*	2.2a
		(8)	
(b)	$(F_{\max} =)\frac{16mg}{13} > \frac{10mg}{13}$	M1	2.1
	…… so A will not move.	A1	2.2a
		(2)	
(c)	• Extensible string • Weight of string • Friction at pulley e.g. rough pulley • Allow for the dimensions of the blocks e.g. "Do not model blocks as particles"; "(include) air resistance";"include rotational effects of forces on blocks i.e. spin"	B1 B1	3.5c 3.5c
		(2)	
		(12)	

Marks		Notes
3a	B1	Normal reaction between A and the plane seen or implied, $\cos\alpha$ does not need to be substituted.
	B1	$F = \frac{2}{3}R$ seen or implied anywhere, including part (b)
	M1	Form an equation of motion for A. Must include all relevant terms. Must be the correct mass but condone consistent missing m's. Condone sign errors and sin/cos confusion
	A1	Correct unsimplified equation (F does not need to be substituted). Allow consistent use of $(-a)$ **N.B**. If $T - 2mg = 2ma$ is seen with no working, M0A0 unless both B1 marks have been scored.
	M1	Form an equation of motion for B. Must be the correct mass on RHS but condone consistent missing m's. Condone sign errors and sin/cos confusion.
	A1	Correct unsimplified equation (F does not need to be substituted). Allow consistent use of $(-a)$
		N.B. Allow the 'whole system' equation to replace the equation for A or B. $3mg - F - 2mg\sin\alpha = 5ma$ Must be the correct mass on RHS but condone consistent missing m's. Condone sign errors and sin/cos confusion.
	M1	Complete method to give an equation in T, m and g only. **N.B.** Allow θ in the equation if they have defined what θ is: e.g. $\theta = \tan^{-1}(\frac{5}{12})$ This is an <u>independent</u> mark but they must have two simultaneous equations in T and a unless one of the equations is the whole system equation in which case one equation will be in T and a and the other equation will be in a only.
	A1*	Obtain the **given answer** from correct working using EXACT trig ratios. (not available if using a decimal angle)
3b	M1	Comparison of their F_{max} ($\frac{2}{3}R$) and their component of weight down the slope, must be comparing numerical values. oe e.g. if they consider the difference **N.B.** Allow comparison of μ and $\tan\alpha$ with numerical values
	A1	Correctly justified conclusion and no errors seen **N.B.** If they equate their difference to an 'ma' term then A0
3c	B1 B1	Deduct 1 mark for each extra (more than 2) incorrect answer up to a maximum of 2 incorrect answers. Ignore extra correct answers. e.g. two correct, one incorrect B1 B0 one correct, one incorrect B1 B0 one correct, two incorrect B0 B0 Ignore incorrect reasons or consequences. Ignore any mention of wind or a general reference to friction.

Question	Scheme	Marks	AO

4(a)	Drum **smooth**, or no friction, (therefore reaction is perpendicular to the ramp)	B1	2.4
		(1)	
(b)	**N.B.** In (b), for a moments equation, if there is an extra $\sin\theta$ or $\cos\theta$ on a length, give M0 for the equation e.g. M(A): $20g\times 4\cos\theta = 5N\sin\theta$ would be given M0A0		
			
	Possible equns ($\nearrow$): $F\cos\theta + R\sin\theta = 20g\sin\theta$ ($\nwarrow$): $N + R\cos\theta = 20g\cos\theta + F\sin\theta$ ($\uparrow$) $R + N\cos\theta = 20g$ ($\rightarrow$): $F = N\sin\theta$ M(A): $20g\times 4\cos\theta = 5N$ M(B): $3N + R\times 8\cos\theta = F\times 8\sin\theta + 20g\times 4\cos\theta$ M(C): $R\times 5\cos\theta = F\times 5\sin\theta + 20g\times\cos\theta$ M(G): $R\times 4\cos\theta = F\times 4\sin\theta + N$	M1 A1 M1 A1 M1 A1	3.3 1.1b 3.4 1.1b 3.4 1.1b
	(The values of the 3 unknowns are: $N = 150.528$; $F = 42.14784$; $R = 51.49312$)		
	Alternative 1: using cpts along ramp (X) and perp to ramp(Y) Possible equations: ($\nearrow$): $X = 20g\sin\theta$ ($\nwarrow$): $Y + N = 20g\cos\theta$ ($\uparrow$): $X\sin\theta + Y\cos\theta + N\cos\theta = 20g$ ($\rightarrow$): $X\cos\theta = Y\sin\theta + N\sin\theta$ M(A): $20g\times 4\cos\theta = 5N$ M(B): $20g\times 4\cos\theta = 8Y + 3N$ M(C): $20g\times\cos\theta = 5Y$ M(G): $4Y = N\times 1$	M1 A1 M1 A1 M1 A1	3.3 1.1b 3.4 1.1b 3.4 1.1b
	(The values of the 3 unknowns are: $N = 150.528$; $X = 54.88$; $Y = 37.632$)		

	Alternative 2: using horizontal cpt (*H*) and cpt perp to ramp (*S*) $(\nearrow): H\cos\theta = 20g\sin\theta$ $(\nwarrow): S + N = H\sin\theta + 20g\cos\theta$ $(\uparrow): S\cos\theta + N\cos\theta = 20g$ $(\rightarrow): H = S\sin\theta + N\sin\theta$ $\text{M}(A): 20g\times 4\cos\theta = 5N$ $\text{M}(B): 20g\times 4\cos\theta + H\times 8\sin\theta = 8S + 3N$ $\text{M}(C): 20g\times\cos\theta + H\times 5\sin\theta = 5S$ $\text{M}(G): 4S = N\times 1 + H\times 4\sin\theta$	M1 A1 M1 A1 M1 A1	3.3 1.1b 3.4 1.1b 3.4 1.1b
	(The values of the 3 unknowns are: $N = 150.528$; $H = 57.1666\ldots$; $S = 53.638666\ldots$)		
	Solve their 3 equations for F and R **OR** X and Y **OR** H and S	M1	1.1b
	$\lvert\text{Force}\rvert = \sqrt{R^2 + F^2}$ Main scheme **OR** $= \sqrt{X^2 + Y^2}$ Alternative 1 **OR** $= \sqrt{(H^2 + S^2 - 2HS\cos(90^o - \theta)}$ Alternative 2	M1	3.1b
	Magnitude = 67 or 66.5 (N)	A1	2.2a
		(9)	
(c)	Magnitude of the normal reaction (at *C*) will **decrease**.	B1	3.5a
		(1)	
		(11)	

Marks		Notes
4a	B1	Ignore any extra incorrect comments.
		Generally 3 independent equations required so **at least one moments equation.**: M1A1M1A1M1A1. More than 3 equations, give marks for the best 3. For each: M1 All terms required. Must be dimensionally correct so if a length is missing from a moments equation it's M0 Condone sin/cos confusion. A1 For a correct equation (trig ratios do not need to be substituted and allow e.g. cos(24/25) if they recover Enter marks on ePEN in order in which equations appear. **N.B.** If reaction at C is not perpendicular to the ramp, can only score marks for M(C) Allow use of (μR) for F
4b	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required, dim correct, condone sin/cos confusion
	A1	Correct unsimplified equation
		N.B. They can find F and R using only TWO equations, the 1st and 7th in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1
Alt 1	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
		N.B. They can find X and Y using only TWO equations, the 1st and 7th in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1
Alt 2	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.

	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct.
	A1	Correct unsimplified equation
		N.B. They can find *H* and *S* using only TWO equations, the 1st and 7th in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1
	M1	Substitute for trig and solve for their two cpts. This is an independent mark <u>but must use 3 equations (</u>unless it's the special case when 2 is sufficient<u>)</u>
	M1	Use Pythagoras to find magnitude (this is an <u>independent</u> M mark but must have found a value for *F* (or *X*) and a value for *R* (or *Y*)) **OR** a complete method to find magnitude e.g. cosine rule but must have found a value for *H* and a value for *S*
	A1	Correct answer only
	B1	Ignore reasons

Question	Scheme	Marks	AO
	In this question mark parts (a) and (b) together.		

7(a)	Resolve vertically	M1	3.1b
	$R + 40\sin\alpha = 20g$	A1	1.1b
	Resolve horizontally	M1	3.1b
	$40\cos\alpha - F = 20a$	A1	1.1b
	$F = 0.14R$	B1	1.2
	$a = 0.396$ or 0.40 (m s^{-2})	A1	2.2a
		(6)	
(b)	Pushing will increase R which will increase available F	B1	2.4
	Increasing F will <u>decrease a</u> * GIVEN ANSWER	B1*	2.4
		(2)	
			(8 marks)

Notes:

(a)

M1: Resolve vertically with usual rules applying

A1: Correct equation**.** Neither g nor $\sin a$ need to be substituted

M1: Apply $F = ma$ horizontally, with usual rules

A1: Neither F nor $\cos a$ need to be substituted

B1: $F = 0.14R$ seen (e.g. on a diagram)

A1: Either answer

(b)

B1: Pushing increases R which produces an increase in available (limiting) friction

B1: F increase produces an <u>a decrease (need to see this)</u>

N.B. It is possible to score B0 B1 but for the B1, some "explanation" is needed to say why friction is increased e.g. by pushing into the ground.

Question	Scheme	Marks	AOs

10(a)	Using the model and vertical motion: $0^2 = (U\sin\alpha)^2 - 2g\times(3-2)$	M1	3.3
	$U^2 = \dfrac{2g}{\sin^2\alpha}$ * GIVEN ANSWER	A1*	2.2a
		(2)	
(b)	Using the model and horizontal motion: $s = ut$	M1	3.4
	$20 = Ut\cos\alpha$	A1	1.1b
	Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$	M1	3.4
	$-\frac{5}{4} = Ut\sin\alpha - \frac{1}{2}gt^2$	A1	1.1b
	sub for t: $-\frac{5}{4} = U\sin\alpha\left(\frac{20}{U\cos\alpha}\right) - \frac{1}{2}g\left(\frac{20}{U\cos\alpha}\right)^2$	M1 (I)	3.1b
	sub for U^2	M1(II)	3.1b
	$-\frac{5}{4} = 20\tan\alpha - 100\tan^2\alpha$	A1(I)	1.1b
	$(4\tan\alpha - 1)(100\tan\alpha + 5) = 0$	M1(III)	1.1b
	$\tan\alpha = \frac{1}{4} \Rightarrow \alpha = 14^0$ or better	A1(II)	2.2a
		(9)	
	N.B. For the last 5 marks, they may set up a quadratic in t, by substituting for $U\sin\alpha$ first, then solve the quadratic to find the value of t, then use $20 = Ut\cos\alpha$ to find α. The marks are the same but earned in a different order. Enter on ePen in the corresponding M and A boxes above, as indicated below.		
	Sub for $U\sin\alpha$ to give equation in t only	M1(II)	
	$-\frac{5}{4} = \sqrt{2g}t - \frac{1}{2}gt^2$	A1(I)	
	Solve for t	M1(III)	
	$t = \dfrac{5}{\sqrt{2g}}$ or 1.1 or 1.13 and use $20 = Ut\cos\alpha$	M1(I)	
	$\alpha = 14^0$ or better	A1(II)	
(b)	**ALTERNATIVE**		

	Using the model and horizontal motion: $s = ut$	M1	3.4
	$20 = Ut\cos\alpha$	A1	1.1b
	A to top: $s = vt - \frac{1}{2}at^2$ and top to T: $s = ut + \frac{1}{2}at^2$		
	$1 = \frac{1}{2}gt_1^2 \Rightarrow t_1 = \sqrt{\frac{2}{g}}$ and $\frac{9}{4} = \frac{1}{2}gt_2^2 \Rightarrow t_2 = \frac{3}{\sqrt{2g}}$ Total time $t = t_1 + t_2$	M1	3.4
	$= \sqrt{\frac{2}{g}} + \frac{3}{\sqrt{2g}}$ $(= \frac{5}{\sqrt{2g}})$	A1	1.1b
	$20 = U\frac{5}{\sqrt{2g}}\cos\alpha$ (sub. for t)	M1	3.1b
	$20 = \sqrt{\frac{2g}{\sin^2\alpha}}\frac{5}{\sqrt{2g}}\cos\alpha$ (sub. for U)	M1	3.1b
	$\tan\alpha = \frac{1}{4}$	A1	1.1b
	Solve for α	M1	1.1b
	$\Rightarrow \alpha = 14^0$ or better	A1	2.2a
		(9)	
(c)	The target will have dimensions so in practice there would be a range of possible values of α **Or** There will be air resistance **Or** The ball will have dimensions **Or** Wind effects **Or** Spin of the ball	B1	3.5b
		(1)	
(d)	Find U using their α e.g. $U = \sqrt{\frac{2g}{\sin^2\alpha}}$	M1	3.1b
	Use $20 = Ut\cos\alpha$ (or use vertical motion equation)	A1 **M1**	1.1b
	$t = \frac{5}{\sqrt{2g}}$ or 1.1 or 1.13	B1 **A1**	1.1b
		(3)	
(d)	**ALTERNATIVE**		

	A to top: $s = vt - \frac{1}{2}at^2$ and top to T: $s = ut + \frac{1}{2}at^2$	M1	3.1b
	$1 = \frac{1}{2}gt_1^2 \Rightarrow t_1 = \sqrt{\frac{2}{g}}$ and $\frac{9}{4} = \frac{1}{2}gt_2^2 \Rightarrow t_2 = \frac{3}{\sqrt{2g}}$ Total time $t = t_1 + t_2$	A1 **M1**	1.1b
	$= = \sqrt{\frac{2}{g}} + \frac{3}{\sqrt{2g}}$ $(= \frac{5}{\sqrt{2g}})$ = 1.1 or 1.13 (s)	B1 **A1**	1.1b
		(3)	
			(15 marks)

Notes:

(a)

M1: Or any other complete method to obtain an equation in U, g and α **only**

A1*: Correct GIVEN ANSWER

(b)

M1: Using horizontal motion

A1: Correct equation

M1: Using vertical motion . N.B. M0 if they use $s = \pm 2$ or ± 3, but allow $s = \pm 1.25$ or ± 0.75 or ± 2.25 or ± 2.75

A1: Correct equation

M1: Using $20 = Ut\cos\alpha$ to sub. for t

M1: Substituting for U^2 using (a)

A1: Correct quadratic equation (in $\tan\alpha$ **or** $\cot\alpha$ **)**

M1: Solve a 3 term quadratic, either by factorisation or formula (or by calculator (implied) if answer is correct) **and find** α

A1: $\alpha = 14^o$ or better (No restriction on accuracy since g's cancel)

N.B. If answer is correct, previous M mark can be implied, but if answer is incorrect, an explicit attempt to solve must be seen to earn the previous M mark.

(b) ALTERNATIVE

M1: Using the model with the usual rules applying to the equation

A1: Correct equation

M1: Using the model to obtain the **total** time from A to T

A1: Correct **total** time t

M1: Substitute for t in $20 = Ut\cos\alpha$

M1: Substitute for U in $20 = Ut\cos\alpha$, using part (a)

A1: Correct equation in $\tan\alpha$ **only**

M1: Solve equation for α

A1: $\alpha = 14^o$ or better (No restriction on accuracy since g's cancel)

N.B. If they quote the equation of the trajectory $y = x\tan\alpha - \dfrac{gx^2}{2U^2\cos^2\alpha}$ oe **AND** put in values for x and y, could score first 5 marks, M1A1M1A1M1 (nothing for the equation only); wrong x value loses first A mark and wrong y value loses second A mark

(c)

B1: Give one limitation of the model e.g. the ball will have dimensions, or there will be air resistance or wind effects or spin

N.B. B0 if any incorrect extra(s) but ignore extra consequences.

(d)

M1: Using their a to find a value for U

A1: Treat as M1: Using their U to find a value for t

B1: Treat as A1 : $t = 1.1$ or 1.10 (since depends on $g = 9.8$)

(d) ALTERNATIVE

M1: Using their a to find a value for U

A1: Treat as M1: Using their U to find a value for t

B1: Treat as A1 : $t = 1.1$ or 1.10 (since depends on $g = 9.8$)

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