adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Integration

Question paper

23 questions

Exam questions and marking instructions remain credited to their board.

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5. (a) Express $\lim_{\delta x \to 0} \sum_{x=1.44}^{2.89} \frac{2}{\sqrt{x}} \delta x$ as an integral.

(1)

(b) Hence show that

$$\lim_{\delta x \to 0} \sum_{x=1.44}^{2.89} \frac{2}{\sqrt{x}} \delta x = k$$

where k is an integer to be found.

(*Solutions relying on calculator technology are not acceptable.*)

(2)

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Question 5 continued

(Total for Question 5 is 3 marks)

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13. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Show that

$$\int_0^2 \frac{x}{(2x+1)^3}\,\mathrm{d}x = \frac{2}{25}$$

(5)

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Question 13 continued

(Total for Question 13 is 5 marks)

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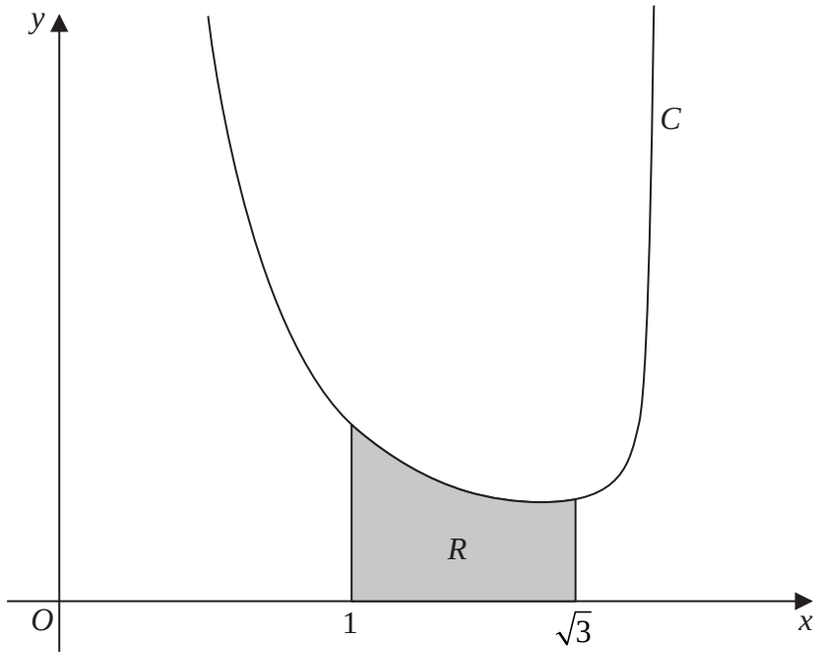


Figure 5

Figure 5 shows a sketch of the curve C with equation

$$y = \frac{1}{x^2\sqrt{4 - x^2}} \qquad 0 < x < 2$$

The region R, shown shaded in Figure 5, is bounded by C, the line with equation $x = 1$, the x-axis and the line with equation $x = \sqrt{3}$

(a) Use the substitution $x = 2\sin u$ to show that the area of R is given by

$$\int_a^b k\,\text{cosec}^2 u\,\text{d}u$$

where a, b and k are constants to be found.

(4)

(b) Hence, using algebraic integration, find the exact area of R.
Give your answer in simplest form.

(3)

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Question 16 continued

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Question 16 continued

(Total for Question 16 is 7 marks)

TOTAL FOR PAPER IS 100 MARKS

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2. Find

$$\int \left(x^4 - 6x^{\frac{1}{2}} - 3 \right) \mathrm{d}x$$

giving the answer in simplest form.

(4)

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Question 2 continued

(Total for Question 2 is 4 marks)

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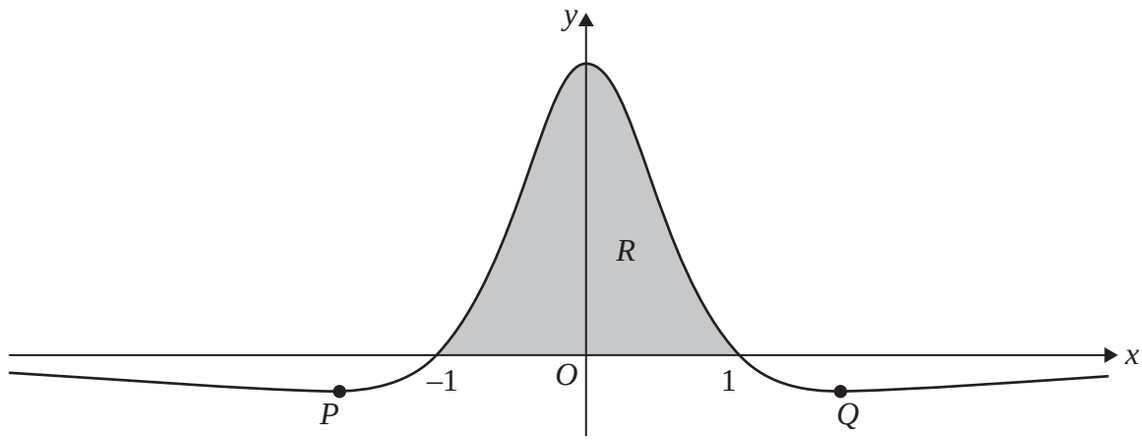


Figure 5

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 5 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{1 - x^2}{\left(1 + x^2\right)^2}$$

The curve

- intersects the x-axis at –1 and 1
- has minimum turning points at P and Q

as shown in Figure 5.

(a) Use calculus to find the exact coordinates of P.

(5)

(b) Using the substitution $x = \tan\theta$ show that

$$\int_{-1}^{1} \mathrm{f}(x)\,\mathrm{d}x = \int_{\alpha}^{\beta} \cos 2\theta\,\mathrm{d}\theta$$

where α and β are constants to be found.

(5)

The finite region R, shown shaded in Figure 5, is bounded by the x-axis and the curve.

(c) Use algebraic integration to find the area of R.

(3)

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Question 15 continued

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Question 15 continued

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Question 15 continued

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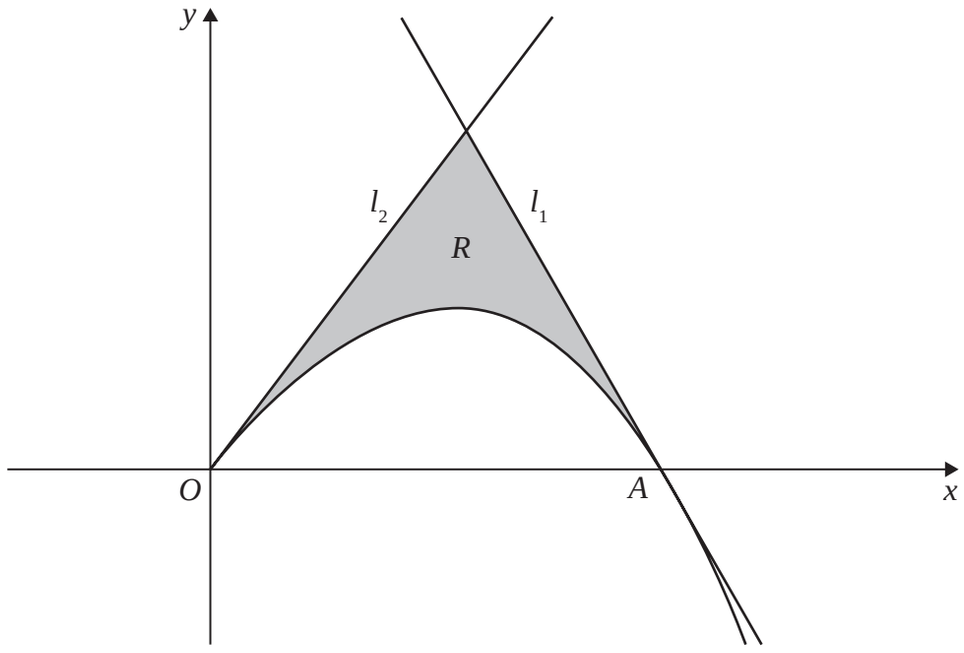


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = 8x - x^{\frac{5}{2}} \qquad x \geqslant 0$$

The curve crosses the x-axis at the point A.

(a) Verify that the x coordinate of A is 4

(1)

The line l_1 is the tangent to the curve at A.

(b) Use calculus to show that an equation of line l_1 is

$$12x + y = 48$$

(3)

The line l_2 has equation $y = 8x$

The region R, shown shaded in Figure 3, is bounded by the curve, the line l_1 and the line l_2

(c) Use algebraic integration to find the exact area of R.

(5)

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Question 10 continued

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Question 10 continued

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Question 10 continued

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13. (a) Given that a is a positive constant, use the substitution $x = a\sin^2\theta$ to show that

$$\int_0^a x^{\frac{1}{2}}\sqrt{a-x}\ \mathrm{d}x = \frac{1}{2}a^2\int_0^{\frac{\pi}{2}}\sin^2 2\theta\ \mathrm{d}\theta$$

(4)

(b) Hence use algebraic integration to show that

$$\int_0^a x^{\frac{1}{2}}\sqrt{a-x}\ \mathrm{d}x = k\pi a^2$$

where k is a constant to be found.

(4)

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Question 13 continued

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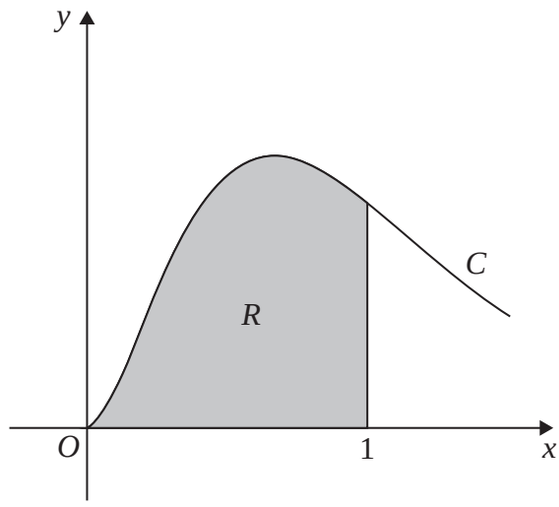


Figure 5

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 5 shows a sketch of part of the curve C with equation

$$y = 8x^2e^{-3x} \qquad x \geqslant 0$$

The finite region R, shown shaded in Figure 5, is bounded by

- the curve C
- the line with equation $x = 1$
- the x-axis

Find the exact area of R, giving your answer in the form

$$A + Be^{-3}$$

where A and B are rational numbers to be found.

(5)

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Question 11 continued

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Question 11 continued

(Total for Question 11 is 5 marks)

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1. Find

$$\int \frac{x^{\frac{1}{2}}(2x-5)}{3}\,\mathrm{d}x$$

writing each term in simplest form.

(4)

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Question 1 continued

(Total for Question 1 is 4 marks)

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10. $$f(x) = \frac{3kx - 18}{(x + 4)(x - 2)}$$ where k is a positive constant

(a) Express $f(x)$ in partial fractions in terms of k.

(3)

(b) Hence find the exact value of k for which

$$\int_{-3}^{1} f(x)\, dx = 21$$

(4)

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Question 10 continued

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13. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$(3 + x)^{-2}$$

writing each term in simplest form.

(4)

(b) Using the answer to part (a) and using algebraic integration, estimate the value of

$$\int_{0.2}^{0.4} \frac{6x}{(3 + x)^2} \, dx$$

giving your answer to 4 significant figures.

(4)

(c) Find, using algebraic integration, the exact value of

$$\int_{0.2}^{0.4} \frac{6x}{(3 + x)^2} \, dx$$

giving your answer in the form $a \ln b + c$, where a, b and c are constants to be found.

(5)

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Question 13 continued

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Question 13 continued

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Question 13 continued

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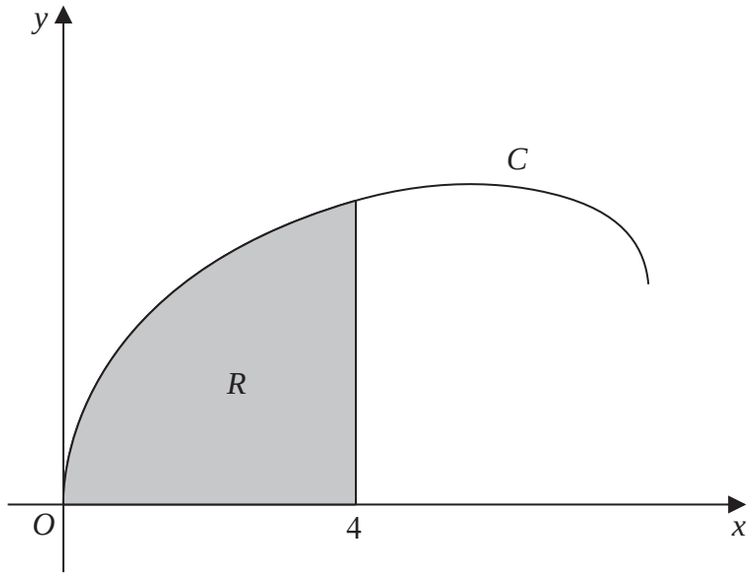


Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 8\sin^2 t \qquad y = 2\sin 2t + 3\sin t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 6, is bounded by C, the x-axis and the line with equation $x = 4$

(a) Show that the area of R is given by

$$\int_0^a \left(8 - 8\cos 4t + 48\sin^2 t \cos t\right) \mathrm{d}t$$

where a is a constant to be found.

(5)

(b) Hence, using algebraic integration, find the exact area of R.

(4)

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Question 16 continued

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Question 16 continued

(Total for Question 16 is 9 marks)

TOTAL FOR PAPER IS 100 MARKS

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5. The table below shows corresponding values of x and y for $y = \log_3 2x$

The values of y are given to 2 decimal places as appropriate.

x	3	4.5	6	7.5	9
y	1.63	2	2.26	2.46	2.63

(a) Using the trapezium rule with all the values of y in the table, find an estimate for

$$\int_3^9 \log_3 2x \, \mathrm{d}x$$

(3)

Using your answer to part (a) and making your method clear, estimate

(b) (i) $\displaystyle\int_3^9 \log_3 (2x)^{10} \, \mathrm{d}x$

(ii) $\displaystyle\int_3^9 \log_3 18x \, \mathrm{d}x$

(3)

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Question 5 continued

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8. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

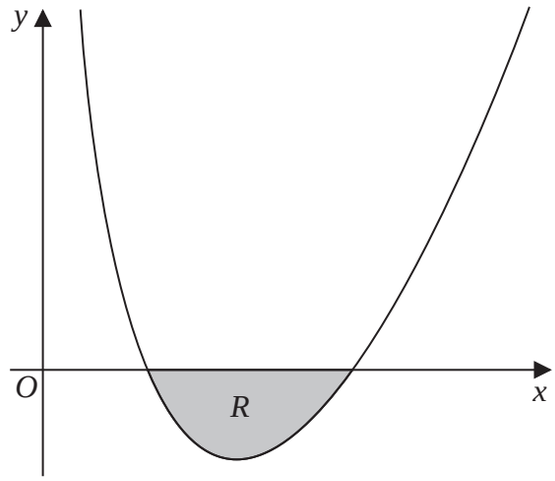


Figure 3

Figure 3 shows a sketch of part of a curve with equation

$$y = \frac{(x-2)(x-4)}{4\sqrt{x}} \qquad x > 0$$

The region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

Find the exact area of R, writing your answer in the form $a\sqrt{2} + b$, where a and b are constants to be found.

(6)

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Question 8 continued

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Question 8 continued

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Question 8 continued

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7. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

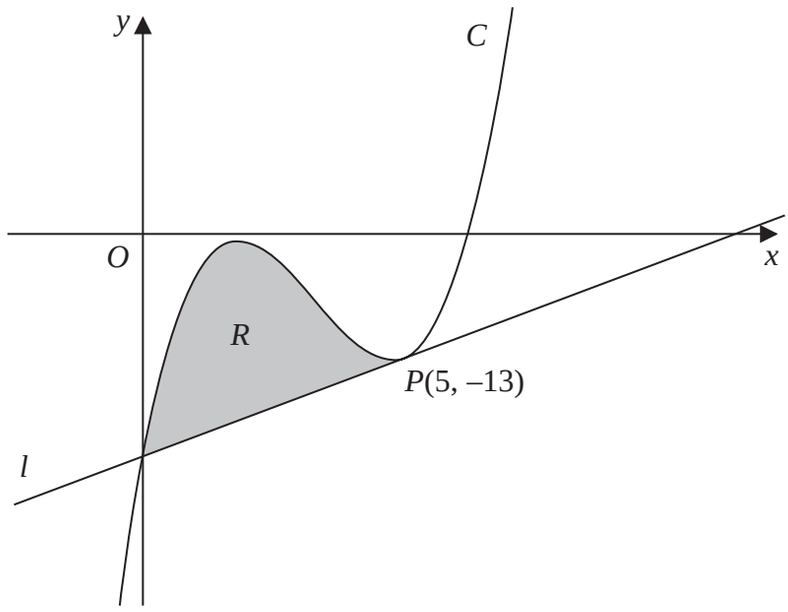


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + 27x - 23$$

The point $P(5, -13)$ lies on C

The line l is the tangent to C at P

(a) Use differentiation to find the equation of l, giving your answer in the form $y = mx + c$ where m and c are integers to be found.

(4)

(b) Hence verify that l meets C again on the y-axis.

(1)

The finite region R, shown shaded in Figure 2, is bounded by the curve C and the line l.

(c) Use algebraic integration to find the exact area of R.

(4)

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Question 7 continued

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Question 7 continued

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12. (a) Use the substitution $u = 1 + \sqrt{x}$ to show that

$$\int_0^{16} \frac{x}{1+\sqrt{x}}\,\mathrm{d}x = \int_p^q \frac{2(u-1)^3}{u}\,\mathrm{d}u$$

where p and q are constants to be found.

(3)

(b) Hence show that

$$\int_0^{16} \frac{x}{1+\sqrt{x}}\,\mathrm{d}x = A - B\ln 5$$

where A and B are constants to be found.

(4)

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Question 12 continued

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Question 12 continued

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Question 12 continued

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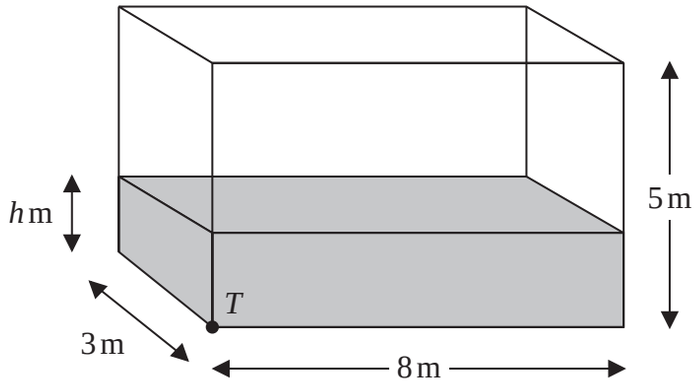


Figure 5

Water flows at a constant rate into a large tank.

The tank is a cuboid, with all sides of negligible thickness.

The base of the tank measures 8 m by 3 m and the height of the tank is 5 m.

There is a tap at a point T at the bottom of the tank, as shown in Figure 5.

At time t minutes after the tap has been opened

- the depth of water in the tank is h metres
- water is flowing into the tank at a constant rate of $0.48\,\text{m}^3$ per minute
- water is modelled as leaving the tank through the tap at a rate of $0.1h\,\text{m}^3$ per minute

(a) Show that, according to the model,

$$1200\,\frac{\mathrm{d}h}{\mathrm{d}t} = 24 - 5h$$

(4)

Given that when the tap was opened, the depth of water in the tank was 2 m,

(b) show that, according to the model,

$$h = A + B\mathrm{e}^{-kt}$$

where A, B and k are constants to be found.

(6)

Given that the tap remains open,

(c) determine, according to the model, whether the tank will ever become full, giving a reason for your answer.

(2)

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Question 14 continued

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Question 14 continued

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8. A new smartphone was released by a company.

The company monitored the total number of phones sold, n, at time t days after the phone was released.

The company observed that, during this time,

the rate of increase of n was proportional to n

Use this information to write down a suitable equation for n in terms of t.

(*You do not need to evaluate any unknown constants in your equation*.)

(2)

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Question 8 continued

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10. (a) Use the substitution $x = u^2 + 1$ to show that

$$\int_5^{10} \frac{3\,\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)} = \int_p^q \frac{6\,\mathrm{d}u}{u(3+2u)}$$

where p and q are positive constants to be found.

(4)

(b) Hence, using algebraic integration, show that

$$\int_5^{10} \frac{3\,\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)} = \ln a$$

where a is a rational constant to be found.

(6)

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Question 10 continued

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Question 10 continued

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Question 10 continued

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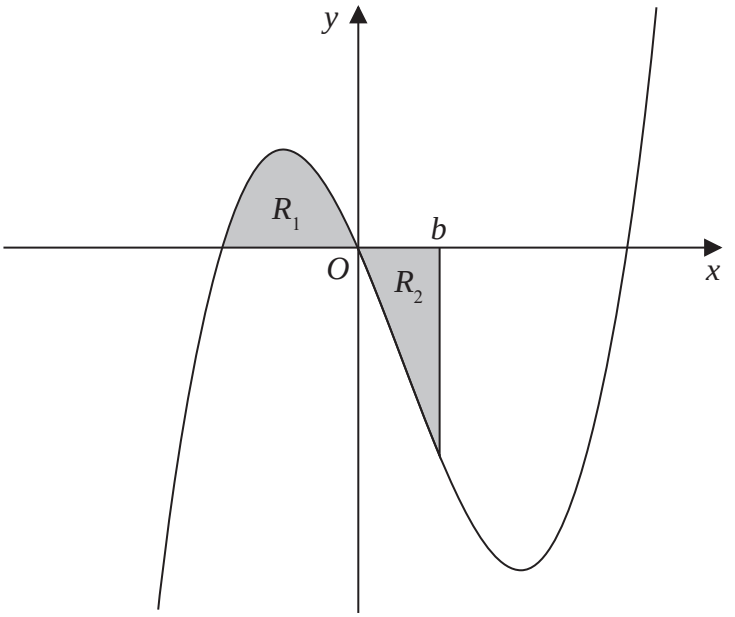


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = x(x + 2)(x - 4)$.

The region R_1 shown shaded in Figure 2 is bounded by the curve and the negative x-axis.

(a) Show that the exact area of R_1 is $\frac{20}{3}$

(4)

The region R_2 also shown shaded in Figure 2 is bounded by the curve, the positive x-axis and the line with equation $x = b$, where b is a positive constant and $0 < b < 4$

Given that the area of R_1 is equal to the area of R_2

(b) verify that b satisfies the equation

$$(b + 2)^2 (3b^2 - 20b + 20) = 0$$

(4)

The roots of the equation $3b^2 - 20b + 20 = 0$ are 1.225 and 5.442 to 3 decimal places.
The value of b is therefore 1.225 to 3 decimal places.

(c) Explain, with the aid of a diagram, the significance of the root 5.442

(2)

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7. Given that $k \in \mathbb{Z}^+$

(a) show that $\int_k^{3k} \frac{2}{(3x-k)} \mathrm{d}x$ is independent of k,

(4)

(b) show that $\int_k^{2k} \frac{2}{(2x-k)^2} \mathrm{d}x$ is inversely proportional to k.

(3)

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13. Show that

$$\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x = \frac{32}{15}\left(2+\sqrt{2}\right)$$

(7)

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10. A spherical mint of radius 5 mm is placed in the mouth and sucked.
Four minutes later, the radius of the mint is 3 mm.

In a simple model, the rate of decrease of the radius of the mint is inversely proportional to the square of the radius.

Using this model and all the information given,

(a) find an equation linking the radius of the mint and the time.
(You should define the variables that you use.)

(5)

(b) Hence find the total time taken for the mint to completely dissolve. Give your answer in minutes and seconds to the nearest second.

(2)

(c) Suggest a limitation of the model.

(1)

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