adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Graphs and transformations

Mark scheme

4 questions

Exam questions and marking instructions remain credited to their board.

Contents

1 June 2024 - Paper 1 - Q6

2 June 2023 - Paper 2 - Q12

3 June 2022 - Paper 2 - Q1

4 June 2019 - Paper 2 - Q6

Question	Scheme	Marks	AOs
6(a)	$x = 2$ **or** $y = 5$	B1	1.1b
	$P(2, 5)$	B1	2.2a
		(2)	
(b)	$16 - 4x = 3(x-2) + 5 \Rightarrow x = ...$	M1	1.1b
	$x = \frac{17}{7}$	A1	2.1
		(2)	
(c)	$k_{\max} = 3$ or $k_{\min} = \frac{"5" - 4}{"2"}$	M1	3.1a
	$\frac{1}{2} < k < 3$	A1	2.5
		(2)	
			(6 marks)

Notes

(a)

B1: One correct coordinate. Either $x = 2$ or $y = 5$ or $(2, \ldots)$ or $(\ldots, 5)$ seen.

B1: Deduces $(2, 5)$ Accept written separately e.g. $x = 2$, $y = 5$ isw after a correct answer. Condone 2, 5 without the brackets.

(b)

M1: Attempts to solve the correct equation without modulus signs $16 - 4x = 3(x-2) + 5 \Rightarrow x = ...$ Must reach a value for x. Ignore attempts at e.g. $16 - 4x = 3(2 - x) + 5$

A1: $x = \frac{17}{7}$ o.e. exact answer and no other values. If other values have been found they must be rejected or the $x = \frac{17}{7}$ clearly selected. Answer only implies both marks.

Note: x = "2.75" coming from $5 = 16 - 4x$ may be found as part of their working to establish which branch of the modulus graph the line $y = 16 - 4x$ intersects. If this is the case it need not be "rejected" provided it is not clearly stated as one of their solutions.

Those that achieve $|x| = \frac{17}{7}$ can score BOD M1A0.

Alternative by squaring:

$$16 - 4x = 3|x-2| + 5 \Rightarrow 11 - 4x = 3|x-2|$$
$$\Rightarrow 16x^2 - 88x + 121 = 9\left(x^2 - 4x + 4\right)$$
$$\Rightarrow 7x^2 - 52x + 85 = 0 \Rightarrow x = 5, \frac{17}{7}$$

M1: Isolates the $|x-2|$ (or $3|x-2|$), squares both sides and solves the resulting 3TQ using the usual rules and may be by calculator, leading to a value for x.

A1: Selects the $\frac{17}{7}$ or rejects any other values as in main scheme.

(c)

M1: Correct method to find either critical value (following through on their P).

Either k {=} 3 or k {=} $\frac{"5"-4}{"2"}$ scores M1 without evidence of an incorrect method.

Note that $k = 3$ occasionally appears from use of the discriminant on $x(k-3)+5=0,$ and scores M0 unless there is an alternative valid reason given.

Allow the use of e.g. m = in place of k = here but do not allow x = or y =

A1: Correct range in terms of k in acceptable notation with no incorrect method seen.

Use of e.g. x is A0. Allow "and" or "$\cap$" to join the regions but not "or" or "," or "$\cup$"

Accept e.g. $(0.5,3)$; $k \in \left(\frac{1}{2},3\right)$; $k<3$ and $k>\frac{1}{2}$; $k>\frac{1}{2} \cap k<3$

but not $\frac{1}{2}<x<3$; $\frac{1}{2}$,, k,, 3 ; $\left[\frac{1}{2},3\right]$; $k>\frac{1}{2} \cup k<3$; $k>\frac{1}{2}, k<3$; $k>\frac{1}{2}$ or $k<3$

Alt 1 via solving simultaneous equations:

$$\text{e.g. } kx+4=3(x-2)+5 \Rightarrow kx+4=3x-1 \Rightarrow x=-\frac{5}{k-3}$$

$$kx+4=3(2-x)+5 \Rightarrow kx+4=11-3x \Rightarrow (k+3)x=7$$

$$\Rightarrow (k+3)\left(\frac{-5}{k-3}\right)=7 \Rightarrow k=\frac{1}{2}$$

M1: Sets $kx+4=3(x-2)+5$ and $kx+4=3(2-x)+5$, eliminates x, and solves for k

A1: As main scheme.

Alt 2 via squaring and the discriminant:

$$kx+4=3|x-2|+5 \Rightarrow kx-1=3|x-2|$$

$$\Rightarrow k^2x^2-2kx+1=9\left(x^2-4x+4\right)$$

$$\Rightarrow \left(k^2-9\right)x^2+(36-2k)x-35=0$$

$$\Rightarrow (36-2k)^2-4\left(k^2-9\right)(-35)=0$$

$$\Rightarrow 144k^2-144k+36=0 \Rightarrow k=\frac{1}{2}$$

M1: Sets $kx+4=3|x-2|+5$, isolates $|x-2|$ (or $3|x-2|$), squares both sides, uses $b^2-4ac\ldots 0$ where … is any equality or inequality, and solves the resulting 3TQ using the usual rules and may be by calculator, leading to a value for k.

Condone slips in expanding the brackets.

A1: As main scheme.

Alt 3 via Domain for the right hand branch of the modulus graph:

$$kx+4=3x-1 \Rightarrow x=\frac{-5}{k-3}>2 \quad \left\{\text{or } x=\frac{5}{3-k}>2\right\}$$

$$\Rightarrow k-3<0 \quad \{\text{and} \quad \Rightarrow -5<2(k-3)\}$$

$$\Rightarrow k<3 \quad \{\text{and} \quad \Rightarrow 0.5<k\}$$

M1: Sets $kx+4=3(x-2)+5$, makes x the subject, sets > 2 and deduces a critical value.

A1: As main scheme.

Question	Scheme	Marks	AOs

12(a)	$N_A - N_B = (3+4)-(8-6) = ...$	**M1**	3.4
	5000 (subscribers)	**A1**	3.2a
		(2)	
(b)	$(T=)3$	**B1**	3.4
	This was the point when company A had the lowest number of subscribers	**B1**	2.4
		(2)	
(c)			
	$-t+7=2t+2$ o.e. or $t+1=14-2t$ o.e.	**B1**	3.1a
	$-t+7=2t+2$ o.e. $\Rightarrow t=...$ or $t+1=14-2t$ o.e. $\Rightarrow t=...$	**M1**	3.4
	One of the two critical values $t=\frac{5}{3}$ or $t=\frac{13}{3}$	**A1**	1.1b
	Chooses the outside region for their **two** values of t Both of $t<"\frac{5}{3}"$, $t>"\frac{13}{3}"$	**A1ft**	2.2a
	$\left\{t\in\mathbb{R}:t<\frac{5}{3}\right\}\cup\left\{t\in\mathbb{R}:t>\frac{13}{3}\right\}$	**A1**	2.5
		(5)	
(d)	The number of subscribers will become negative (when $t>7$)	**B1**	3.5b
		(1)	
			(10 marks)

Notes

(a)

M1: Uses the models to find the difference when $t=0$. Allow slips in evaluating N_A and N_B but it must be clear that $t = 0$ is being used. Just 5 with no working implies M1.

A1: 5000 or 5 thousand (subscribers) (5 is A0)

(b)

B1: $(t/T=)3$ Just look for the number 3 so e.g. $t > 3$ or e.g. "just after 3" is acceptable.

If more than one value is offered then score B0 unless it is clear that the 3 is intended.
Must be seen in (b) not just on their diagram.

B1: Any acceptable reason e.g.

- This was the point when company A had the lowest number of subscribers
- After this point the number of subscribers started to increase
- It is the minimum
- Condone "it is the turning point"
- The graph changes direction
- It is the vertex
- The gradient becomes positive
- N_A increased

Allow this mark even if the first B mark was not scored e.g. $T = 3.5$ because the graph starts to increase scores B0B1

Do not allow contradictory statements.

Do not allow:

- The graph reflects at $t = 3$ on its own without further clarification

(c)

B1: Forms one valid equation (allow an equation or any inequality sign)

M1: Attempts to solve one valid equation (allow an equation or any inequality sign)

A1: For either $t=\frac{5}{3}$ or $t=\frac{13}{3}$ only (allow an equation or any inequality sign) or exact equivalent

Must be seen or used in part (c).

See notes below for attempts that use "squaring" to find the values of *t*.

A1ft: Chooses the outside region for their **two** values of t where $t > 0$.

So for $t = a$ and $t = b$ where $0 < a < b$ should be $t < a$, $t > b$. Allow , /or/and/ $\cup$ / $\cap$

Condone if incorrectly combined e.g. $"\frac{13}{3}" < t < "\frac{5}{3}"$ but **not** $"\frac{5}{3}" < t < "\frac{13}{3}"$

A1: Fully correct solution in the form $\left\{t : t < \frac{5}{3}\right\} \cup \left\{t : t > \frac{13}{3}\right\}$ or $\left\{t \middle| t < \frac{5}{3}\right\} \cup \left\{t \middle| t > \frac{13}{3}\right\}$ or

$\left(0,\ \frac{5}{3}\right) \cup \left(\frac{13}{3},\ 5\right)$ either way around but condone $\left\{t < \frac{5}{3}\right\} \cup \left\{t > \frac{13}{3}\right\}$, $\left\{t : t < \frac{5}{3} \cup t > \frac{13}{3}\right\}$,

$\left\{t < \frac{5}{3} \cup t > \frac{13}{3}\right\}$ or $\left(-\infty,\ \frac{5}{3}\right) \cup \left(\frac{13}{3},\ \infty\right)$.

It is not necessary to mention R , e.g. $\left\{t : t \in \mathrm{R}, t > \frac{13}{3}\right\} \cup \left\{t : t \in \mathrm{R}, t < \frac{5}{3}\right\}$

Look for $\{\ \}$ and $\cup$ or condone $\left(-\infty,\ \frac{5}{3}\right) \cup \left(\frac{13}{3},\ \infty\right)$

Do not allow solutions not in set notation such as $t < \frac{5}{3}$ or $t > \frac{13}{3}$.

Note that a lower bound for $t < \frac{5}{3}$ and an upper bound for $t > \frac{13}{3}$ are not required but may be included e.g. $\left\{t \in \square : 0 < t < \frac{5}{3}\right\} \cup \left\{t \in \square : \frac{13}{3} < t < 5\right\}$ or $\left\{t \in \square : 0 \text{ „ } t < \frac{5}{3}\right\} \cup \left\{t \in \square : \frac{13}{3} < t \text{ „ } 5\right\}$

Note that the marks in this part require **valid** equations to be solved. They must have removed the mod brackets and arrived at an equation equivalent to $-t + 7 = 2t + 2$ or $t + 1 = 14 - 2t$ (all you need to check initially is whether their equation **without mod brackets** is equivalent to one of these).

Note that $\left\{t : t < \frac{5}{3},\ t > \frac{13}{3}\right\}$ is condoned for the A1ft but not for the final A1.

If x is used in their set notation then final A0, but we would condone this for the penultimate A1ft.

See notes below for answers given with no working.

(d)

B1: Requires any indication that the number of subscribers will become negative. E.g.

- It allows negative subscribers (which isn't possible)
- $8 - |2t - 6| \ldots 0 \Rightarrow t$ „ 7 so not valid after $t = 7$ but condone not valid for t after (any value above 7)

But not

- Subscribers will become zero

Guidance for attempts that use "squaring" to find the values of *t* in (c):

Way 1:

$(-t+7)^2=(2t+2)^2$ o.e. or $(t+1)^2=(14-2t)^2$ o.e.	**B1**	3.1a
$(-t+7)^2=(2t+2)^2 \Rightarrow t=...$ o.e. (Gives –9 and $\frac{5}{3}$) or $(t+1)^2=(14-2t)^2 \Rightarrow t=...$ o.e. (Gives 15 and $\frac{13}{3}$)	**M1**	3.4
One of the two critical values $t=\frac{5}{3}$ or $t=\frac{13}{3}$	**A1**	1.1b
Chooses the outside region for their **two** values of *t* Both of $t<"\frac{5}{3}"$, $t>"\frac{13}{3}"$	**A1ft**	2.2a
$\left\{t\in\mathbb{R}:t<\frac{5}{3}\right\}\cup\left\{t\in\mathbb{R}:t>\frac{13}{3}\right\}$	**A1**	2.5

Way 2:

$\lvert t-3\rvert+4=8-\lvert 2t-6\rvert \Rightarrow \lvert t-3\rvert+\lvert 2t-6\rvert=4 \Rightarrow 3t-9=4$ o.e.	**B1**	3.1a
$(3t-9)^2=4^2 \Rightarrow 9t^2-54t+81=16 \Rightarrow 9t^2-54t+65=0 \Rightarrow t=...$ (Gives $\frac{5}{3}$ and $\frac{13}{3}$)	**M1**	3.4
One of the two critical values $t=\frac{5}{3}$ or $t=\frac{13}{3}$	**A1**	1.1b
Chooses the outside region for their **two** values of *t* Both of $t<"\frac{5}{3}"$, $t>"\frac{13}{3}"$	**A1ft**	2.2a
$\left\{t\in\mathbb{R}:t<\frac{5}{3}\right\}\cup\left\{t\in\mathbb{R}:t>\frac{13}{3}\right\}$	**A1**	2.5

B1: Forms one valid equation and squares both sides (allow an equation or any inequality sign)

May be implied by e.g. $(t-3+4)^2=(8-(2t-6))^2$

Alternatively, arrives at $3t-9=4$ (o.e.) as in way 2.

M1: Attempts to solve one valid equation after squaring both sides (allow an equation or any inequality sign). Note that it is acceptable to just solve $3t-9=4$

A1: As in main scheme. **A1ft:** As in main scheme. **A1:** As in main scheme.

Note: the following is common and scores 00000.

$$\lvert t-3\rvert+4=8-\lvert 2t-6\rvert \Rightarrow (t-3)^2+4=8-(2t-6)^2$$

Which typically leads to

$$t=\frac{15\pm 4\sqrt{15}}{5}$$

Guidance for answers only in part (c):

t...awrt 1.7 or t...awrt 4.3 where … is any inequality or equation scores **11000**

$t...\frac{5}{3}$ or $t...\frac{13}{3}$ where … is any inequality or equation scores **11100** for one correct c.v.

Both $t <$ awrt 1.7 and $t > b$ where $\left\{b > \frac{5}{3}\right\}$ scores **11010** for outside region.

Both $t < a$ and $t >$ awrt 4.3 where $\left\{a < \frac{13}{3}\right\}$ scores **11010** for outside region.

Both $t < \frac{5}{3}$ and $t > b$ where $\left\{b > \frac{5}{3}\right\}$ scores **11110** for outside region with one correct.

Both $t < a$ and $t > \frac{13}{3}$ where $\left\{a < \frac{13}{3}\right\}$ scores **11110** for outside region with one correct.

Both $t < \frac{5}{3}$ and $t > \frac{13}{3}$ scores **11110** for outside region with one correct.

Fully correct e.g. $\left\{t : t < \frac{5}{3}\right\} \cup \left\{t : t > \frac{13}{3}\right\}$ scores **11111**

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
1	For an attempt to solve **Either** $3-2x=7+x \Rightarrow x=...$ **or** $2x-3=7+x \Rightarrow x=...$	M1	1.1b
	Either $x=-\frac{4}{3}$ **or** $x=10$	A1	1.1b
	For an attempt to solve **Both** $3-2x=7+x \Rightarrow x=...$ **and** $2x-3=7+x \Rightarrow x=...$	dM1	1.1b
	For **both** $x=-\frac{4}{3}$ **and** $x=10$ with no extra solutions	A1	1.1b
		(4)	
ALT	**Alternative by squaring:**		
	$(3-2x)^2=(7+x)^2 \Rightarrow 9-12x+4x^2=49+14x+x^2$	M1	1.1b
	$3x^2-26x-40=0$	A1	1.1b
	$3x^2-26x-40=0 \Rightarrow x=...$	dM1	1.1b
	For **both** $x=-\frac{4}{3}$ **and** $x=10$ with no extra solutions	A1	1.1b
			(4 marks)

Notes:

Note this question requires working to be shown not <u>just answers written down</u>. But correct equations seen followed by the correct answers can score full marks.

M1: Attempts to solve either correct equation.

Allow equivalent equations e.g. $3-2x=-7-x \Rightarrow x=...$

A1: One correct solution. Allow exact equivalents for $-\frac{4}{3}$ e.g. $-1\frac{1}{3}$ or $-1.\dot{3}$ but not e.g. -1.33

dM1: Attempts to solve both correct equations.

Allow equivalent equations e.g. $3-2x=-7-x \Rightarrow x=...$ **Depends on the first method mark.**

A1: For both $x=-\frac{4}{3}$ and $x=10$ with no extra solutions and neither clearly rejected but ignore any attempts to find the *y* coordinates whether correct or otherwise and ignore reference to e.g. x = -7 (from where $y = 7 + x$ intersects the *x*-axis) or $x=1.5$ (from finding the value of *x* at the vertex) as "extras". Allow exact equivalents for $-\frac{4}{3}$ e.g. $-1\frac{1}{3}$ or $-1.\dot{3}$ but not rounded e.g. -1.33

Isw if necessary e.g. ignore subsequent attempts to put the values in an inequality e.g. $-\frac{4}{3}<x<10$

But if e.g. $x=-\frac{4}{3}$ is obtained and a candidate states $x=\left|-\frac{4}{3}\right|$ then score A0

Alternative solution via squaring

M1: Attempts to square both sides. Condone poor squaring e.g. $(3-2x)^2 = 9 \pm 4x^2$ or $9 \pm 2x^2$

A1: Correct quadratic equation $3x^2 - 26x - 40 = 0$. The "= 0" may be implied by their attempt to solve. Terms must be collected but not necessarily all on one side so allow e.g. $3x^2 - 26x = 40$

dM1: Correct attempt to solve a **3 term** quadratic. See general guidance for solving a quadratic equation. The roots can be written down from a calculator so the method may be implied by their values. **Depends on the first method mark**.

A1: For both $x = -\frac{4}{3}$ and $x = 10$ with no extra solutions and neither clearly rejected but ignore any attempts to find the *y* coordinates and do not count e.g. $x = -7$ (from where $y = 7 + x$ intersects the *x*-axis) or $x = 1.5$ (from finding the value of *x* at the vertex) as "extras". Allow exact equivalents for $-\frac{4}{3}$ e.g. $-1\frac{1}{3}$ or $-1.\dot{3}$ but not e.g. -1.33

Isw if necessary e.g. ignore subsequent attempts to put the values in an inequality e.g. $-\frac{4}{3} < x < 10$

But if e.g. $x = -\frac{4}{3}$ is obtained and a candidate states $x = \left|-\frac{4}{3}\right|$ then score A0

Question	Scheme	Marks	AOs

6 (a)	$gg(0) = g((0-2)^2+1) = g(5) = 4(5)-7 = 13$	M1	2.1
		A1	1.1b
		(2)	
(b)	Solves either $(x-2)^2+1=28 \Rightarrow x=...$ or $4x-7=28 \Rightarrow x=...$	M1	1.1b
	At least one critical value $x=2-3\sqrt{3}$ or $x=\frac{35}{4}$ is correct	A1	1.1b
	Solves both $(x-2)^2+1=28 \Rightarrow x=...$ and $4x-7=28 \Rightarrow x=...$	M1	1.1b
	Correct final answer of '$x<2-3\sqrt{3}$, $x>\frac{35}{4}$,	A1	2.1
	Note: Writing awrt -3.20 or a truncated -3.19 or a truncated -3.2 in place of $2-3\sqrt{3}$ is accepted for any of the A marks	**(4)**	
(c)	h is a one-one {function (or mapping) so has an inverse} g is a many-one {function (or mapping) so does not have an inverse}	B1	2.4
		(1)	
(d) **Way 1**	$\left\{h^{-1}(x)=-\frac{1}{2} \Rightarrow\right\} \quad x=h\left(-\frac{1}{2}\right)$	M1 B1 on epen	1.1b
	$x=\left(-\frac{1}{2}-2\right)^2+1$ **Note:** Condone $x=\left(\frac{1}{2}-2\right)^2+1$	M1	1.1b
	$\Rightarrow x=7.25$ only **cso**	A1	2.2a
		(3)	
(d) **Way 2**	$\{\text{their } h^{-1}(x)\} = \pm 2 \pm \sqrt{x \pm 1}$	M1	1.1b
	Attempts to solve $\pm 2 \pm \sqrt{x \pm 1} = -\frac{1}{2} \Rightarrow \pm\sqrt{x \pm 1} = ...$	M1	1.1b
	$\Rightarrow x=7.25$ only **cso**	A1	2.2a
		(3)	
			(10 marks)

Notes for Question 6

(a)	
M1:	Uses a complete method to find $gg(0)$. E.g. • Substituting $x=0$ into $(0-2)^2+1$ and the result of this into the relevant part of $g(x)$ • Attempts to substitute $x=0$ into $4((x-2)^2+1)-7$ or $4(x-2)^2-3$
A1:	$gg(0)=13$
(b)	
M1:	See scheme
A1:	See scheme
M1:	See scheme
A1:	Brings all the strands of the problem together to give a correct solution.
Note:	You can ignore inequality symbols for any of the M marks
Note:	If a 3TQ is formed (e.g. $x^2-4x-23=0$) then a correct method for solving a 3TQ is required for the relevant method mark to be given.
Note:	Writing $(x-2)^2+1=28 \Rightarrow (x-2)+1=\sqrt{28} \Rightarrow x=-1+\sqrt{28}$ (i.e. taking the square-root of each term to solve $(x-2)^2+1=28$ is not considered to be an acceptable method)
Note:	Allow set notation. E.g. $\{x \in \mathbb{R}: x<2-3\sqrt{3} \cup x>8.75\}$ is fine for the final A mark

Notes for Question 6 Continued	
(b)	***continued***
Note:	Give final A0 for $\{ x \in \mathbb{R} : x < 2-3\sqrt{3} \cap x > 8.75 \}$
Note:	Give final A0 for $2-3\sqrt{3} > x > 8.75$
Note:	Allow final A1 for their writing a final answer of "$x < 2-3\sqrt{3}$ and $x > \frac{35}{4}$"
Note:	Allow final A1 for a final answer of $x < 2-3\sqrt{3}$, $x > \frac{35}{4}$
Note:	Writing $2-\sqrt{27}$ in place of $2-3\sqrt{3}$ is accepted for any of the A marks
Note:	Allow final A1 for a final answer of $x < -3.20$, $x > 8.75$
Note:	Using 29 instead of 28 is M0 A0 M0 A0
(c)	
B1:	A correct explanation that conveys the underlined points
Note:	A minimal acceptable reason is "h is a one-one and g is a many-one"
Note:	Give B1 for "h^{-1} is one-one and g^{-1} is one-many"
Note:	Give B1 for "h is a one-one and g is not"
Note:	Allow B1 for "g is a many-one and h is not"
(d)	**Way 1**
M1:	Writes $x = \mathrm{h}\left(-\frac{1}{2}\right)$
M1:	See scheme
A1:	Uses $x = \mathrm{h}\left(-\frac{1}{2}\right)$ to deduce that $x = 7.25$ only, **cso**
(d)	**Way 2**
M1:	See scheme
M1:	See scheme
A1:	Use a correct $\mathrm{h}^{-1}(x) = 2-\sqrt{x-1}$ to deduce that $x = 7.25$ only, **cso**
Note:	Give final A0 cso for $2+\sqrt{x-1} = -\frac{1}{2} \Rightarrow \sqrt{x-1} = -\frac{5}{2} \Rightarrow x-1 = \frac{25}{4} \Rightarrow x = 7.25$
Note:	Give final A0 cso for $2 \pm \sqrt{x-1} = -\frac{1}{2} \Rightarrow \sqrt{x-1} = -\frac{5}{2} \Rightarrow x-1 = \frac{25}{4} \Rightarrow x = 7.25$
Note:	Give final A1 cso for $2 \pm \sqrt{x-1} = -\frac{1}{2} \Rightarrow -\sqrt{x-1} = -\frac{5}{2} \Rightarrow x-1 = \frac{25}{4} \Rightarrow x = 7.25$
Note:	Allow final A1 for $2 \pm \sqrt{x-1} = -\frac{1}{2} \Rightarrow \pm\sqrt{x-1} = -\frac{5}{2} \Rightarrow x-1 = \frac{25}{4} \Rightarrow x = 7.25$

Question	Scheme	Marks	AOs