adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Kinematics

Mark scheme

12 questions

Exam questions and marking instructions remain credited to their board.

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4(a)	$\mathbf{r} = (32\mathbf{i} - 16\mathbf{j})$	B1	1.1b
		(1)	
4(b)	Use Pythagoras: $\sqrt{32^2 + 16^2}$ oe for their **r**	M1	3.1a
	$16\sqrt{5}$ (m)	A1	1.1b
		(2)	
4(c)	Differentiate **r** wrt *t* to obtain **v**	M1	3.4
	$6t^{\frac{1}{2}}\mathbf{i} - 2t\mathbf{j}$ oe (m s^{-1})	A1	1.1b
		(2)	
4(d)	Differentiate their **v** wrt *t* to obtain **a**	M1	3.4
	$3t^{-\frac{1}{2}}\mathbf{i} - 2\mathbf{j}$	A1	1.1b
	$\frac{3T^{-\frac{1}{2}}}{-2} = -\frac{1}{3}$ must see $\frac{\mathbf{i} \text{ component of their } \mathbf{a}}{\mathbf{j} \text{ component of their } \mathbf{a}} = -\frac{1}{3}$ oe	M1	2.1
	$(T =) \frac{81}{4}$ oe	A1	1.1b
		(4)	
			(9 marks)

Notes: N.B. Accept column vectors throughout apart from in the answer to (c).

4(a)	B1	cao. **N.B.** Must be using correct vector notation, including brackets if using column vectors.
4(b)	M1	For an unsimplified expression, using their **r**, with the square root
	A1	Accept any surd equivalent isw **N.B**. Must come from $\mathbf{r} = (32\mathbf{i} - 16\mathbf{j})$
4(c)	M1	Both powers of *t* decreasing by 1 (but not just division by *t*) **N.B.** M0 if **i** and/or **j** are missing and never reappear.
	A1	Must be in terms of *t*, **i** and **j**
4(d)	M1	Both powers of *t* decreasing by 1 (but not just division by *t*) **If no i's and/or j's,** can score M1 if **EITHER** they have 2 separate components, provided they are clearly treated as such in the subsequent working **OR** the **i's** and **j's** reappear **otherwise** M0.
	A1	Correct vector **or** 2 correct separate components, provided they are clearly treated as such in the subsequent working

	M1	Complete method to form a correct equation in T (t) only, for their **a,** using the 2 components from their **a**
	A1	Accept 20.3 or 20.25 or any equivalent fraction (allow t instead of T)

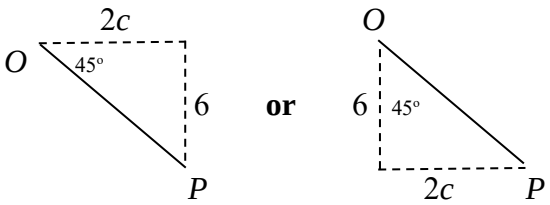
Question	Scheme	Marks	AOs

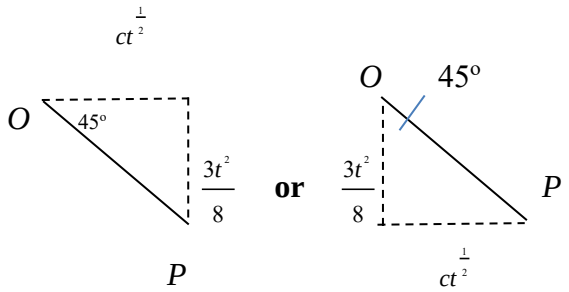
2(a)	$\frac{10}{4}$	M1	3.4
	2.5, $\frac{5}{2}, \frac{10}{4}$ m s^{-2} units needed.	A1	1.1b
		(2)	
2(b)	Find the area, with correct structure, from $t = 0$ to 18	M1	3.1b
	$\frac{1}{2}\times 4\times 10+(14\times 10)$ triangle + rectangle **or** $\frac{1}{2}\times 10\times(14+18)$ trapezium **or** $(18\times 10)-\frac{1}{2}\times 4\times 10$ rectangle – triangle **N.B.** $\frac{1}{2}\times 4\times 10$ may be replaced by $\frac{1}{2}\times 2.5\times 4^2$ using $s=ut+\frac{1}{2}at^2$ or by $\frac{10^2-0^2}{2\times 2.5}$ using $v^2=u^2+2as$	A1	1.1b
	160 (m)	A1	1.1b
		(3)	
2(c)	Using area, from $t = 18$ to $t = 24$, = (200 – their (b)) with correct structure **OR** s = (200 – their (b)), using *suvat* to find s **N.B.** If their (b) is incorrect and they don't use it, allow a correct restart.	M1	3.1b
	$6U+\frac{1}{2}\times 6\times(10-U)=200-$ their (b) rectangle + triangle **or** $\frac{1}{2}\times 6\times(10+U)=200-$ their (b) trapezium ($s=\left(\frac{u+v}{2}\right)t$ **or** $(6\times 10)-\frac{1}{2}\times 6\times(10-U)=200-$ their (b) rectangle – triangle **or** $(10\times 6)+\frac{1}{2}\left(-\frac{(10-U)}{6}\right)\times 6^2=200-$ their (b) $s=ut+\frac{1}{2}at^2$ **or** $(U\times 6)-\frac{1}{2}\left(-\frac{(10-U)}{6}\right)\times 6^2=200-$ their (b) $s=vt-\frac{1}{2}at^2$ **N.B.** Two stage *suvat* method: $(10\times 6)+\frac{1}{2}a\times 6^2=200-$ their (b) => **AND** $U=10+6\times$ their a	A1**ft**	1.1b

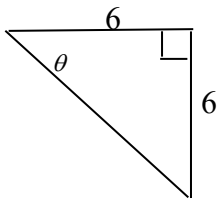
		$\frac{10}{3} = 3\frac{1}{3}$ oe	A1	1.1b
			(3)	
				(8 marks)

Notes:		
2a	M1	Any complete *suvat* method to find *a* e.g. use $s = 20$ and $20 = \frac{1}{2}a \times 4^2$ **N.B**. Ignore units at this stage
	A1	**Any** equivalent number **with** correct units. Accept m/s^2, m/s/s, m per s per s.
2b	M1	Complete method, they may use s*uvat* on one or more sections, to find the **TOTAL** area. M0 if a single *suvat* equation is used for the whole motion M0 if $\frac{1}{2}$ not seen used in an area method
	A1	Correct unsimplified expression.
	A1	cao. Ignore units. **N.B.** Correct answer, **with no working,** can score all 3 marks.
2c	M1	Complete method, using area or *suvat*, to give an equation in *U* only, with correct structure M0 if $\frac{1}{2}$ not seen used in an area method M0 if 10 is used instead of $(10 - U)$ or $(10 - U)$ is used instead of $(10 + U)$ in any equation
	A1**ft**	Correct unsimplified equation in *U* only (allow *V* or *v* instead of *U*), **ft on their 160**.
	A1	Accept 3.3 or better. Ignore units. Allow use of *V* throughout instead of *U*, <u>including in the answer</u>. **N.B.** Correct answer, **with no working,** can score all 3 marks.

Question	Scheme	Marks	AOs

4(a)	**ALTERNATIVES** when $t = 4$ is substituted at the beginning.		
	$2c\mathbf{i} - 6\mathbf{j}$ or as a column vector, seen or implied.	B1	1.1b
	ALT 1  **AND** **either** $\tan 45^o = \frac{2c}{6} \Rightarrow 2c = 6$ **or** **states** isosceles triangle so $2c = 6$ **N.B.** In both of the above,we must see the justification for the equation. **ALT 2** $\tan 135^o = \frac{2c}{-6} \Rightarrow 2c = 6$ **N.B.** M0 if they are using the wrong bearing.	M1	3.1a
	$c = 3$ *	A1*	2.2a
	SC 1 M1A0: no right-angled triangle $2c\mathbf{i} - 6\mathbf{j} = k(\mathbf{i} - \mathbf{j}) \Rightarrow 2c = 6$ **or** **i**-cpt = – **j**-cpt $\Rightarrow 2c = 6$ **N.B.** In both of the above,we must see the justification for the equation. **SC 2 M1A0:** no right-angled triangle $\tan 45^o = \frac{2c}{6}$ or $\frac{6}{2c} \Rightarrow 2c = 6$ **N.B.** In the above,we must see the justification for the equation.		

	ALTERNATIVES when $t = 4$ is substituted at the end:		
	$ct^{\frac{1}{2}} = 2c$ **and** $(-)\frac{3t^2}{8} = (-)6$ when $t = 4$, seen or implied	B1	
	ALT 3  **AND**	M1	
	either $\tan 45^o = \frac{\frac{3t^2}{8}}{ct^{\frac{1}{2}}}$ => $2c = 6$ when $t = 4$ **or** **states** isosceles triangle, so $ct^{\frac{1}{2}} = \frac{3t^2}{8}$ => $2c = 6$ when $t = 4$ **N.B.** In both of the above,we must see the justification for the equation. **N.B.** M0 if they are using the wrong bearing.		
	$c = 3$	A1*	
	SC 3 M1A0: no right-angled triangle $(ct^{\frac{1}{2}}\mathbf{i} - \frac{3t^2}{8}\mathbf{j}) = k(\mathbf{i} - \mathbf{j})$ => $2c = 6$ when $t = 4$ **or** **i**-cpt = – **j**-cpt => $ct^{\frac{1}{2}} = \frac{3t^2}{8}$ => $2c = 6$ when $t = 4$ **N.B.** In both of the above,we must see the justification for the equation. **SC 4 M1A0:** no right-angled triangle $\tan 45^o = \frac{\frac{3t^2}{8}}{ct^{\frac{1}{2}}}$ => $2c = 6$ when $t = 4$.		

	N.B. Allow a **verification**: i.e. use $c = 3$ and $t = 4$ to show that P is on a bearing of 135° from O. $6\mathbf{i} - 6\mathbf{j}$ then a diagram:  **AND** $\tan\theta = \frac{6}{6}$ or isosceles triangle => $\theta = 45°$ **N.B.** In the above,we must see the justification for the equation.	B1 M1	
	bearing = 45° + 90° = 135°	A1*	
		(3)	
4(b)	Differentiate **r** wrt t to obtain **v**	M1	2.1
	$\mathbf{v} = 3\times\frac{1}{2}t^{-\frac{1}{2}}\mathbf{i} - \frac{3}{8}\times 2t\mathbf{j} = \frac{3}{2}t^{-\frac{1}{2}}\mathbf{i} - \frac{3}{4}t\mathbf{j}$ oe	A1	1.1b
	Put $t = 4$ into **both** components and use Pythagoras: $\sqrt{\left(\frac{3}{4}\right)^2 + (-3)^2}$	M1	3.1a
	$\sqrt{\frac{153}{16}}$ or $\frac{\sqrt{153}}{4}$ or $\frac{3\sqrt{17}}{4}$ or $3\sqrt{\frac{17}{16}} = 3.0923..$ (m s^{-1})	A1	1.1b
		(4)	
4(c)	Differentiate their **v** wrt t to obtain **a**	M1	3.4
	$\mathbf{a} = -\frac{3}{4}t^{-\frac{3}{2}}\mathbf{i} - \frac{3}{4}\mathbf{j}$	A1	1.1b
	$\frac{-\frac{3}{4}T^{-\frac{3}{2}}}{-\frac{3}{4}} = \frac{-1}{-27}$ oe	M1	2.1
	$(T =)\ 9$	A1	1.1b
		(4)	

		(11 marks)
Notes: Accept column vectors throughout		
4a	B1	$2c\mathbf{i} - 6\mathbf{j}$ seen or implied. B0 for r = $2c - 6$ if no evidence of components.
	M1	**ALT 1:** Use the bearing to obtain a CORRECT diagram showing a right-angled triangle with at least one 45° angle marked or clearly explained (i.e. 135° marked on the diagram and either $135^o - 90^o = 45^o$ or $180^o - 135^o = 45^o$), and $2c$ and ± 6 marked **AND** use of isosceles triangle or tan or (sin/cos and Pythag) to obtain $2c = 6$ **ALT 2: No diagram required** Use $\tan 135^\circ = \frac{2c}{-6} \Rightarrow 2c = 6$
	A1*	Given answer correctly obtained
		ALTERNATIVE when $t = 4$ is substituted at the end:
	B1	$ct^{\frac{1}{2}} = 2c$ **and** $(-)\frac{3t^2}{8} = (-)6$ when $t = 4$, seen or implied
	M1	**ALT 3:** Use the bearing to obtain a CORRECT diagram showing a right-angled triangle with at least one 45° angle marked or clearly explained (i.e. 135° marked on the diagram and either $135^o - 90^o = 45^o$ or $180^o - 135^o = 45^o$), and $ct^{\frac{1}{2}}$ and $\pm \frac{3t^2}{8}$ marked **AND** use of isosceles triangle or tan or (sin/cos and Pythag) to obtain $2c = 6$ when $t = 4$
	A1*	Given answer correctly obtained
4b	M1	Both powers of t decreasing by 1 (M0 if **i** or **j** is missing but allow recovery or working with components only) **N.B.** This mark is available if c has not been substituted for.
	A1	Correct unsimplified derivative or two correct components
	M1	Put $t = 4$ in their **v** (must be using an attempted derivative of **r**) and then use Pythagoras with the root, allow a missing $-$ sign **N.B.** If they **state** $t = 4$, allow a slip when they substitute in, for this M mark. This mark is available if c has not been substituted for.
	A1	Accept 3.1 or better
4c	M1	Both powers of t decreasing by 1 **N.B.** This mark is available if c has not been substituted for (M0 if **i** or **j** is missing but allow recovery or working with components only).
	A1	Correct unsimplified derivative
	M1	Use of an appropriate **ratio** (must be using an attempted derivative of **their v)**, condone sign error and the **reciprocal,** to obtain an equation in t or ***T*** **only**. **N.B.** If they state that $-\frac{3}{4}T^{-\frac{3}{2}}\mathbf{i} - \frac{3}{4}\mathbf{j} = k(-\mathbf{i} - 27\mathbf{j})$ and then equate coefficients to give two simultaneous equations in k and T, these need to be used to produce an equation in ***T*** **only**, before the M mark is earned.
	A1	cao (allow t instead of T)

Question	Scheme	Marks	AOs

3(a)	**7i – 3j** seen or implied by Pythagoras	B1	1.1b
	Use Pythagoras: $\sqrt{7^2+(-3)^2}$	M1	3.1a
	$\sqrt{58}$, 7.6 or better ($\mathrm{m\,s^{-1}}$)	A1	1.1b
		(3)	
3(b)	$t^2-3t+7=2t^2-3$ **OR** $\dfrac{t^2-3t+7}{2t^2-3}=\dfrac{1}{1}=1$	M1	2.1
	$t = 2$ only	A1	1.1b
		(2)	
3(c)	Differentiate **v** wrt t to give a vector.	M1	3.1a
	$(2t-3)\mathbf{i}+4t\mathbf{j}$	A1	1.1b
		(2)	
3(d)	$2t-3=0$	M1	3.1a
	$t = 1.5$	A1	1.1b
		(2)	
			(9 marks)

Notes: Allow column vectors throughout.

3a	B1	cao
	M1	Use of Pythagoras, including the square root, on a **velocity** vector at $t = 0$
	A1	cao. Must come from a correct **v**.
3b	M1	Equating **i** and **j** components of **v** or a ratio of 1:1 to obtain a quadratic in t only. If they use a constant, e.g. $t^2-3t+7=k$ and $2t^2-3=k$, k must be eliminated to earn this mark. **N.B.** M0 (since wrong working seen) if they write down $\mathbf{i}+\mathbf{j}=\left(t^2-3t+7\right)\mathbf{i}+\left(2t^2-3\right)\mathbf{j}$ **OR** $\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}t^2-3t+7\\2t^2-3\end{pmatrix}$ **OR** $t^2-3t+7=1$ and $2t^2-3=1$

		and then $t^2 - 3t + 7 = 2t^2 - 3$
	A1	$t = 2$
		N.B. Allow M1A1 for a **correct** trial and error method where they obtain $\mathbf{v} = 5\mathbf{i} + 5\mathbf{j}$ when t = 2 but M0 if they don't get t = 2
3c	M1	At least one power decreasing by 1 in **each** component in their **v** (M0 if clearly dividing by t) **Both i** and **j** needed in their answer or a column vector Allow recovery if the **i** and **j** disappear and then reappear.
	A1	cao (must be a vector) isw e.g. if they find the magnitude or put t = 0 or differentiate again **i**'s and **j**'s do not need to be collected. **N.B**. Allow M1A0 for $2t - 3\mathbf{i} + 4t\mathbf{j}$
3d	M1	$2t - 3 = 0$ or (their **derivative** of the **i**-component of **v)** = 0 **N.B**. M0 if they equate the derivative of both components of **v** to zero.
	A1	cao **N.B.** Correct answer, with no working, can score both marks.

1(a)	Put $t = 2$ in **v** and use Pythagoras: $\sqrt{12^2 + (-6\sqrt{2})^2}$	M1	3.1a
	$\sqrt{216}, 6\sqrt{6}$ or 15 or better (m s^{-1})	A1	1.1b
		(2)	
1(b)	Differentiate **v** wrt t to obtain **a**	M1	3.4
	$6t\mathbf{i} - 3t^{-\frac{1}{2}}\mathbf{j}$ oe (m s^{-2}) isw	A1	1.1b
		(2)	
1(c)	Integrate **v** wrt t to obtain **r**	M1	3.4
	$\mathbf{r} = t^3\mathbf{i} - 4t^{\frac{3}{2}}\mathbf{j}\ (+\mathbf{C})$	A1	1.1b
	$(\mathbf{i} - 4\mathbf{j}) = 4^3\mathbf{i} - 4\times 4^{\frac{3}{2}}\mathbf{j} + \mathbf{C}$	M1	3.1a
	$(-62\mathbf{i} + 24\mathbf{j})$ (m) isw e.g. if they go on to find the distance.	A1	1.1b
		(4)	
			(8 marks)

Notes: Accept column vectors throughout apart from the answer to (b).

1a	M1	Need square root but -ve sign not required. Allow **i**'s and/or **j**'s to go missing from their **v** at $t = 2$, provided they have applied Pythagoras correctly.
	A1	cao **N.B**. Correct answer with no working can score 2 marks.
1b	M1	Both powers decreasing by 1. Allow a column vector. M0 if **i** or **j** is missing but allow recovery in (b).
	A1	cao. Do not accept a column vector.
1c	M1	Both powers increasing by 1 M0 if **i** or **j** is missing but allow recovery.
	A1	(**r** =) not required
	M1	Putting **r** = $(\mathbf{i} - 4\mathbf{j})$ and $t = 4$ into their displacement **vector** expression which must have **C** (allow *C*) to give an equation in **C** only, seen or implied. Must have attempted to integrate **v** for this mark to be available. **N.B**. **C** does not need to be found and this is a method mark, so allow slips.
	A1	cao

Question	Scheme	Marks	AOs

5(a)	Using horizontal motion	M1	3.3
	Whole Motion **Half way**		
	$U\cos\alpha \times t = 120$ $U\cos\alpha \times t = 60$	A1	1.1b
	Using vertical motion **OR**	M1	3.4
	$U\sin\alpha \times t - \frac{1}{2}gt^2 = 0$ $0 = U\sin\alpha - gt$	A1	1.1b
	Attempt to solve problem by eliminating t	DM1	3.1b
	$U^2\sin\alpha\cos\alpha = 588$ *	A1*	2.2a
		(6)	
	N.B. No credit given if they use the given answer from (b).		
5(b)	Using vertical motion **OR** conservation of energy	M1	3.4
	$0^2 = (U\sin\alpha)^2 - 2g\times 10$ $\frac{1}{2}mU^2 - \frac{1}{2}m(U\cos\alpha)^2 = mg\times 10$	A1	1.1b
	ALTERNATIVE 1: If t is time to top: use of $10 = \frac{1}{2}gt^2$ oe ($t = \frac{10}{7}$) to obtain an equation in U and α only M1 $U\sin\alpha = 14$ or $U\cos\alpha = 42$ A1 **ALTERNATIVE 2:** If t is time to top: use of : $10 = U\sin\alpha\ t - \frac{1}{2}gt^2$ with $t = \frac{60}{U\cos\alpha}$ substituted to obtain an equation in U and α only : M1 $10 = U\sin\alpha \times \frac{60}{U\cos\alpha} - \frac{1}{2}g\left(\frac{60}{U\cos\alpha}\right)^2$ A1		
	Attempt to solve problem by eliminating α : e.g. $U\sin\alpha = 14$ => $U\cos\alpha = 42$, from part (a) or from using $t = \frac{10}{7}$, then square and add to give result **OR:** $U^2\sin^2\alpha = 20g = 196$ and $U^2\sin\alpha\cos\alpha = 588$, divide to give $\tan\alpha = \frac{1}{3}$ then $\sin^2\alpha = \frac{1}{10}$, hence result **OR in ALTERNATIVE 2**: sub for U^2 using part (a), to give $\tan\alpha = \frac{1}{3}$ then $\sin^2\alpha = \frac{1}{10}$, hence result	DM1	3.1b

	N.B. Just stating that $\sin\alpha = \frac{4}{10}$, with no working is DM0A0.		
	$U^2 = 1960$ *	A1*	2.2a
	N.B. Verification (i.e. starting with $U^2 = 1960$ and trying to work backwards) is not an acceptable method for this question.		
		(4)	
5(c)	V, since air resistance has to be overcome, or just 'because of <u>air resistance' isw</u>	B1	3.5a
		(1)	
5(d)	e.g. wind effects, more accurate value of g, spin of ball, size of ball, shape of ball, dimensions of ball, not a particle, variable acceleration, surface area of ball, humidity. Allow wind resistance and rotational resistance (Ignore any mention of air resistance or drag)	B1	3.5c
		(1)	
			(12 marks)

Notes:

5a		**N.B**. Could score 2/6 for any one of the 4 given equations if there is no corresponding second equation or there is an attempt but it's incorrect.
	M1	Complete method to give equation in U, α and t only, condone sin/cos confusion and sign errors, each term that needs to be resolved must be resolved
	A1	Correct equation
	M1	Complete method to give equation in U, α and t only, condone sin/cos confusion and sign errors, each term that needs to be resolved must be resolved
	A1	Correct equation
	DM 1	Eliminate t, dependent on first and second M1's
	A1*	Given answer correctly obtained, <u>with no wrong working seen.</u> Allow $588 = U^2 \sin\alpha\cos\alpha$ but nothing else
5b	M1	Complete method to give equation in U and α only with correct no. of terms, condone sin/cos confusion and sign errors, each term that needs to be resolved must be resolved
	A1	Correct equation
	DM 1	Eliminate α and rearrange, dependent on first M1
	A1*	Given answer correctly obtained with <u>no wrong working seen</u> (**N.B**. If they use a value for α (18.43.°.) they lose the final A1*)
5c	B1	Clear statement isw
5d	B1	B0 if there is an incorrect extra e.g. mass or weight

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5(a)	Differentiate **v** wrt t	M1	3.1a
	$\frac{3}{2}t^{-\frac{1}{2}}\mathbf{i}-2\mathbf{j}$ isw	A1	1.1b
		(2)	
5(b)	$3t^{\frac{1}{2}}=2t$	M1	2.1
	Solve for t	DM1	1.1b
	$t=\frac{9}{4}$	A1	1.1b
		(3)	
5(c)	Integrate **v** wrt t	M1	3.1a
	$\mathbf{r}=2t^{\frac{3}{2}}\mathbf{i}-t^2\mathbf{j}(+\mathbf{C})$	A1	1.1b
	$t = 1$, $\mathbf{r} = -\mathbf{j}$ => $\mathbf{C} = -2\mathbf{i}$ so $\mathbf{r}=2t^{\frac{3}{2}}\mathbf{i}-t^2\mathbf{j}-2\mathbf{i}$	A1	2.2a
		(3)	
5(d)	$\sqrt{(3t^{\frac{1}{2}})^2+(2t)^2}=10$ or $(3t^{\frac{1}{2}})^2+(2t)^2=10^2$	M1	2.1
	$9t+4t^2=100$	M(A)1	1.1b
	$t = 4$	A1	1.1b
	$\mathbf{r}=14\mathbf{i}-16\mathbf{j}$	M1	1.1b
	$\sqrt{14^2+(-16)^2}$	M1	3.1a
	$\sqrt{452}$ $(2\sqrt{113})$ (m)	A1	1.1b
		(6)	
			(14 marks)

Notes:

5a	M1	Both powers decreasing by 1 (M0 if vector(s) disappear but allow recovery)
	A1	cao
5b	M1	Complete method, using **v**, to obtain an equation in t only, allow a sign error
	DM1	Dependent on M1,solve for t

	A1	cao
5c	M1	Both powers increasing by 1 (M0 if vectors disappear but allow recovery)
	A1	Correct expression without **C**
	A1	cao
5d	M1	Use of Pythagoras on **v** and 10 to set up equation in *t*
	M(A)1	Correct 3 term quadratic in *t*
	A1	cao
	M1	Substitute their numerical *t* value into their **r**
	M1	Use of Pythagoras to find the magnitude of their **r**
	A1	cso

1(a)	Differentiate **v**	M1	1.1a
	$(\mathbf{a}=)6\mathbf{i}-\frac{15}{2}t^{\frac{1}{2}}\mathbf{j}$	A1	1.1b
	$=6\mathbf{i}-15\mathbf{j}$ (m s^{-2})	A1	1.1b
		(3)	
1(b)	Integrate **v**	M1	1.1a
	$(\mathbf{r}=)(\mathbf{r}_0)+3t^2\mathbf{i}-2t^{\frac{5}{2}}\mathbf{j}$	A1	1.1b
	$=(-20\mathbf{i}+20\mathbf{j})+(48\mathbf{i}-64\mathbf{j})=28\mathbf{i}-44\mathbf{j}$ (m)	A1	2.2a
		(3)	
		(6)	

Marks		**Notes**
		N.B. Accept column vectors throughout and condone missing brackets in working but they must be there in final answers
1a	M1	Use of $\mathbf{a}=\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}$ with attempt to differentiate (both powers decreasing by 1) M0 if **i**'s and **j**'s omitted and they don't recover
	A1	Correct differentiation in any form
	A1	Correct and simplified. Ignore subsequent working (ISW) if they go on and find the magnitude.
1b	M1	Use of $\mathbf{r}=\int\mathbf{v}\mathrm{d}t$ with attempt to integrate (both powers increasing by 1) M0 if **i**'s and **j**'s omitted and they don't recover
	A1	Correct integration in any form. Condone $\mathbf{r}_0$ not present
	A1	Correct and simplified.

Question	Scheme	Marks	AO

2(a)	$(\mathbf{v}=)\mathbf{C}+(2\mathbf{i}-3\mathbf{j})t$	M1	3.1a
	$(\mathbf{v}=)(-\mathbf{i}+4\mathbf{j})+(2\mathbf{i}-3\mathbf{j})t$	A1	1.1b
	$\frac{4-3T}{-1+2T}=\frac{-4}{3}$ oe	M1	3.1a
	$T=8$	A1	1.1b
		(4)	
(b)	$(\mathbf{s}=)\mathbf{C}t+(2\mathbf{i}-3\mathbf{j})\frac{1}{2}t^2$ $(+\mathbf{D})$	M1	3.1a
	$(\mathbf{s}=)(-\mathbf{i}+4\mathbf{j})t+\frac{1}{2}(2\mathbf{i}-3\mathbf{j})t^2$ $(+\mathbf{D})$	A1	1.1b
	$AB=\sqrt{12^2+8^2}$ **N.B. Beware you may see 4(2i – 3j) which leads to** $\sqrt{(8^2+12^2)}$ this is M0A0M0A0.	M1	3.1a
	$=4\sqrt{13}\,(=14.422051....)$ (m)	A1**cso**	1.1b
		(4)	
		(8)	

Marks		**Notes**
2a	M1	Use of $\mathbf{v}=\mathbf{u}+\mathbf{a}t$ **OR** integration to give an expression of the form $\mathbf{C}+(2\mathbf{i}-3\mathbf{j})t$ **, where C is a non-zero constant <u>vector</u>** M0 if **u** and **a** are reversed Condone use of **a** = (2**i** + 3**j**) for this M mark
	A1	Any correct unsimplified expression seen or implied
	M1	Correct use of ratios, <u>using a velocity vector</u> (must be using $\frac{-4}{3}$) to give equation <u>in T only</u> M0 if they equate $4-3T=-4$ and/or $-1+2T=3$ and therefore M0 if they then divide to produce their equation
	A1	Correct only
		N.B. (i) Can score the second M1A1 if they get $T = 8$, using a calculator to solve two simultaneous equations, but if answer is wrong, and no equation in T only, second M0 (ii) Can score M1A1 M1A1 if they get $T = 8$, using trial and error, but if they don't get $T = 8$, can only score max M1A1M0A0

2b	M1	Use of $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with **a = (2i – 3j)** **OR** integration to give an expression of the form $\mathbf{C}t + (2\mathbf{i} - 3\mathbf{j})\frac{1}{2}t^2$ **, where C is their non-zero constant <u>vector</u> from (a)** Condone use of **a** = (2**i** + 3**j)** for this M mark **OR** any other complete method using vector **suva***t* equations
	A1	Correct unsimplified expression seen or implied
	M1	Use of $t = 4$ in their **s** (which must be a **displacement vector**) and then Pythagoras with the root sign **N.B.** This M mark can be implied by a correct answer, otherwise we need to see Pythagoras used, with the root sign, for the M mark.
	A1**cso**	Any surd form or 14 or better

Question	Scheme	Marks	AO

5(a)	Horizontal speed $= 20\cos 30°$	B1	3.4
	Vertical velocity <u>at $t = 2$</u>	M1	3.4
	$= 20\sin 30° - 2g$	A1	1.1b
	$\theta = \tan^{-1}\left(\pm\dfrac{9.6}{10\sqrt{3}}\right)$	M1	1.1b
	Speed $= \sqrt{100\times 3 + 9.6^2}$ or e.g. speed $= \dfrac{9.6}{\sin\theta}$	M1	1.1b
	19.8 or 20 $\left(\text{m s}^{-1}\right)$ at 29.0° or 29° to the horizontal oe	A1	2.2a
		(6)	
(b)	Using sum of horizontal distances $= 50$ at $t = 2$	M1	3.3
	$(u\cos\theta)\times 2 + (20\cos 30°)\times 2 = 50$ $(u\cos\theta = 25 - 20\cos 30°)$	A1	1.1b
	Vertical distances equal	M1	3.4
	$\Rightarrow (20\sin 30°)\times 2 - \dfrac{g}{2}\times 4 = (u\sin\theta)\times 2 - \dfrac{g}{2}\times 4$ $(20\sin 30° = u\sin\theta)$	A1	1.1b
	Solving for both θ and u	M1	3.1b
	$\theta = 52°$ or better (52.47756849….°) $u = 13$ or better (12.6085128…)	A1	2.2a
		(6)	
(c)	It does not take account of the fact that they are not particles (moving freely under gravity) It does not take account of the size(s) of the balls It does not take account of the spin of the balls It does not take account of the wind g is not exactly 9.8 m s^{-2} **N.B.** If they refer to the mass or weight of the balls give B0	B1	3.5b
		(1)	
		(13)	

Marks		**Notes**		
5a	B1	Seen or implied, possibly on a diagram		
	M1	Use of $v = u + at$ or any other complete method <u>using $t = 2$</u> Condone sign errors and sin/cos confusion.		
	A1	Correct unsimplified equation in v or v^2		
	M1	Correct use of trig to find a relevant angle for the direction. Must have found a horizontal and a vertical velocity component		
	M1	Use Pythagoras or trig to find the magnitude Must have found a horizontal and a vertical velocity component		
	A1	Or equivalent. Need magnitude **and** direction stated or implied in a diagram. (0.506 or 0.51 rads)		
5b	M1	First equation,in terms of u and θ (could be implied by subsequent working), using the horizontal motion with $t = 2$ used Condone sign errors and sin/cos confusion		
	A1	Correct unsimplified equation – any equivalent form		
	M1	Second equation, in terms of u and θ (could be implied by subsequent working), using the vertical motion – equating distances or just vertical components of velocities. Condone sign errors and sin/cos confusion		
	A1	Correct unsimplified equation – any equivalent form		
	M1	Complete strategy: all necessary equations formed and solve for u and θ **N.B.** This is an independent method mark but can only be earned if 50 m has been used in their solution.		
	A1	Both values correct. (Here we accept 2SF or better, since the g's cancel) Allow radians for θ: 0.92 or better (0.915906..) rads.		
5c	B1	Any factor **related to the model** as stated in the question. Penalise incorrect extras but ignore consequences e.g. 'AB (or the ground) is not horizontal' should be penalised or 'they do not move in a vertical plane' should be penalised		

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7(a)	Resolve vertically	M1	3.1b
	$R + 40\sin\alpha = 20g$	A1	1.1b
	Resolve horizontally	M1	3.1b
	$40\cos\alpha - F = 20a$	A1	1.1b
	$F = 0.14R$	B1	1.2
	$a = 0.396$ or 0.40 (m s^{-2})	A1	2.2a
		(6)	
(b)	Pushing will increase R which will increase available F	B1	2.4
	Increasing F will <u>decrease a</u> * GIVEN ANSWER	B1*	2.4
		(2)	
			(8 marks)

Notes:

(a)

M1: Resolve vertically with usual rules applying

A1: Correct equation**.** Neither g nor $\sin a$ need to be substituted

M1: Apply $F = ma$ horizontally, with usual rules

A1: Neither F nor $\cos a$ need to be substituted

B1: $F = 0.14R$ seen (e.g. on a diagram)

A1: Either answer

(b)

B1: Pushing increases R which produces an increase in available (limiting) friction

B1: F increase produces an <u>a decrease (need to see this)</u>

N.B. It is possible to score B0 B1 but for the B1, some "explanation" is needed to say why friction is increased e.g. by pushing into the ground.

Question	Scheme	Marks	AOs

10(a)	Using the model and vertical motion: $0^2 = (U\sin\alpha)^2 - 2g\times(3-2)$	M1	3.3
	$U^2 = \dfrac{2g}{\sin^2\alpha}$ * GIVEN ANSWER	A1*	2.2a
		(2)	
(b)	Using the model and horizontal motion: $s = ut$	M1	3.4
	$20 = Ut\cos\alpha$	A1	1.1b
	Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$	M1	3.4
	$-\frac{5}{4} = Ut\sin\alpha - \frac{1}{2}gt^2$	A1	1.1b
	sub for t: $-\frac{5}{4} = U\sin\alpha\left(\frac{20}{U\cos\alpha}\right) - \frac{1}{2}g\left(\frac{20}{U\cos\alpha}\right)^2$	M1 (I)	3.1b
	sub for U^2	M1(II)	3.1b
	$-\frac{5}{4} = 20\tan\alpha - 100\tan^2\alpha$	A1(I)	1.1b
	$(4\tan\alpha - 1)(100\tan\alpha + 5) = 0$	M1(III)	1.1b
	$\tan\alpha = \frac{1}{4} \Rightarrow \alpha = 14^0$ or better	A1(II)	2.2a
		(9)	
	N.B. For the last 5 marks, they may set up a quadratic in t, by substituting for $U\sin\alpha$ first, then solve the quadratic to find the value of t, then use $20 = Ut\cos\alpha$ to find α. The marks are the same but earned in a different order. Enter on ePen in the corresponding M and A boxes above, as indicated below.		
	Sub for $U\sin\alpha$ to give equation in t only	M1(II)	
	$-\frac{5}{4} = \sqrt{2g}t - \frac{1}{2}gt^2$	A1(I)	
	Solve for t	M1(III)	
	$t = \dfrac{5}{\sqrt{2g}}$ or 1.1 or 1.13 and use $20 = Ut\cos\alpha$	M1(I)	
	$\alpha = 14^0$ or better	A1(II)	
(b)	**ALTERNATIVE**		

	Using the model and horizontal motion: $s = ut$	M1	3.4
	$20 = Ut\cos\alpha$	A1	1.1b
	A to top: $s = vt - \frac{1}{2}at^2$ and top to T: $s = ut + \frac{1}{2}at^2$		
	$1 = \frac{1}{2}gt_1^2 \Rightarrow t_1 = \sqrt{\frac{2}{g}}$ and $\frac{9}{4} = \frac{1}{2}gt_2^2 \Rightarrow t_2 = \frac{3}{\sqrt{2g}}$ Total time $t = t_1 + t_2$	M1	3.4
	$= \sqrt{\frac{2}{g}} + \frac{3}{\sqrt{2g}}$ $(= \frac{5}{\sqrt{2g}})$	A1	1.1b
	$20 = U\frac{5}{\sqrt{2g}}\cos\alpha$ (sub. for t)	M1	3.1b
	$20 = \sqrt{\frac{2g}{\sin^2\alpha}}\frac{5}{\sqrt{2g}}\cos\alpha$ (sub. for U)	M1	3.1b
	$\tan\alpha = \frac{1}{4}$	A1	1.1b
	Solve for α	M1	1.1b
	$\Rightarrow \alpha = 14^0$ or better	A1	2.2a
		(9)	
(c)	The target will have dimensions so in practice there would be a range of possible values of α **Or** There will be air resistance **Or** The ball will have dimensions **Or** Wind effects **Or** Spin of the ball	B1	3.5b
		(1)	
(d)	Find U using their α e.g. $U = \sqrt{\frac{2g}{\sin^2\alpha}}$	M1	3.1b
	Use $20 = Ut\cos\alpha$ (or use vertical motion equation)	A1 **M1**	1.1b
	$t = \frac{5}{\sqrt{2g}}$ or 1.1 or 1.13	B1 **A1**	1.1b
		(3)	
(d)	**ALTERNATIVE**		

	A to top: $s = vt - \frac{1}{2}at^2$ and top to T: $s = ut + \frac{1}{2}at^2$	M1	3.1b
	$1 = \frac{1}{2}gt_1^2 \Rightarrow t_1 = \sqrt{\frac{2}{g}}$ and $\frac{9}{4} = \frac{1}{2}gt_2^2 \Rightarrow t_2 = \frac{3}{\sqrt{2g}}$ Total time $t = t_1 + t_2$	A1 **M1**	1.1b
	$= = \sqrt{\frac{2}{g}} + \frac{3}{\sqrt{2g}}$ $(= \frac{5}{\sqrt{2g}})$ = 1.1 or 1.13 (s)	B1 **A1**	1.1b
		(3)	
			(15 marks)

Notes:

(a)

M1: Or any other complete method to obtain an equation in U, g and α **only**

A1*: Correct GIVEN ANSWER

(b)

M1: Using horizontal motion

A1: Correct equation

M1: Using vertical motion . N.B. M0 if they use $s = \pm 2$ or ± 3, but allow $s = \pm 1.25$ or ± 0.75 or ± 2.25 or ± 2.75

A1: Correct equation

M1: Using $20 = Ut\cos\alpha$ to sub. for t

M1: Substituting for U^2 using (a)

A1: Correct quadratic equation (in $\tan\alpha$ **or** $\cot\alpha$ **)**

M1: Solve a 3 term quadratic, either by factorisation or formula (or by calculator (implied) if answer is correct) **and find** α

A1: $\alpha = 14^o$ or better (No restriction on accuracy since g's cancel)

N.B. If answer is correct, previous M mark can be implied, but if answer is incorrect, an explicit attempt to solve must be seen to earn the previous M mark.

(b) ALTERNATIVE

M1: Using the model with the usual rules applying to the equation

A1: Correct equation

M1: Using the model to obtain the **total** time from A to T

A1: Correct **total** time t

M1: Substitute for t in $20 = Ut\cos\alpha$

M1: Substitute for U in $20 = Ut\cos\alpha$, using part (a)

A1: Correct equation in $\tan\alpha$ **only**

M1: Solve equation for α

A1: $\alpha = 14^o$ or better (No restriction on accuracy since g's cancel)

N.B. If they quote the equation of the trajectory $y = x\tan\alpha - \dfrac{gx^2}{2U^2\cos^2\alpha}$ oe **AND** put in values for x and y, could score first 5 marks, M1A1M1A1M1 (nothing for the equation only); wrong x value loses first A mark and wrong y value loses second A mark

(c)

B1: Give one limitation of the model e.g. the ball will have dimensions, or there will be air resistance or wind effects or spin

N.B. B0 if any incorrect extra(s) but ignore extra consequences.

(d)

M1: Using their a to find a value for U

A1: Treat as M1: Using their U to find a value for t

B1: Treat as A1 : $t = 1.1$ or 1.10 (since depends on $g = 9.8$)

(d) ALTERNATIVE

M1: Using their a to find a value for U

A1: Treat as M1: Using their U to find a value for t

B1: Treat as A1 : $t = 1.1$ or 1.10 (since depends on $g = 9.8$)

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