adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

# Hypothesis testing

## Mark scheme

3 questions

Exam questions and marking instructions remain credited to their board.

# Contents


1 June 2024 - Paper 3 Statistics - Q4

2 June 2022 - Paper 3 Statistics - Q4

3 June 2019 - Paper 3 Statistics - Q5

| Qu 4 | Scheme | Marks | AO |
|---|---|---|---|
| **(a)** | $H_0 : p = 0.1$ $H_1 : p \neq 0.1$ [Allow 10% for 0.1] | B1 | 2.5 |
| | [ $X \sim B(40, 0.1)$] $\Rightarrow$ $P(X = 0) = 0.0148$ [Allow any letter for $X$] | M1 | 3.3 |
| | $P(X \ldots 9) = 1 - P(X \,„\, 8) = 1 - 0.9845 = 0.0155$ | A1 | 1.1b |
| | Critical region is $\{X = 0\} \cup \{X \ldots 9\}$ (o.e.) | A1 | 1.1b |
| | | **(4)** | |
| **(b)** | ["0.0148" + "0.0155" ] = 0.0303 | B1ft | 1.1b |
| | | **(1)** | |
| **(c)** | [Provided 7 is not in their CR ] insufficient evidence to support Freya's belief | B1 | 2.2b |
| | | **(1)** | |
| | | **( 6 marks)** | |

| | Notes |
|---|---|
| | **Mark (a) and (b) together for sight of the probabilities.** |
| **(a)** | B1 for both hypotheses in terms of $p$ or $\pi$. Must be attached to $H_0$ and $H_1$ |
| | M1 for use of the correct model. Implied by sight of at least one probability truncated or rounded to at least 2sf from: 0.0155, 0.0148, 0.0805, 0.9845, 0.9581, 0.9949<br>Implied by sight of fully correct CR (with no probs) so e.g. $X = 0$, $X > 8$ scores M1A0A0 |
| | 1st A1 for at least one correct probability (to at least 3sf) with its probability statement<br>i.e. for $P(X = 0)$ = awrt 0.0148 or $P(X \ldots 9)$ = awrt 0.0155 |
| | 2nd A1(**dep on M1 and 1st A1** but not on B1)<br>for both correct probs (to at least 3 sf) and the correct critical region<br>Do not need set notation. Allow $X < 1$ and $X > 8$ or words e.g. "0 or greater than 8" etc<br>Allow "," or "and" or "or" or "$\cap$" between $X \,„\, 0$ and $X \ldots 9$<br>$P(X = 0)$ and $P(X \ldots 9)$ is 2nd A0 |
| **(b)** | B1ft for awrt 3.03% or correct sum of their two probabilities (provided each is less than 0.5)<br>Their probabilities must be to at least 2sf and relate to their CR |
| | **To score in (c) they must have a CR of the form ($X = 0$ or $X < a$)and $X > b$, where $b$ is …7**<br>**May be implied by $P(X < a)$ and $P(X > b)$ i.e. 2nd A0 in (a) but correct form. Need $b > a$** |
| **(c)** | B1 for a suitable comment in context that suggests no support for Freya's belief/claim<br>**or** e.g. insufficient evidence of change in proportion/percentage of left-handed adults<br>**or** e.g. proportion/percentage of left-handed adults is not different from 10% ( **or** …is 10% )<br>**or** e.g. 10% of adults in the country are left-handed<br>Do not allow contradictory comments e.g. "in CR so no support for Freya's belief" is B0 |
| **NB** | A correct contextual answer in (c) using an **acceptance region** please send to review. |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **4(a)** | $H_0: p = 0.1$ $H_1: p \neq 0.1$ | B1 | 2.5 |
| | | **(1)** | |
| **(b)** | Use of $X \sim B(50, 0.1)$<br>implied by sight of one of awrt 0.0052 or awrt 0.9755 or awrt 0.0245 | M1 | 3.4 |
| | Critical regions $X = 0$ or $X \geqslant 10$ | A1 | 1.1b |
| | $X = 0$ and $X \geqslant 10$ plus<br>$P(X = 0) =$ awrt 0.0052 and $P(X \geqslant 10) =$ awrt 0.0245 | A1 | 1.1b |
| | **SC**: Both CR correct with no probabilities and no distribution seen scores M0A1A0 | | |
| | | **(3)** | |
| **(c)** | 0.0297 | B1ft | 1.1b |
| | | **(1)** | |
| **(d)** | 15 is <u>in the critical region</u> therefore there is evidence to support the **<u>manager</u>**'s belief | B1ft | 2.2b |
| | | **(1)** | |
| | | | **(6 marks)** |

**Notes**

| | | |
|---|---|---|
| **(a)** | **B1** | For both hypotheses in terms of $p$ or $\pi$. Connected to $H_0$ and $H_1$ correctly<br>Condone 10% but not 10 |
| **(b)** | **M1** | Using correct distribution to find the probability associated with one tail of the CR<br>If the correct distribution is <u>stated</u> (may be seen in part(a)) allow for one tail of the correct CR **or** one of (awrt 0.025 or awrt 0.005 or awrt 0.975) seen connected to a correct probability statement |
| | **A1** | Lower CR $X = 0$ / $X < 1$ / $X \leqslant 0$ / [condone eg $P(X = 0)$ labelled as CR]<br>Or Upper CR $X \geqslant 10$ or $X > 9$ [condone $P(X \geqslant 10)$ oe labelled as CR] |
| | **A1** | Both CR's correct with the relevant probabilities Allow $\cup$ for "and" and $X > 9$, $X < 1$, $X \leqslant 0$ [**do not allow** $P(X = 0)$ or $P(X \geqslant 10)$ oe]<br>Allow CR in different form eg $(9, \infty)$, $[10, \infty)$ |
| **(c)** | **B1ft** | awrt 0.0297 or 2.97% or ft for the sum of the probabilities in (b) for "their 2 critical regions" if seen. If none seen it must be awrt 0.0297<br>**SC M0 in (b) for a one tail test** Allow B1ft for their one tail CR in (b) eg 0.0338 or 0.0245 or 0.0579 |
| **(d)** | **B1ft** | A correct statement about 15 and "their CR" or sight $P(X \geqslant 15) = 0.0000738...$ and comparison with "their 0.0245"<br>**and** a compatible correct statement in context. eg There is evidence that there has been a change in the **<u>proportion/probability</u>** arriving **<u>late</u>**<br>Condone increase rather than change<br>Do not allow contradicting statements.<br>NB No CR given in (b) then B0 |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 5(a) | $\frac{24.63-25}{'\sigma'} = -1.0364$ | M1 | 3.1b |
| | $[\sigma =]0.357$ (must come from compatible signs) | A1 | 1.1b |
| | $P(D > k) = 0.4$ or $P(D < k) = 0.6$ | B1 | 1.1b |
| | $\frac{k-25}{'0.357'} = 0.2533$ | M1 | 3.4 |
| | $k$ = awrt **25.09** | A1 | 1.1b |
| | | **(5)** | |
| **(b)** | $[Y \sim B(200, 0.45) \rightarrow]$ $W \sim N(90, 49.5)$ | B1 | 3.3 |
| | $P(Y < 100) \approx P(W < 99.5)$ $\left[ = P\left( Z < \frac{99.5-90}{\sqrt{49.5}} \right) \right]$ | M1 | 3.4 |
| | = 0.9115… awrt **0.912** | A1 | 1.1b |
| | | **(3)** | |
| **(c)** | $H_0 : \mu = 25$ $H_1 : \mu < 25$ | B1 | 2.5 |
| | $[\bar{D} \sim] N\left(25, \frac{0.16^2}{20}\right)$ | M1 | 3.3 |
| | $P(\bar{D} < 24.94)[= P(Z < -1.677...)] = 0.046766...$ | A1 | 3.4 |
| | $p = 0.047 < 0.05$ or $z = -1.677\ldots < -1.6449$<br>or $24.94 < 24.94115\ldots$<br>or reject $H_0$ /in the critical region/significant | M1 | 1.1b |
| | There is sufficient evidence to support Hannah's belief. | A1 | 2.2b |
| | | **(5)** | |
| | | | **(13 marks)** |

**Notes**

**(a)**

**M1:** for standardising 24.63, 25 and '$\sigma$' (ignore label) and setting = to $z$ where $1 < |z| < 2$
**A1:** [$\sigma =$] awrt 0.36. Do **not** award this mark if signs are not compatible.
**B1:** for either correct probability statement (may be implied by correct answer)
this mark may be scored for a correct region shown on a diagram
**M1:** for a correct expression with $z$ = awrt 0.253 (may be implied by correct answer)
**A1:** awrt 25.09 (Correct answer with no incorrect working scores 5 out of 5)

**(b)**

**B1:** setting up normal distribution approximation of binomial N(90, 49.5) (may be implied by a correct answer) Look out for e.g. $\sigma = \frac{3\sqrt{22}}{2}$ or $\sigma =$ awrt 7.04
**M1:** attempting a probability using a continuity correction i.e. $P(W < 100.5)$, $P(W < 99.5)$ or $P(W < 98.5)$ condone $\leq$ (The continuity correction may be seen in a standardisation).
**A1:** awrt 0.912 [Note: 0.911299… from binomial scores 0 out of 3]

**(c)**

**B1:** for both hypotheses in terms of $\mu$
**M1:** selecting suitable model must see N(ormal), mean 25, sd = $\frac{0.16}{\sqrt{20}}$ (o.e.) or var = $\frac{4}{3125}$ (o.e.)
Condone N(25, $\frac{0.16}{\sqrt{20}}$) if $\frac{0.16}{\sqrt{20}}$ then used as s.d.
**A1:** $p$ value = awrt 0.047 or **test statistic** awrt –1.68 or **CV** awrt 24.941
(any of these values imply the M1 provided they do not come from Normal mean = 24.94)
**M1:** a correct comparison (including compatible signs) **or** correct non-contextual conclusion (f.t. their $p$ value, test statistic or critical value in the comparison)
M1 may be implied by a correct contextual statement
**NB** Any contradictory non contextual statements/comparisons score M0A0 e.g. '$p < 0.05$, not significant'
**A1:** correct conclusion in context mentioning Hannah's belief
or the mean amount/liquid in each bottle is now less than 25ml (**dep on M1A1M1**)

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