adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Composite and inverse functions

Mark scheme

3 questions

Exam questions and marking instructions remain credited to their board.

Contents

1 June 2024 - Paper 1 - Q8

2 June 2023 - Paper 1 - Q7

3 October 2021 - Paper 2 - Q2

Question	Scheme	Marks	AOs
8(a)	$\text{fg}(2) = 4 - 3\left(\dfrac{5}{2(2)-9}\right)^2 = \ldots$	M1	1.1b
	$\text{fg}(2) = 1$	A1	1.1b
		(2)	
(b)	$y = \dfrac{5}{2x-9} \Rightarrow 2xy - 9y = 5 \Rightarrow 2xy = 5 + 9y$	M1	1.1b
	$2xy = 5 + 9y \Rightarrow x = \dfrac{5+9y}{2y}$	A1	2.1
	$g^{-1}(x) = \dfrac{5+9x}{2x} x \neq 0 \{x \in \mathbb{R}\}$	A1	2.5
		(3)	
(c)(i)	$\{\text{gf}(x) =\}\ \dfrac{5}{2\left(4-3x^2\right)-9}$	M1	1.1b
	$= \dfrac{5}{-1-6x^2}$ or $\dfrac{-5}{1+6x^2}$	A1	1.1b
(ii)	-5 ,, $\text{gf}(x) < 0$	B1	2.2a
		(3)	
(d)	$\text{f}(x) = \text{h}(x) \Rightarrow 4 - 3x^2 = 2x^2 - 6x + k$ $\Rightarrow 5x^2 - 6x + k - 4 = 0$	M1	1.1b
	$b^2 - 4ac < 0 \Rightarrow 36 - 4(5)(k-4) < 0 \Rightarrow k > \ldots$	dM1	3.1a
	$k > 5.8$ o.e.	A1	2.2a
		(3)	
			(11 marks)

Notes

(a)

M1: Correct method, e.g. attempts to find g(2) $\left(= \dfrac{5}{4-9}\right)$ and substitutes its value into f to achieve a value.

Alternatively, attempts $\text{fg}(x) = 4 - 3\left(\dfrac{5}{2x-9}\right)^2$, condoning slips, and substitutes $x = 2$ to achieve a value.

A1: Correct answer only. If gf(2) is also attempted, then mark the final attempt which is the most complete.

(b)

M1: Eliminates the fraction and puts the *xy* term (or *x* term) onto one side of the equation. Alternatively swaps *x*'s and *y*' s, eliminates the fraction and puts the *xy* term (or *y* term) onto one side of the equation. Condone minor slips in rearranging e.g. $-9y$ instead of $+9y$

A1: Correct expression for the inverse, *x* in terms of *y* or *y* in terms of *x*. Need not be simplified.

Note that $y = \dfrac{5}{2x-9} \Rightarrow 2x - 9 = \dfrac{5}{y} \Rightarrow 2x = \dfrac{5}{y} + 9$ is M1 and $\Rightarrow x = \dfrac{\frac{5}{y} + 9}{2}$ is A1

A1: Fully correct notation for the inverse including its domain and including the e.g. $g^{-1} =$.
Condone $x \neq 0$ without $x \in \mathrm{R}$ Need not be simplified.
Do not be too worried about g^{-1} looking a bit like y^{-1} due to poor handwriting but if it is clearly y^{-1} then withhold this mark.

Accept e.g. $g^{-1}(x) = \dfrac{5+9x}{2x}$ $x \in \square$, $x \neq 0$ **or** $g^{-1}(x) = \dfrac{5}{2x} + \dfrac{9}{2}$ $x \neq 0$ **or** $g^{-1} = \dfrac{\frac{5}{x}+9}{2}$ $x \neq 0$
Ignore any reference to the range of g.

(c)(i)
M1: Correct method. Attempts to substitute f into g, condoning slips, e.g. missing the 3.
A1: Correct simplified fraction. Ignore any reference to domains. Do not isw.
There is no need to include the gf(x) =
If fg(x) is also attempted, then mark the final attempt which is the most complete.

(ii)
B1: Deduces the correct range. May be scored even if gf(x) is incorrect (but not a follow through).
Allow e.g. -5 ,, $y < 0$, $y \in [-5, 0)$, $[-5, 0)$
Do not allow e.g. -5 ,, $x < 0$, $y \in (-5, 0)$, -5 ,, $f(x) < 0$, -5 ,, $g(x) < 0$

(d)
M1: Sets f(x) = h(x) and attempts to collect terms to obtain a 3TQ = 0
The = 0 may be implied by use of the discriminant. Condone copying slips in f(x) and h(x).

dM1: Recognises the need to use "$b^2 - 4ac \ldots 0$" on their 3TQ and uses this to establish a value or range of values for k. Allow for an attempt to solve $b^2 - 4ac \ldots 0$ or $b^2 \ldots 4ac$, which must be in terms of k only, where ... is an equality or any inequality.

(Alt 1) Attempts to complete the square for their 3TQ (usual rules) and uses its minimum value set … 0 to establish a value or range of values for k. Their expression for the minimum value must be in terms of k only. Condone any equality or inequality when comparing their minimum value to 0.

e.g. $5x^2 - 6x + k - 4 \to 5\left(x - \dfrac{3}{5}\right)^2 - \dfrac{29}{5} + k \to "-\dfrac{29}{5} + k" > 0 \Rightarrow k > \ldots$ scores dM1

(Alt 2) Differentiates their 3TQ with respect to x to achieve a linear expression, sets = 0 (which may be implied), solves for x and substitutes x back into their 3TQ set … 0 to establish a value or range of values for k. Here … can be any equality or inequality.
e.g. $5x^2 - 6x + k - 4 \to 10x - 6 \to x = 0.6 \Rightarrow 5(0.6)^2 - 6(0.6) + k - 4 > 0 \Rightarrow k > \ldots$ scores dM1

A1: Deduces the correct range for k, e.g. $k > \dfrac{29}{5}$ o.e. Must be in terms of k and not e.g. x

Accept e.g. $k \in (5.8, \infty)$ or just $\left(\dfrac{29}{5}, \infty\right)$ but **not** e.g. $k \ldots \dfrac{29}{5}$ or $x > \dfrac{29}{5}$ or $[5.8, \infty)$

Question	Scheme	Marks	AOs

7(a)	$f(x) > 3$	B1	1.1b
		(1)	
(b)	$y = 3 + \sqrt{x-2} \Rightarrow x = ...$	M1	1.1b
	$f^{-1}(x) = (x-3)^2 + 2$	A1	1.1b
	$x > 3$	B1ft	2.2a
		(3)	
(c)	$f(6) = 3 + \sqrt{6-2} = 5 \Rightarrow g("5") = \frac{15}{"5"-3} = ...$	M1	1.1b
	$= \frac{15}{2}$	A1	1.1b
		(2)	
(d)	$3 + \sqrt{a^2+2-2} = \frac{15}{a-3} \Rightarrow "a^2 - 9 = 15"$	M1	1.1b
	$a = 2\sqrt{6}$	A1	2.2a
		(2)	
			(8 marks)

Notes

(a)

B1: $f(x) > 3$ o.e.

e.g. $y > 3$, range > 3, $f(x) \in (3, \infty)$, $\{f(x) : f(x) > 3\}$, $f > 3$ **but not e.g.** $x > 3$, $f(x) \geqslant 3$, $[3, \infty)$

(b)

M1: Sets $y = 3 + \sqrt{x-2}$ and attempts to make x the subject (or vice versa). Look for the correct order of operations so score for an expression of the form $(x =)\ (y \pm 3)^2 \pm 2$ or $(y =)\ (x \pm 3)^2 \pm 2$

A1: $f^{-1}(x) = (x-3)^2 + 2$ Also accept $f^{-1} : x \to (x-3)^2 + 2$. Condone $f^{-1} = (x-3)^2 + 2$ (or $f^{-1} = y = (x-3)^2 + 2$) but do not allow just $y = ...$ or $f^{-1} : y =$

Also accept other equivalent expressions such as $f^{-1}(x) = x^2 - 6x + 11$ (simplified or unsimplified)

B1ft: $x > 3$ or follow through on their part (a). The omission of $x \in \mathbb{R}$ is condoned.

Allow equivalent answers such as $x \in ("3", \infty)$ or $\{x : x > "3"\}$

Note: It is also acceptable to define f^{-1} in any variable e.g. as $f^{-1}(t) = (t-3)^2 + 2 \quad t > 3$ as long as the variable is used consistently to score M1A1B1. If another variable is used other than x it must be fully defined e.g. $f^{-1}(t) = ...$ not just $f^{-1} = ...$

(c)

M1: Substitutes $x = 6$ into f and substitutes the result into g to find a value for $gf(6)$.

Allow an attempt to substitute $x = 6$ into $gf(x) = \frac{15}{\sqrt{x-2}}$ condoning slips. They must proceed to find a value. Condone arithmetical slips and bracket errors/omissions. Condone for M1 attempts where when dealing with $\sqrt{x-2}$ leads to two different answers e.g. $\frac{15}{\sqrt{6-2}} \to \pm\frac{15}{2}$

A1: $\frac{15}{2}$ only oe isw once a correct answer is seen

(d)

M1: Attempts to form the equation $3+\sqrt{a^2+2-2}=\dfrac{15}{a-3}$, and proceeds to a quadratic in a (usually $a^2=k$ or $a^2-k=15$ but condone arithmetical, miscopying and sign slips. Condone equations which would lead to complex roots.
May be implied by a correct exact answer.

Alternatively, they attempt to form the equation $a^2+2=\text{f}^{-1}\text{g}(a)\Rightarrow a^2+2=\left(\dfrac{15}{a-3}-3\right)^2+2$

$\Rightarrow (a+3)(a-3)=15\Rightarrow a^2-9=15$ (condone slips)

They should be square rooting both sides so that $\sqrt{a^2+2-2}\rightarrow a$, before multiplying both sides by $a-3$ and rearranging so that the a^2 term comes from their "$(a+3)(a-3)$"

May be implied by a correct exact answer for their quadratic in a but a correct decimal answer does not imply this mark.

A1: $(a=)\ 2\sqrt{6}$ or accept $\sqrt{24}$ (they must reject the negative solution if found as $\text{f}\left(a^2+2\right)\neq\text{g}(a)$ when $a=-2\sqrt{6}$) $\sqrt{6}\times\sqrt{4}$ is A0
isw $\sqrt{24}$ followed by $4\sqrt{6}$ (incorrect manipulation of the surd) but not followed by $\pm\sqrt{24}$ o.e.
A decimal answer on its own or multiple answers e.g. $\pm\sqrt{24}$ score A0.

Question	Scheme	Marks	AOs

2(a)	$y \leqslant 7$	B1	2.5
		(1)	
(b)	$\text{f}(1.8) = 7 - 2\times 1.8^2 = 0.52 \Rightarrow \text{gf}(1.8) = \text{g}(0.52) = \dfrac{3\times 0.52}{5\times 0.52 - 1} = \ldots$	M1	1.1b
	$\text{gf}(1.8) = 0.975$ oe e.g. $\dfrac{39}{40}$	A1	1.1b
		(2)	
(c)	$y = \dfrac{3x}{5x-1} \Rightarrow 5xy - y = 3x \Rightarrow x(5y-3) = y$	M1	1.1b
	$\left(\text{g}^{-1}(x) =\right) \dfrac{x}{5x-3}$	A1	2.2a
		(2)	
			(5 marks)

Notes

(a)

B1: Correct range. Allow f(x) or f for y. Allow e.g. $\{y \in \mathbb{R} : y \leqslant 7\}$, $-\infty < y \leqslant 7$, $(-\infty, 7]$

(b)

M1: Full method to find f(1.8) and substitutes the result into g to obtain a value.
Also allow for an attempt to substitute $x = 1.8$ into an attempt at gf(x).

E.g. $\text{gf}(x) = \dfrac{3(7-2x^2)}{5(7-2x^2)-1} = \dfrac{3(7-2(1.8)^2)}{5(7-2\times(1.8)^2)-1} = \ldots$

A1: Correct value

(c)

M1: Correct attempt to cross multiply, followed by an attempt to factorise out x from an xy term and an x term.
If they swap x and y at the start then it will be for an attempt to cross multiply followed by an attempt to factorise out y from an xy term and a y term.

A1: Correct expression. Allow equivalent correct expressions e.g. $\dfrac{-x}{3-5x}$, $\dfrac{1}{5} + \dfrac{3}{25x-15}$

Ignore any domain if given.

Question	Scheme	Marks	AOs