adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Geometric sequences and series

Mark scheme

2 questions

Exam questions and marking instructions remain credited to their board.

Contents

1 June 2024 - Paper 1 - Q9

2 October 2021 - Paper 1 - Q5

Question	Scheme	Marks	AOs
9(a)	$\dfrac{9^{7-2k}}{3^{4k-5}}=\dfrac{3^{2(k-1)}}{9^{7-2k}} \Rightarrow \dfrac{3^{2(7-2k)}}{3^{4k-5}}=\dfrac{3^{2(k-1)}}{3^{2(7-2k)}}$	M1	3.1a
	$\left(3^{2(7-2k)}\right)^2=3^{4k-5}\times 3^{2(k-1)}$ $\Rightarrow 28-8k=6k-7 \Rightarrow k=\ldots$ $3^{2(7-2k)-(4k-5)}=3^{2(k-1)-2(7-2k)}$ $\Rightarrow 19-8k=6k-16 \Rightarrow k=\ldots$	dM1	1.1b
	$k=\dfrac{5}{2}$*	A1*	2.1
		(3)	
(b)	$a=3^{4(2.5)-5}$ and $r=\dfrac{9^{7-2(2.5)}}{3^{4(2.5)-5}} \Rightarrow$ one of $a=243$ or $r=\dfrac{1}{3}$	M1	2.2a
	$S_\infty=\dfrac{a}{1-r}=\dfrac{"243"}{1-"\frac{1}{3}"}$	M1	1.1b
	$S_\infty=\dfrac{729}{2}(364.5)$ cao	A1	1.1b
		(3)	
			(6 marks)

Notes

(a) **Special cases:**

SC 1: For those that verify rather than prove a SC 100 is awarded for substituting $k=\dfrac{5}{2}$ into all three terms to correctly obtain 243, 81 and 27 **with** a statement that this is **geometric** with $r=\dfrac{1}{3}$ (or equivalent reason). All statements must be correct.

SC 2: Be aware that e.g. $\dfrac{9^{7-2k}}{3^{4k-5}}=\dfrac{3^{2(k-1)}}{9^{7-2k}} \Rightarrow 81^{2(7-2k)}=9^{6k-7}$ is an incorrect process (without some indication that they have intentionally squared both sides) that fortuitously leads to the correct answer and may score maximum SC 010.

M1: Uses the 3 terms to set up an equation in k **and**

- **either** reaches a common base by replacing 9 with 3^2 or by replacing 3 with $9^{0.5}$ and uses the power law of indices correctly
- **or** uses the laws of indices correctly to reach $9^{14-4k}=3^{6k-7}$ condoning slips in e.g. expanding brackets.

Writing down e.g. $2(7-2k)-(4k-5)=2(k-1)-2(7-2k)$ is sufficient to imply the M1.

dM1: Correct processing leading to a value for k.

A1*: Correct value following correct working. Allow $k=2.5$ in place of $k=\dfrac{5}{2}$

Condone missing/invisible brackets provided they are recovered correctly.

Alt 1 Using Base 9:

$$\frac{9^{7-2k}}{3^{4k-5}}=\frac{3^{2(k-1)}}{9^{7-2k}} \Rightarrow \frac{9^{7-2k}}{9^{2k-2.5}}=\frac{9^{k-1}}{9^{7-2k}} \text{ o.e. scores M1}$$

$$\Rightarrow 9^{9.5-4k}=9^{3k-8} \Rightarrow 9.5-4k=3k-8 \Rightarrow 7k=17.5 \Rightarrow k=2.5$$

Score dM1 when a value of k is achieved using a correct process and A1* if fully correct.

Alt 2 Finding r in terms of k and using e.g. $u_3 = ar^2$:

$$r = \frac{9^{7-2k}}{3^{4k-5}} = \frac{3^{2(7-2k)}}{3^{4k-5}} \left\{= 3^{19-8k}\right\} \text{ or } r = \frac{3^{2(k-1)}}{9^{7-2k}} = \frac{3^{2k-2}}{3^{2(7-2k)}} \left\{= 3^{6k-16}\right\}$$

$$\Rightarrow 3^{4k-5} \times \left(3^{19-8k}\right)^2 = 3^{2k-2} \text{ or } \Rightarrow 3^{4k-5} \times \left(3^{6k-16}\right)^2 = 3^{2k-2} \text{ scores M1}$$

$$\Rightarrow 3^{4k-5} \times 3^{2(19-8k)} = 3^{2k-2} \Rightarrow 33 - 12k = 2k - 2 \Rightarrow 14k = 35 \Rightarrow k = 2.5$$

Score dM1 when a value of k is achieved using a correct process and A1* if fully correct.

Alt 3 Using Logs Way 1:

$$\frac{9^{7-2k}}{3^{4k-5}} = \frac{3^{2(k-1)}}{9^{7-2k}} \Rightarrow \left(9^{7-2k}\right)^2 = 3^{6k-7} \Rightarrow 9^{14-4k} = 3^{6k-7} \text{ scores M1}$$

$$\Rightarrow (14-4k)\log_3 9 = 6k - 7$$

$$\Rightarrow 2(14-4k) = 6k - 7$$

$$\Rightarrow k = 2.5$$

Score dM1 when a value of k is achieved using a correct process and A1* if fully correct.

Alt 4 Using Logs Way 2:

$$\frac{9^{7-2k}}{3^{4k-5}} = \frac{3^{2(k-1)}}{9^{7-2k}}$$

$$\Rightarrow (7-2k)\log_3 9 - (4k-5)\log_3 3 = (2k-2)\log_3 3 - (7-2k)\log_3 9 \quad \text{scores M1}$$

$$\Rightarrow 2(7-2k) - (4k-5) = 2k - 2 - 2(7-2k)$$

$$\Rightarrow 19 - 8k = 6k - 16$$

$$\Rightarrow k = 2.5$$

Score dM1 when a value of k is achieved using a correct process and A1* if fully correct.

Alt 5 Recognising that taking $\log_3$ forms an Arithmetic Sequence:

$$\{\log_3\} u_1 = 4k - 5,\ \{\log_3\} u_2 = 2(7-2k),\ \{\log_3\} u_3 = 2(k-1)$$

$$\Rightarrow 2(7-2k) - (4k-5) = 2(k-1) - 2(7-2k) \quad \text{scores M1}$$

$$\Rightarrow 19 - 8k = 6k - 16$$

$$\Rightarrow k = 2.5$$

Score dM1 when a value of k is achieved using a correct process and A1* if fully correct.
There is no need to see any mention of log in this approach.

(b)

M1: Deduces expressions for the first term **and** the common ratio using $k = \frac{5}{2}$ in the <u>correct formulae</u> **and** finds at least one of $a = 243$ or $r = \frac{1}{3}$. Allow if seen in (a). May be implied by correct values for a and r. For reference, $a = 3^{4(2.5)-5}$ and $r = \frac{9^{7-2(2.5)}}{3^{4(2.5)-5}}$ $\left\{\text{or } r = \frac{3^{2(2.5-1)}}{9^{7-2(2.5)}}\right\}$

M1: Recalls the sum to infinity formula and substitutes their values for a and r provided $|r| < 1$
Dependent on a correct attempt to find both a and r using $k = 2.5$ but allow if neither value is correct or if they are unprocessed e.g. $\frac{3^{4(2.5)-5}}{1 - \frac{9^{7-2(2.5)}}{3^{4(2.5)-5}}}$ scores this mark.

A1: cao. Correct sum to infinity. Answer only (with no working) scores full marks. Apply isw.

Question	Scheme	Marks	AOs

5(a)	$u_3 = £20\,000 \times 1.08^2 = (£)23328*$	B1*	1.1b
		(1)	
(b)	$20\,000 \times 1.08^{n-1} > 65\,000$	M1	1.1b
	$1.08^{n-1} > \frac{13}{4} \Rightarrow n-1 > \frac{\ln(3.25)}{\ln(1.08)}$ or e.g. $1.08^{n-1} > \frac{13}{4} \Rightarrow n-1 > \log_{1.08}\left(\frac{13}{4}\right)$	M1	3.1b
	Year 17	A1	3.2a
		(3)	
(c)	$S_{20} = \frac{20\,000\left(1-1.08^{20}\right)}{1-1.08}$	M1	3.4
	Awrt (£) 915 000	A1	1.1b
		(2)	
			(6 marks)
Notes			

(a)

B1*: Uses a correct method to show that the Profit in Year 3 will be £23 328. Condone missing units

E.g. $£20\,000 \times 1.08^2$ or $£20\,000 \times 108\% \times 108\%$

This may be obtained in two steps. E.g $\frac{8}{100} \times 20000 = 1600$ followed by $\frac{8}{100} \times 21600 = 1728$ with the calculations $21600 + 1728 = 23328$ seen.

Condone calculations seen as 8% of $20000 = 1600$.

This is a show that question and the method must be seen.

It is not enough to state Year 1 = £21 600, Year 2 = £ 23 328

(b)

M1: Sets up an inequality or an equation that will allow the problem to be solved.

Allow for example N or n for $n-1$. So award for $20\,000\times1.08^{n-1}>65\,000$,

$20\,000\times1.08^{n}=65\,000$ or $20\,000\times(108\%)^{n}\geqslant 65\,000$ amongst others.

Condone **slips** on the 20 000 and 65 000 but the 1.08 o.e. must be correct

M1: Uses a correct strategy involving logs in an attempt to solve a type of equation or inequality of the form seen above. It cannot be awarded from a sum formula

The equation/inequality must contain an index of $n-1$, N, n etc.

Again condone **slips** on the 20 000 and 65 000 but additionally condone an error on the 1.08, which may appear as 1.8 for example

E.g. $20\,000\times1.08^{n}=65\,000\Rightarrow n\log 1.08=\log\dfrac{65000}{20000}\Rightarrow n=...$

E.g. $20\,00\times1.8^{n}=65\,000\Rightarrow \log 2000+n\log 1.8=\log 65000\Rightarrow n=...$

A1: Interprets their decimal value and gives the correct year number. Year 17

The demand of the question dictates that solutions relying entirely on calculator technology are not acceptable, BUT allow a solution that appreciates **a correct term** formula or the entire set of calculations where you may see the numbers as part of a larger list

E.g. Uses, or implies the use of, an acceptable calculation and finds value(s)

for M1: $(n=16)\Rightarrow P=20\,000\times1.08^{15}$=awrt 63400 **or** $(n=17)\Rightarrow P=20\,000\times1.08^{16}$ = awrt 68500

M1: $(n=16)\Rightarrow P=20\,000\times1.08^{15}$=awrt 63400 **and** $(n=17)\Rightarrow P=20\,000\times1.08^{16}$ = awrt 68500

A1: 17 years following correct method and both M's

(c)

M1: Attempts to use the model with a **correct** sum formula to find the total profit for the 20 years.

You may see an attempt to find the sum of 20 terms via a list. This is acceptable provided there are 20 terms with $u_n=1.08\times u_{n-1}$ seen at least 4 times and the sum attempted.

Condone a slip on the 20 000 (e.g appearing as 2 000) and/or a slip on the 1.08 with it being the same "r" as in (b) . Do not condone 20 appearing as 19 for instance

A1: awrt £915 000 but condone missing unit

The demand of the question dictates that all stages of working should be seen. An answer without working scores M0 A0

Question	Scheme	Marks	AOs