adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Parametric equations

Mark scheme

7 questions

Exam questions and marking instructions remain credited to their board.

Contents

1 June 2025 - Paper 2 - Q5

2 June 2024 - Paper 2 - Q10

3 June 2023 - Paper 2 - Q9

4 June 2022 - Paper 1 - Q16

5 October 2021 - Paper 1 - Q13

6 October 2021 - Paper 2 - Q13

7 June 2018 - Paper 1 - Q14

Question	Scheme	Marks	AOs
5(a)	$-3=\frac{t-1}{2}\Rightarrow t=-5\Rightarrow y=5\left("-5"+2\right)^4$	M1	1.1b
	$\left(y=\right)405$	A1	1.1b
		(2)	
(b)	$x=\frac{t-1}{2}\Rightarrow t=2x+1\Rightarrow y=5\left("2x+1"+2\right)^4$	M1	1.1b
	$y=5\left(2x+3\right)^4$	A1	1.1b
		(2)	
(c)	$\left("2x+3"\right)^n\rightarrow ...\left("2x+3"\right)^{n-1}$	M1	1.1b
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)40\left(2\left(-3\right)+3\right)^3$	dM1	1.1b
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)-1080$	A1	1.1b
		(3)	
			(7 marks)

Notes

(a) **Mark parts (a) and (b) together.**

M1: Substitutes $x=-3$ into $x=\frac{t-1}{2}$, attempts to find *t*, and substitutes their *t* into $y=5\left(t+2\right)^4$

Condone slips. May be implied by substitution of $t=-5$ into *y* or by 405 or by $y=5\left(-3\right)^4$

Alternatively, they may substitute $x=-3$ into their $y=\mathrm{f}\left(x\right)$

A1: cao Answer only scores full marks and ISW after seeing 405 e.g. (–5, 405)
Accept (–3, 405) or e.g. $P=405$

(b) **Mark parts (a) and (b) together.**

M1: Attempts to make *t* the subject of $x=\frac{t-1}{2}$ using the correct **order** of operations and substitutes into $y=5\left(t+2\right)^4$ Condone slips e.g. dividing by 2 first: $t=\frac{x}{2}+1$ or missing the +2 in $y=5\left(t+2\right)^4$

Alt 1: Writes $t=\left(\frac{y}{5}\right)^{\frac{1}{4}}-2$, substitutes into $x=\frac{t-1}{2}$ and attempts to make *y* the subject using the correct **order** of operations, i.e., $2x=\left(\frac{y}{5}\right)^{\frac{1}{4}}-3\Rightarrow\left(2x+3\right)^4=\frac{y}{5}\Rightarrow y=5\left(2x+3\right)^4$

Alt 2: Makes *t* the subject and then substitutes into an expanded $5\left(t+2\right)^4$
Do not be too concerned about their expansion, but it must contain t^4 and a constant term.

A1: cao and ISW after a correct answer seen. May be scored for $y=5\left(2x+1+2\right)^4$
Allow e.g. $y=80x^4+480x^3+1080x^2+1080x+405$ or e.g.
$y=5\left(2x+1\right)^4+40\left(2x+1\right)^3+120\left(2x+1\right)^2+160\left(2x+1\right)+80$
Their RHS may be unsimplified but do not allow if e.g. binomial coefficients are still present.
Do not accept f (*x*) = … It must be *y* = …

(c) Note: Differentiation seen in (a) or (b) can score marks if used in (c)

M1: Reduces the power of their $\left("2x+3"\right)^n$ by one to $Q\left("2x+3"\right)^{n-1}$ where Q is a constant and could be 1. There should be no other terms using this method.

Alternatively, attempts to expand their $\left("2x+3"\right)^n$ (may have been expanded in (b)) and reduces the power of x by one in at least one term.

They may use parametric differentiation, i.e., $\frac{\mathrm{d}y}{\mathrm{d}t}=a(t+2)^3$ and $\frac{\mathrm{d}x}{\mathrm{d}t}=b$ where a and b are constants, leading to $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{a\left(t+2\right)^3}{b}$ i.e. they must divide the correct way round.

Condone attempts at the chain rule that reach e.g. $y'=\ldots u^3$ but make a slip when substituting back in for x or t, e.g., $\frac{\mathrm{d}y}{\mathrm{d}x}=\left(5x+3\right)^3$ or $\frac{\mathrm{d}y}{\mathrm{d}t}=\left(t+2\right)^2$, provided the intention is clear.

dM1: Substitutes $x=-3$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x which may be implied by their answer.

Using parametric differentiation, substitutes their t (found from an attempt at substituting $x=-3$ into $x=\frac{t-1}{2}$ which may have been seen in (a)) into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.

Note for reference, if correct, the parametric differentiation is $\frac{\mathrm{d}y}{\mathrm{d}t}=20(t+2)^3$ and $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{1}{2}$ leading to $\frac{\mathrm{d}y}{\mathrm{d}x}=40\left(t+2\right)^3$

They may substitute their value of t into $\frac{\mathrm{d}y}{\mathrm{d}t}$ first before using the chain rule to reach $\frac{\mathrm{d}y}{\mathrm{d}x}$ which is acceptable and implies the first M mark.

A1: $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)-1080$

Correct answer only scores full marks.

May be seen labelled as m or something else.

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AO
10(a)	$x=4, y=2 \Rightarrow t=-1$	B1	2.2a
	$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}=-3t^2\times\frac{1}{2(t+3)}$	M1	1.1b
	$\frac{dy}{dx}=-3(-1)^2\times\frac{1}{2(-1+3)}=-\frac{3}{4}$	M1	1.1b
	$\Rightarrow y-2=-\frac{3}{4}(x-4)$ or $\Rightarrow y=-\frac{3}{4}x+c \rightarrow 2=-\frac{3}{4}\times 4+c \Rightarrow c...$	ddM1	2.1
	$y-2=-\frac{3}{4}(x-4)\Rightarrow 4y-8=-3x+12$ or $c=5\Rightarrow y=-\frac{3}{4}x+5$ $\Rightarrow 3x+4y=20*$	A1*	1.1b
		(5)	
(b)	Maximum height is 9m	B1	3.4
		(1)	
			(6 marks)

Notes

(a) **If parametric differentiation is not used in part (a) (e.g. uses Cartesian form) then only the B mark is available but see alternative below.**

B1: Uses the given Cartesian coordinates to deduce the correct value for *t*.
If more than one value for *t* e.g. $t=-5$ is given and $t=-1$ is not "selected" score B0 but if **just** $t=-1$ is used subsequently allow recovery and score B1

M1: Attempts to use $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$ or equivalent with their differentiated equations.
There must be an attempt to differentiate both parameters, however poor, and divide or multiply correctly so using $\frac{dy}{dx}=\frac{y}{x}$ scores M0. Both parameters must be "changed".

Condone confusion with the variables e.g. referring to $\frac{dy}{dt}$ as $\frac{dy}{dx}$ if the intention is clear.

This may be implied by e.g. $\frac{dy}{dt}=-3t^2$, $\frac{dx}{dt}=2(t+3), t=-1, \frac{dy}{dt}=-3, \frac{dx}{dt}=4\Rightarrow\frac{dy}{dx}=-\frac{3}{4}$

M1: Uses their numerical value of *t* (not 4) in their $\frac{dy}{dx}$ to obtain a value.
Condone attempts with different values of *t* e.g. $t=-1$ and $t=-5$

ddM1: Applies a correct straight line method with their value of $\frac{dy}{dx}$ which has come from an attempt to use parametric differentiation with their value of *t* (not 4) and with $x = 4$ and $y = 2$ correctly placed. An attempt at the equation of the normal is M0.
If using $y = mx + c$ they must reach as far as $c = \ldots$
Depends on both previous M marks.

A1*: Correct equation as printed with no errors but condone $4y+3x=20*$
Allow equivalents e.g. $20=4y+3x*$ or $3x+4y=20*$
This is a printed answer so there must be at least one intermediate step as shown in the main scheme.

Alternative for (a) using parametric differentiation but avoids the need for a value for *t*:

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}x}=-3t^2\times\frac{1}{2(t+3)}$$

$$-3t^2\times\frac{1}{2(t+3)}=-3(1-y)^{\frac{2}{3}}\times\frac{1}{2\sqrt{x}}=-3(1-2)^{\frac{2}{3}}\times\frac{1}{2\sqrt{4}}=-\frac{3}{4}$$

or

$$-3t^2\times\frac{1}{2(t+3)}=-3\left(\sqrt{x}-3\right)^2\times\frac{1}{2\sqrt{x}}=-3(2-3)^2\times\frac{1}{2\sqrt{4}}=-\frac{3}{4}$$

or

$$-3t^2\times\frac{1}{2(t+3)}=-3(1-y)^{\frac{2}{3}}\times\frac{1}{2\left((1-y)^{\frac{1}{3}}+3\right)}=-3(1-2)^2\times\frac{1}{2\times2}=-\frac{3}{4}$$

$$\Rightarrow y-2=-\frac{3}{4}(x-4)\Rightarrow 3x+4y=20*$$

B1: **Either** a correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and/or y following a correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t **or** for $t=-1$ seen anywhere.

M1: Attempts to use $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}x}$ with their differentiated equations.

There must be an attempt to differentiate both parameters, however poor, and divide or multiply correctly so using $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y}{x}$ scores M0

Condone confusion with the variables e.g. referring to $\frac{\mathrm{d}y}{\mathrm{d}t}$ as $\frac{\mathrm{d}y}{\mathrm{d}x}$ if the intention is clear.

This may be implied by e.g. $\frac{\mathrm{d}y}{\mathrm{d}t}=-3t^2$, $\frac{\mathrm{d}x}{\mathrm{d}t}=2(t+3), t=-1$, $\frac{\mathrm{d}y}{\mathrm{d}t}=-3, \frac{\mathrm{d}x}{\mathrm{d}t}=4\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{3}{4}$

M1: Attempts to express their $\frac{\mathrm{d}y}{\mathrm{d}x}$ which is in terms of t, in terms of x and/or y **and** uses $x = 4$ and $y = 2$ correctly placed in an attempt to find the gradient of the tangent.

ddM1: Applies a correct straight line method with their value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ which has come from an attempt to use parametric differentiation with their gradient and with $x = 4$ and $y = 2$ correctly placed.

If using $y = mx + c$ they must reach as far as $c = \ldots$

Depends on both previous M marks.

A1*: Correct equation as printed with no errors.

This is a printed answer so there must be at least one intermediate step as shown in the main scheme.

(b)

B1: 9m or equivalent **including correct units**. Accept e.g. 9 metres, 900cm etc.

Question	Scheme	Marks	AOs

9(a) Way 1	$x=(t+3)^2-25$	**M1**	1.1b
	$\Rightarrow x+25=(t+3)^2 \Rightarrow (x+25)^{\frac{1}{2}}=(t+3) \Rightarrow y=...$	**M1**	2.1
	$y=6\ln(x+25)^{\frac{1}{2}} \Rightarrow y=3\ln(x+25)$	**A1cso**	1.1b
		(3)	
	(a) Way 2		
	$y=6\ln(t+3)=3\ln(t+3)^2$	**M1**	1.1b
	$y=3\ln(t+3)^2=3\ln(t^2+6t+9)=3\ln(x+16+9)$	**M1**	2.1
	$y=3\ln(x+25)$	**A1cso**	1.1b
	(a) Way 3		
	$y=6\ln(t+3) \Rightarrow \frac{y}{6}=\ln(t+3) \Rightarrow t+3=e^{\frac{y}{6}} \Rightarrow t=e^{\frac{y}{6}}-3$	**M1**	1.1b
	$x=\left(e^{\frac{y}{6}}-3\right)^2+6\left(e^{\frac{y}{6}}-3\right)-16 \Rightarrow y=...$ or $x=\left(e^{\frac{y}{6}}-3+8\right)\left(e^{\frac{y}{6}}-3-2\right) \Rightarrow y=...$	**M1**	2.1
	$y=3\ln(x+25)$	**A1cso**	1.1b
	(a) Way 4		
	$x=(t+3)^2-25$	**M1**	1.1b
	$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\left(\frac{6}{t+3}\right)}{2t+6} \Rightarrow \frac{3}{(t+3)^2}=\frac{3}{x+25} \Rightarrow y=3\ln(x+25)(+c)$	**M1**	2.1
	e.g. $t=0 \Rightarrow x=-16, y=6\ln 3 \Rightarrow 6\ln 3=3\ln(9) \Rightarrow c=0$ $y=3\ln(x+25)$	**A1cso**	1.1b
(b)	$x=0,\ y=3\ln 25$ oe e.g. $6\ln 5$	**B1ft**	2.2a
	$\frac{dy}{dx}=\frac{3}{x+"25"} \Rightarrow \frac{dy}{dx}=\frac{3}{0+"25"} \quad \left(=\frac{3}{25}\right)$ **or** $\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\left(\frac{6}{t+3}\right)}{2t+6} \Rightarrow \frac{6}{2\times 2+6} \quad \left(=\frac{6}{50}=\frac{3}{25}\right)$	**M1**	2.1
	$y-"3\ln 25"="\frac{3}{25}"(x\{-0\})$	**dM1**	3.1a
	$25y-3x=150\ln 5$	**A1**	2.2a
		(4)	
			(7 marks)
	Notes		
	Choose the mark scheme that best matches their chosen method.		

(a) **Way 1**

M1: Attempts to complete the square. Award for sight of $x=(t+3)^2 \pm ...$ where $\ldots \neq 0$

M1: Rearranges their $x=(t+3)^2-25$ to either $(t+3)=...$ or $(t+3)^2=...$ and then substitutes correctly their expression into the parametric equation for y. So e.g., $t=\sqrt{x+25}-3 \rightarrow y=6\ln\left(\sqrt{x+25}-3\right)$ is M0.

A1cso: $y=3\ln(x+25)$ including brackets with all stages of working shown.
The "$y=$" must appear at some point.

Way 2

M1: Attempts to use the power rule for logarithms $y=6\ln(t+3)=...\ln(t+3)^2$ where $\ldots \neq 6$

M1: Writes $y=6\ln(t+3)$ as $3\ln(t+3)^2$ and then multiplies out and substitutes correctly in for t to obtain a Cartesian equation for C

A1cso: $y=3\ln(x+25)$ including brackets with all stages of working shown.
The "$y=$" must appear at some point.

Way 3

M1: Attempts to make t the subject for $y=6\ln(t+3)$ to obtain $t=\mathrm{e}^{\frac{y}{6}} \pm ...$ where $\ldots \neq 0$

M1: Substitutes $t=\mathrm{e}^{\mathrm{f}(y)} \pm ...$ correctly into $x=t^2+6t-16$ and rearranges to make y the subject.

A1cso: $y=3\ln(x+25)$ including brackets with all stages of working shown.
The "$y=$" must appear at some point.

Way 4

M1: Attempts to complete the square. Award for sight of $x=(t+3)^2 \pm ...$ where $\ldots \neq 0$

M1: Attempts to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ where $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\left(\frac{...}{t+3}\right)}{at+b}$, $a,b \neq 0$ and uses the completed square form to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and then integrates to obtain a Cartesian equation for C

A1cso: A complete method using any correct point on the curve to show that $c = 0$ and obtain $y=3\ln(x+25)$ with all stages of working shown. The "$y=$" must appear at some point.

Note that a common incorrect approach in (a) is:

$$x=t^2+6t-16=(t-2)(t+8) \Rightarrow x=t-2 \Rightarrow t=x+2 \Rightarrow y=6\ln(x+5)$$

which scores no marks.

(b)

B1ft: Deduces $y = 3\ln 25$ oe e.g $y = 6\ln 5$ but allow follow through on their Cartesian equation with $x = 0$ and apply isw after a correct value or ft value for y

M1: Attempts to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 0$ so score for obtaining $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\ldots}{x + "25"}$ and substituting in $x = 0$

Allow this mark if they use the letters A and B e.g. $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\ldots}{x + B} = \frac{\ldots}{0 + B}$ or allow a "made up" A and B.

or

Attempts to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $t = 2$ by finding $\frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}} = \frac{\left(\frac{6}{t+3}\right)}{2t+6} \Rightarrow \frac{\frac{6}{5}}{2\times 2+6} \left(= \frac{6}{50} = \frac{3}{25}\right)$

For the derivative look for $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\left(\frac{\ldots}{t+3}\right)}{at+b}$ oe e.g. $\left(\frac{\ldots}{t+3}\right) \times \frac{1}{at+b}$ $a, b \neq 0$

NOTE if candidates find $\frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}} = \frac{\left(\frac{6}{t+3}\right)}{2t+6} = \frac{6(2t+6)}{t+3} = 12$ we will give BOD that $t = 2$ has been used unless there is clear evidence that $t = 2$ has not been used.

dM1: Attempts to find the equation of the tangent. Score for sight of $y - "3\ln 25" = "\frac{3}{25}"(x\{-0\})$ or if they use $y = mx + c$ they must proceed as far as $c = \ldots$ **It is dependent on the previous method mark.**

Must have numeric A and B now.

A1: $25y - 3x = 150\ln 5$ or any integer multiple of this equation in the form $ax + by = c\ln 5$

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
16 (a)	Attempts $y.\dfrac{\mathrm{d}x}{\mathrm{d}t}=\left(2\sin 2t+3\sin t\right)\times 16\sin t\cos t$ and uses $\sin 2t=2\sin t\cos t$	M1	2.1
	Correct expanded integrand. **Usually** for one of $\left(R\right)=\int \underline{\underline{48\sin^2 t\cos t+16\sin^2 2t}}\ \mathrm{d}t$ $\left(R\right)=\int \underline{\underline{48\sin^2 t\cos t+64\sin^2 t\cos^2 t}}\ \mathrm{d}t$ $\left(R\right)=\int \underline{\underline{24\sin 2t\sin t+16\sin^2 2t}}\ \mathrm{d}t$	A1	1.1b
	Attempts to use $\cos 4t=1-2\sin^2 2t=\left(1-8\sin^2 t\cos^2 t\right)$	M1	1.1b
	$R=\int_0^a 8-8\cos 4t+48\sin^2 t\cos t\ \mathrm{d}t$ *	A1*	2.1
	Deduces $a=\dfrac{\pi}{4}$	B1	2.2a
		(5)	
(b)	$\int 8-8\cos 4t+48\sin^2 t\cos t\ \mathrm{d}t=8t-2\sin 4t+16\sin^3 t$	M1 A1	2.1 1.1b
	$\left[8t-2\sin 4t+16\sin^3 t\right]_0^{\frac{\pi}{4}}=2\pi+4\sqrt{2}$	M1 A1	2.1 1.1b
		(4)	
			(9 marks)
Notes:			

(a) Condone work in another variable, say $\theta\leftrightarrow t$ if used consistently for the first 3 marks

M1: For the key step in attempting $y.\dfrac{\mathrm{d}x}{\mathrm{d}t}=\left(2\sin 2t+3\sin t\right)\times 16\sin t\cos t$ with an attempt to use $\sin 2t=2\sin t\cos t$ Condone slips in finding $\dfrac{\mathrm{d}x}{\mathrm{d}t}$ but it must be of the form $k\sin t\cos t$

E.g. I $\quad y.\dfrac{\mathrm{d}x}{\mathrm{d}t}=\left(2\sin 2t+3\sin t\right)\times k\sin t\cos t=\left(4\sin t\cos t+3\sin t\right)\times k\sin t\cos t$

E.g. II $\quad y.\dfrac{\mathrm{d}x}{\mathrm{d}t}=\left(2\sin 2t+3\sin t\right)\times k\sin t\cos t=\left(2\sin 2t+3\sin t\right)\times \dfrac{k}{2}\sin 2t$

A1: A correct (expanded) integrand in t . Don't be concerned by the absence of $\int$ or dt or limits

$\left(R\right)=\int \underline{\underline{48\sin^2 t\cos t+16\sin^2 2t}}\ \mathrm{d}t$ or $\left(R\right)=\int \underline{\underline{48\sin^2 t\cos t+64\sin^2 t\cos^2 t}}\ \mathrm{d}t$

but watch for other correct versions such as $\left(R\right)=\int \underline{\underline{24\sin 2t\sin t+16\sin^2 2t}}\ \mathrm{d}t$

If they have the form $P\sin^2 2t$ it is acceptable to write $P\sin^2 2t = \frac{P}{2}\left(\pm 1 \pm \cos 4t\right)$

If they have the form $Q\sin^2 t\cos^2 t$ sight and use of $\sin 2t$ and/or $\cos 2t$ will usually be seen first.

There are many ways to do this, below is such an example

$$Q\sin^2 t\cos^2 t = Q\left(\frac{1-\cos 2t}{2}\right)\left(\frac{1+\cos 2t}{2}\right) = Q\left(\frac{1-\cos^2 2t}{4}\right) = Q\left(\frac{1}{4}-\frac{\cos^2 2t}{4}\right) = Q\left(\frac{1}{4}-\frac{1+\cos 4t}{8}\right)$$

Allow candidates to start with the given answer and work backwards using the same rules. So expect to see $\cos 4t = \pm 1 \pm 2\times \sin^2 2t$ or $\cos 4t = \pm 2\times \cos^2 2t \pm 1$ before double angle identities for $\sin 2t$ or $\cos 2t$ are used.

A1*: Proceeds to the given answer with correct working. The order of the terms is not important. Ignore limits for this mark. The integration sign and the dt must be seen on their final answer. If they have worked backwards there must be a concluding statement to the effect that they know that they have shown it. The integration sign and the dt must also be seen

E.g. Reaches $\int 48\sin^2 t\cos t + 64\sin^2 t\cos^2 t \ \mathrm{d}t$

Answer is $\int 8 - 8\cos 4t + 48\sin^2 t\cos t \ \mathrm{d}t$

$= \int 8 - 8\left(1-2\sin^2 2t\right) + 48\sin^2 t\cos t \ \mathrm{d}t$

$= \int 16\sin^2 2t + 48\sin^2 t\cos t \ \mathrm{d}t$

$= \int 64\sin^2 t\cos^2 t + 48\sin^2 t\cos t \ \mathrm{d}t$ which is the same, ✓

B1: Deduces $a = \frac{\pi}{4}$. It may be awarded from the upper limit and can be awarded from (b)

(b)

M1: For the key process in using a correct approach to integrating the trigonometric terms. May be done separately.

There may be lots of intermediate steps (e.g. let $u = \sin t$) .

There are other more complicated methods so look carefully at what they are doing.

$\int 8 - 8\cos 4t + 48\sin^2 t\cos t \ \mathrm{d}t = ... \pm P\sin 4t \pm Q\sin^3 t$ where P and Q are constants

A1: $\int 8 - 8\cos 4t + 48\sin^2 t\cos t \ \mathrm{d}t = 8t - 2\sin 4t + 16\sin^3 t\left(+c\right)$

If they have written $16\sin^3 t$ as $16\sin t^3$ only award if further work implies a correct answer.

Similarly, 8t may be written as 8x. Award if further work implies 8t, e.g. substituting in their limits. Do not penalise this sort of slip at all, these are intermediate answers.

M1: Uses the limits their a and 0 where $a = \frac{\pi}{6}, \frac{\pi}{4}$ or $\frac{\pi}{3}$ in an expression of the form $kt \pm P\sin 4t \pm Q\sin^3 t$ leading to an exact answer. Ignore evidence at lower limit as terms are 0

A1: CSO $2\pi + 4\sqrt{2}$ or exact **simplified** equivalent such as $2\pi + \frac{8}{\sqrt{2}}$ or $2\pi + \sqrt{32}$.

Be aware that $\int_0^{\frac{\pi}{4}} 8 - 8\cos 4t + 48\sin^2 t\cos t \ \mathrm{d}t = 8t + \lambda\sin 4t + 16\sin^3 t\left(+c\right)$ would lead to the correct answer but would only score M1 A0 M1 A0

Question	Scheme	Marks	AOs
13	$(x-3)^2+y^2=\left(\dfrac{t^2+5}{t^2+1}-3\right)^2+\left(\dfrac{4t}{t^2+1}\right)^2$	M1	3.1a
	$=\dfrac{\left(2-2t^2\right)^2+16t^2}{\left(t^2+1\right)^2}=\dfrac{4+8t^2+4t^4}{\left(t^2+1\right)^2}$	dM1	1.1b
	$\dfrac{4\left(t^4+2t^2+1\right)}{\left(t^2+1\right)^2}=\dfrac{4\left(t^2+1\right)^2}{\left(t^2+1\right)^2}=4^*$	A1*	2.1
		(3)	

M1: Attempts to substitute the given parametric forms into the Cartesian equation or the lhs of the Cartesian equation. There may have been an (incorrect) attempt to multiply out the $(x-3)^2$ term.

dM1: Attempts to combine (at least the lhs) using correct processing into a single fraction, multiplies out and collects terms on the numerator.

A1*: Fully correct proof showing all key steps

Question	Scheme	Marks	AOs
Alt	$x=\dfrac{t^2+5}{t^2+1}\Rightarrow xt^2+x=t^2+5\Rightarrow t^2=\dfrac{5-x}{x-1}$ $y=\dfrac{4t}{t^2+1}\Rightarrow y^2=\dfrac{16t^2}{\left(t^2+1\right)^2}=\dfrac{16\left(\dfrac{5-x}{x-1}\right)}{\left(\dfrac{5-x}{x-1}+1\right)^2}$	M1	3.1a
	$y^2=\dfrac{16\left(\dfrac{5-x}{x-1}\right)}{\left(\dfrac{5-x}{x-1}+1\right)^2}=16\left(\dfrac{5-x}{x-1}\right)\times\left(\dfrac{(x-1)}{5-x+x-1}\right)^2\Rightarrow y^2=(5-x)(x-1)$	dM1	1.1b
	$y^2=(5-x)(x-1)\Rightarrow y^2=6x-x^2-5$ $\Rightarrow y^2=4-(x-3)^2$ or other intermediate step $\Rightarrow (x-3)^2+y^2=4^*$	A1*	2.1
		(3)	
			(3 marks)
Notes			

M1: Adopts a correct strategy for eliminating t to obtain an equation in terms of x and y only. See scheme.

Other methods exist which also lead to an appropriate equation. E.g using $t=\dfrac{y}{x-1}$

dM1: Uses correct processing to eliminate the fractions and start to simplify

A1*: Fully correct proof showing all key steps

Question	Scheme	Marks	AOs

13(a)	$y = \text{cosec}^3\theta \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}\theta} = -3\text{cosec}^2\theta\text{cosec}\,\theta\cot\theta$	B1	1.1b
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}\theta} \div \dfrac{\mathrm{d}x}{\mathrm{d}\theta}$	M1	1.1b
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-3\text{cosec}^3\theta\cot\theta}{2\cos 2\theta}$	A1	1.1b
		(3)	
(b)	$y = 8 \Rightarrow \text{cosec}^3\theta = 8 \Rightarrow \sin^3\theta = \dfrac{1}{8} \Rightarrow \sin\theta = \dfrac{1}{2}$	M1	3.1a
	$\theta = \dfrac{\pi}{6} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-3\text{cosec}^3\left(\frac{\pi}{6}\right)\cot\left(\frac{\pi}{6}\right)}{2\cos\left(\frac{2\pi}{6}\right)} = \ldots$ **or** $\sin\theta = \dfrac{1}{2} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\frac{-3}{\sin^3\theta}\times\frac{\cos\theta}{\sin\theta}}{2\left(1-2\sin^2\theta\right)} = \dfrac{-3\times 8\times\frac{\sqrt{3}/2}{1/2}}{2\left(1-2\times\frac{1}{4}\right)}$	M1	2.1
	$= -24\sqrt{3}$	A1	2.2a
		(3)	
			(6 marks)

Notes

(a)

B1: Correct expression for $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}$ seen or implied in any form e.g. $\dfrac{-3\cos\theta}{\sin^4\theta}$

M1: Obtains $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = k\cos 2\theta$ or $\alpha\cos^2\theta + \beta\sin^2\theta$ (from product rule on $\sin\theta\cos\theta$)

and attempts $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}\theta} \div \dfrac{\mathrm{d}x}{\mathrm{d}\theta}$

A1: Correct expression in any form.

May see e.g. $\dfrac{-3\cos\theta}{2\sin^4\theta\cos 2\theta}$, $-\dfrac{3}{4\sin^4\theta\cos\theta - 2\sin^3\theta\tan\theta}$

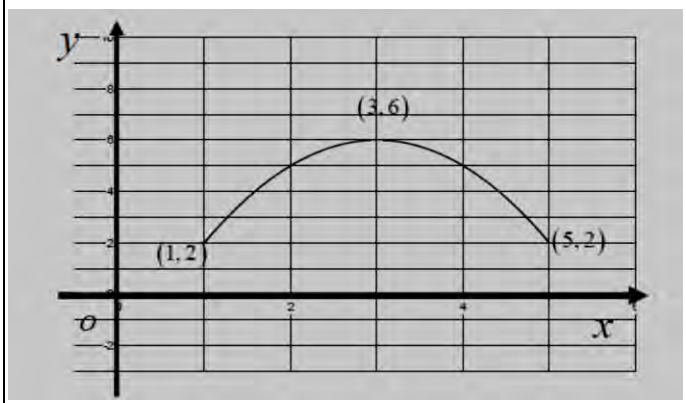
(b)

M1: Recognises the need to find the value of $\sin\theta$ or θ when $y = 8$ and uses the y parameter to establish its value. This should be correct work leading to $\sin\theta = \dfrac{1}{2}$ or e.g. $\theta = \dfrac{\pi}{6}$ or 30°.

M1: Uses their value of $\sin\theta$ or θ in their $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ from part (a) (working in exact form) in an attempt to obtain an exact value for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. May be implied by a correct exact answer.

If no working is shown but an exact answer is given you may need to check that this follows their $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

A1: Deduces the correct gradient

Question	Scheme	Marks	AOs
14(a)	Attempts to use $\cos 2t = 1-2\sin^2 t \Rightarrow \frac{y-4}{2} = 1-2\left(\frac{x-3}{2}\right)^2$	M1	2.1
	$\Rightarrow y-4 = 2-4\times\frac{(x-3)^2}{4} \Rightarrow y = 6-(x-3)^2$ *	A1*	1.1b
		(2)	
(b)	 ∩ shaped parabola	M1	1.1b
	Fully correct with 'ends' at (1,2) & (5,2)	A1	1.1b
	Suitable reason : Eg states as $x = 3+2\sin t$, $1 \leqslant x \leqslant 5$	B1	2.4
		(3)	
(c)	Either finds the lower value for $k = 7$ or deduces that $k < \frac{37}{4}$	B1	2.2a
	Finds where $x+y=k$ meets $y = 6-(x-3)^2$ $\Rightarrow k-x = 6-(x-3)^2$ and proceeds to 3TQ in x or y	M1	3.1a
	Correct 3TQ in x $\quad x^2-7x+(k+3)=0$ Or y $\quad y^2+(7-2k)y+(k^2-6k+3)=0$	A1	1.1b
	Uses $b^2-4ac=0 \Rightarrow \quad 49-4\times1\times(k+3)=0 \Rightarrow k=\left(\frac{37}{4}\right)$ or $(7-2k)^2-4\times1\times(k^2-6k+3)=0 \Rightarrow k=\left(\frac{37}{4}\right)$	M1	2.1
	Range of values for $k = \left\{k : 7 \leqslant k < \frac{37}{4}\right\}$	A1	2.5
		(5)	
			(10 marks)

(a)

M1: Uses $\cos 2t = 1-2\sin^2 t$ in an attempt to eliminate t

A1 : Proceeds to $y=6-(x-3)$ without any errors

Allow a proof where they start with $y=6-(x-3)^2$ and substitute the parametric coordinates. M1 is scored for a correct $\cos 2t = 1-2\sin^2 t$ but A1 is only scored when both sides are seen to be the same AND a comment is made, hence proven, or similar .

(b)

M1: For sketching a $\cap$ parabola with a maximum in quadrant one. It does not need to be symmetrical

A1: For sketching a $\cap$ parabola with a maximum in quadrant one and with end coordinates of $(1,2)$ and $(5,2)$

B1: Any suitable explanation as to why *C* does not include all points of $y=6-(x-3)^2, \quad x\in\mathbb{R}$

This should include a reference to **the limits on sin or cos** with a **link to a restriction on *x* or *y***.

For example

'As $-1\leq \sin t \leq 1$ then $1\leqslant x\leqslant 5$' Condone in words '*x* lies between 1 and 5' and strict inequalities

'As $\sin t \leq 1$ then $x\leqslant 5$' Condone in words '*x* is less than 5'

'As $-1\leq \cos(2t)\leq 1$ then $2\leqslant y\leqslant 6$ ' Condone in words '*y* lies between 2 and 6'

Withhold if the statement is incorrect Eg "because the domain is $2\leqslant x\leqslant 5$"

Do not allow a statement on the top limit of *y* as this is the same for both curves

(c)

B1: Deduces either

- the correct that the lower value of $k=7$ This can be found by substituting into $(5,2)$
  $x+y=k \Rightarrow k=7$ or substituting $x=5$ into $x^2-7x+(k+3)=0 \Rightarrow 25-35+k+3=0$
  $\Rightarrow k=7$
- or deduces that $k<\frac{37}{4}$ This may be awarded from later work

M1: For an attempt at the upper value for *k*.

Finds where $x+y=k$ meets $y=6-(x-3)^2$ once by using an appropriate method.

Eg. Sets $k-x=6-(x-3)^2$ and proceeds to a 3TQ

A1: Correct 3TQ $x^2-7x+(k+3)=0$ The = 0 may be implied by subsequent work

M1: Uses the "discriminant" condition. Accept use of $b^2=4ac$ oe or $b^2...4ac$ where ... is any inequality leading to a critical value for *k*. Eg. one root $\Rightarrow 49-4\times1\times(k+3)=0 \Rightarrow k=\frac{37}{4}$

A1: Range of values for *k* = $\left\{k: 7\leqslant k<\frac{37}{4}\right\}$ Accept $k\in\left[7,\frac{37}{4}\right)$ or exact equivalent

ALT	As above	B1	2.2a
	Finds where $x+y=k$ meets $y=6-(x-3)^2$ once by using an appropriate method. Eg. Sets gradient of $y=6-(x-3)^2$ equal to -1	M1	3.1a
	$-2x+6=-1 \Rightarrow x=3.5$	A1	1.1b
	Finds point of intersection and uses this to find upper value of *k*. $y=6-(3.5-3)^2=5.75$ Hence using $k=3.5+5.75=9.25$	M1	2.1
	Range of values for *k* = $\{k: 7\leqslant k<9.25\}$	A1	2.5

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