adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Probability

Mark scheme

13 questions

Exam questions and marking instructions remain credited to their board.

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3(a)	[Let D = the number of defective components] $D \sim B(36, 0.015)$	M1	3.3
(i)	[$P(D = 2)$ =] 0.084792… awrt **<u>0.0848</u>**	A1	1.1b
(ii)	[$P(D > 3) = 1 - P(D \leq 3) = 1 - 0.9979\ldots$ =] 0.002032… awrt **<u>0.00203</u>**	A1	1.1b
		(3)	
3(b)	[Let X = the number of defective components] $X \sim B(260, 0.015)$	M1	3.3
	[$P(X \geqslant 8)$ =] 0.0441(1…)	A1	1.1b
	[0.04 > 0.025 so not significant] <u>Insufficient</u> evidence that <u>proportion/probability</u> of defective <u>components</u> has <u>changed</u> (o.e.)	A1	2.2b
		(3)	
			(6 marks)

Notes:

(a)	M1	for selecting B(36, 0.015) sight of or use of. Must be B(…) [Implied by sight of Po(0.54)] May be implied by sight of: 0.0848 <u>or</u> awrt 0.085 <u>or</u> 0.00203 <u>or</u> 0.998 <u>or</u> 0.002
(i)	A1	for awrt 0.0848
(ii)	A1	for awrt 0.00203 [Allow 2.03×10^{-3}]
	Use of Poisson Po(0.54) (i) 0.08496 scores A0 (ii) 0.002309… awrt 0.00231 scores A1	
Notation: We do not accept "calculator speak" e.g. $N = 36$, $p = 0.015$ instead of B(36, 0.015)		
(b)	M1	for sight of B(260, 0.015) may be implied by probability of awrt 0.0441 <u>or</u> awrt 0.0264 or awrt 0.956 <u>or</u> correct CR $X = 0$ [or ≤ 0] **and** $X \geq 9$ <u>or</u> $P(X \geq 8) = 0.04$ or better
	A1	for probability of awrt 0.0441 (selected if more probabilities seen) Condone $P(X = 8) = 0.0441\ldots$ If $P(X \geq 8)$ is explicitly seen allow 0.04 or better <u>or</u> for the correct CR $X = 0$ [or ≤ 0] **and** $X \geq 9$. Allow acceptance region [1, 8] (o.e.)
	A1	**dep on M1A1** for a correct conclusion in context. Allow e.g: <u>proportion/probability</u> of defective <u>components</u> has <u>not</u> <u>changed</u> <u>proportion/probability</u> of defective <u>components</u> is still <u>0.015</u> Allow "defectives" instead of "components" If a comparison is seen (other than 0.956 < 0.975) it must be with 0.025 Incorrect comparisons score A0 (condone missing comparison) Do not accept contradictory statements e.g. "significant so insufficient evidence of change in proportion" would score A0 "Insufficient evidence that $p \neq 0.015$" is A0, we need the words.
SC	Poisson Approximation (If you see Po(3.9) or probability 0.0454…)	
	B2 and mark as M0A1A1	
	B1	for probability of awrt 0.0454 <u>or</u> 0.04… if $P(X \geq 8)$ seen <u>or</u> CR as above
	B1	for correct conclusion in context (same rules as above)

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
6(a)	Use sum of probs = 1: $1=\frac{1}{k}(2+4+...+2n)$ Allow $1=\frac{S_r}{k}$ where $S_r=\sum_{r=1}^{n}2r$	M1	3.1a
	Sum e.g. $\frac{n}{2}[4+2(n-1)]$ or $\frac{n}{2}[2+2n]$ or use $\sum_{1}^{n}r=\frac{n(n+1)}{2}$	M1	2.1
	Use sum formula: e.g. $1=\frac{1}{k}\times\frac{n}{2}[4+2(n-1)]$ or $1=\frac{n}{2}\left[\frac{2}{k}+\frac{2n}{k}\right]$	A1	1.1b
	$k = n(n + 1)$ *	A1*cso	1.1b
		(4)	
6(b)	Cases: R = 16, 18, 20, 22 and 24	M1	3.4
	Probability = $\left[\frac{16+18+20+22+24}{20\times21}\right]=\frac{5}{21}$	A1	1.1b
		(2)	
6(c)	[No real roots $\Rightarrow\Delta<0$] so $g^2-4(3g-5)[<0]$	M1	3.1a
	$[g^2-12g+20<0\Rightarrow]$ $(g-10)(g-2)[<0]$	M1	2.1
	So require $2 < g < 10$ or $4\le g\le 8$	M1	1.1b
	So require g = 4, 6, 8 only	dM1	1.1b
	Probability = $\left[\frac{4+6+8}{20\times21}\right]=\frac{18}{420}=\frac{3}{70}$	A1	3.2a
		(5)	
			(11 marks)

Notes:

(a)	M1	for clear attempt/intention to use sum of probs = 1 for n terms. Must see k and "= 1" Condone missing … $\sum_{2}^{2n}r$ is M0 since not n terms
	M1	for using an arithmetic series or Σr to find sum to n terms. No need for k
	A1	(dep on M1M1) for a correct working leading to a correct equation in k and n
	A1*	cso for correct solution with both Ms clearly scored and no incorrect working seen
	SC	Start with $1=\frac{n}{2}\left[\frac{2}{k}+\frac{2n}{k}\right]$ or $1=\frac{1}{k}\times\frac{n}{2}[4+2(n-1)]$ and no mention of a and l or a and d Score **SC B2** (M1M0A1A0) 1st B1 for the start and 2nd B1cso for completing to printed answer.
(b)	M1	for identifying the correct values of R
	A1	for a correct probability (any exact equivalent) Allow $0.\dot{2}38095\dot{2}$ Correct answer with no working scores 2 marks
(c)	M1	for attempt to use discriminant [< 0 not needed here]. Condone g^2-12g
	M1	for attempting to factorise or find critical values for their 3TQ (ignore their < & allow = 0)
	M1	for choosing the correct (inside) region, ft their critical values from a 3TQ with < 0
	dM1	(dep on 3rd M1) for choosing/identifying the appropriate values of g (ft their critical values from 3TQ with < 0 used)
	A1	for an exact probability in any form. Allow $0.0\dot{4}28571\dot{4}$ A correct answer with no incorrect working will score full marks
Acc	**If scored A0 for an answer of awrt 0.238 in (b), allow A1 in (c) for awrt 0.0429**	

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Qu 1	Scheme	Marks	AO
(a)	X~B(10, $\frac{1}{6}$) [Allow 0.167 or better for $\frac{1}{6}$]	M1	3.3
(i)	[P(X = 3) =] 0.155045… **awrt <u>0.155</u>**	A1	1.1b
(ii)	[P(X < 3) = P(X „ 2) =] 0.775226… **awrt <u>0.775</u>**	A1	1.1b
		(3)	
(b)	[Let D = no. of days when X = 3] D~B(60, "0.155")	M1	3.3
	P(D …12) = 1 – P(D „ 11) [Allow 1 – P(D < 12)]	M1	3.4
	= 1 – 0.78819… **awrt <u>0.212</u>**	A1	1.1b
		(3)	
(c)	[n = 600, p = $\frac{1}{6}$] estimate = **<u>100</u>**	B1	3.4
		(1)	
(d)	[S = total no. of sixes over 60 days.] $S \approx T \sim N\left("100", \sqrt{\frac{5}{6}\times 100}^{2}\right)$	M1A1	3.3,1.1b
	P(S > 95) ≈ P([T >]95.5) <u>or</u> $P\left([Z>]\frac{95.5-"100"}{"9.128..."}\right)$ <u>or</u> P([Z >] – 0.49..)	M1	3.4
	= 0.688976… **awrt <u>0.689</u>**	A1	1.1b
		(4)	
		(11 marks)	

	Notes
	If you see any attempt using an *n*-sided die with *n* not equal to 6 please send to review.
(a)	M1 for sight or use of the correct distribution. <u>Must</u> have B, or Bin or Bpd or Bcd **and** the correct value for n and p, just n = 10, p = $\frac{1}{6}$ is M0
	Implied by one answer correct to 2dp <u>or</u> by sight of $\binom{10}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^7$ or one of :
	[P(X = 0) =] **0.16**(1…),[P(X = 1) =] **0.32**(3…), [P(X = 2) =] **0.29**(0…), [P(X „ 3) =] **0.93**(0…)
(i)	1st A1 for awrt 0.155
(ii)	2nd A1 for awrt 0.775
(b)	1st M1 for selecting a correct model. Sight or use of correct binomial, ft their (a)(i)
	May be implied by sight of [P(D „ 11) =] 0.78… or 0.79 <u>or</u> [P(D „ 12) =] 0.87…
	2nd M1 for correct interpretation of "at least 12" and writing or using 1 – P(D „ 11)
	We are <u>not</u> attempting to ft their incorrect 0.155 on our calculators here.
	A1 for awrt 0.212 [Answer only 3/3]
(c)	B1 for 100 but must be seen in part (c) i.e. between (b) and (d)
(d)	1st M1 for attempting normal **with mean = 100** <u>or</u> ft their answer to (c)
	May be implied by the correct mean and a correctly labelled s.d. (σ) or var (σ^2)
	1st A1 for correctly labelled standard deviation allow $\sqrt{\frac{250}{3}} = \sqrt{83.3...}$ = 9.1(28….) <u>or</u>
	correctly labelled variance. Implied by N($\mu, \frac{250}{3}$) <u>or</u> correct answer.
	2nd M1 for attempt at continuity correction i.e. sight of 95 <u>+</u> 0.5
	2nd A1 for awrt 0.689 [Answer only 4/4]
NB	**If they don't state the model for 1st M1 but just give probabilities with probability statements (*Y* is any letter):**
$\sigma = \frac{250}{3} =$	1st M1 implied by: P(Y > 94.5) = 0.52(63…), P(Y > 95) = 0.52(39…), P(Y > 95.5) = 0.52(15..)
No cc	1st M1 1st A1 implied by: P(T > 95) = 0.70(805…)
	1st M1 1st A1 2nd M1 implied by: P(T > 94.5) = 0.72(657…)
	Exact binomial gives 0.68567… and will likely score 0/4

Qu 4	Scheme	Marks	AO
(a)	$H_0 : p = 0.1$ $H_1 : p \neq 0.1$ [Allow 10% for 0.1]	B1	2.5
	[$X \sim B(40, 0.1)$] $\Rightarrow$ $P(X = 0) = 0.0148$ [Allow any letter for X]	M1	3.3
	$P(X \ldots 9) = 1 - P(X \,„\, 8) = 1 - 0.9845 = 0.0155$	A1	1.1b
	Critical region is $\{X = 0\} \cup \{X \ldots 9\}$ (o.e.)	A1	1.1b
		(4)	
(b)	["0.0148" + "0.0155"] = 0.0303	B1ft	1.1b
		(1)	
(c)	[Provided 7 is not in their CR] insufficient evidence to support Freya's belief	B1	2.2b
		(1)	
		(6 marks)	

	Notes
	Mark (a) and (b) together for sight of the probabilities.
(a)	B1 for both hypotheses in terms of p or π. Must be attached to H_0 and H_1
	M1 for use of the correct model. Implied by sight of at least one probability truncated or rounded to at least 2sf from: 0.0155, 0.0148, 0.0805, 0.9845, 0.9581, 0.9949 Implied by sight of fully correct CR (with no probs) so e.g. $X = 0$, $X > 8$ scores M1A0A0
	1st A1 for at least one correct probability (to at least 3sf) with its probability statement i.e. for $P(X = 0)$ = awrt 0.0148 or $P(X \ldots 9)$ = awrt 0.0155
	2nd A1(**dep on M1 and 1st A1** but not on B1) for both correct probs (to at least 3 sf) and the correct critical region Do not need set notation. Allow $X < 1$ and $X > 8$ or words e.g. "0 or greater than 8" etc Allow "," or "and" or "or" or "$\cap$" between $X \,„\, 0$ and $X \ldots 9$ $P(X = 0)$ and $P(X \ldots 9)$ is 2nd A0
(b)	B1ft for awrt 3.03% or correct sum of their two probabilities (provided each is less than 0.5) Their probabilities must be to at least 2sf and relate to their CR
	To score in (c) they must have a CR of the form ($X = 0$ or $X < a$)and $X > b$, where b is …7 **May be implied by $P(X < a)$ and $P(X > b)$ i.e. 2nd A0 in (a) but correct form. Need $b > a$**
(c)	B1 for a suitable comment in context that suggests no support for Freya's belief/claim **or** e.g. insufficient evidence of change in proportion/percentage of left-handed adults **or** e.g. proportion/percentage of left-handed adults is not different from 10% (**or** …is 10%) **or** e.g. 10% of adults in the country are left-handed Do not allow contradictory comments e.g. "in CR so no support for Freya's belief" is B0
NB	A correct contextual answer in (c) using an **acceptance region** please send to review.

Qu 1	Scheme	Marks	AO
(a)	[0.13 + 0.25 =] **0.38**	B1 **(1)**	1.1b
(b)	Independence implies: e.g. $\left[\mathrm{P}(B\cap C)=\mathrm{P}(B)\times\mathrm{P}(C) \Rightarrow\right]$ $0.3=(0.3+0.05+0.25)\times(0.3+p)$	M1	1.1b
	So p = **0.2**	A1	1.1b
	[Sum of probabilities = 1 gives] q = **0.07**	B1ft **(3)**	1.1b
(c)	$[\mathrm{P}(A\mid B')=]\dfrac{\mathrm{P}(A\cap B')}{\mathrm{P}(B')}$ or $\dfrac{0.13}{(1-0.6)}$ or $\dfrac{0.13}{(0.13+"0.2"+"0.07")}$	M1	1.1b
	$=\dfrac{13}{40}$ or **0.325**	A1 **(2)**	1.1b
		(6 marks)	

Notes

(a) B1 for 0.38 (or exact equivalent)

If answers are given on Venn Diagram and in the script then the script takes precedence.

(b) M1 for a correct equation in p or P(C) only.
May be implied by an answer of $p = 0.2$ **provided** this does not come from incorrect working.
Condone missing brackets if they get 0.2
Other rules for independence will give simple rearrangements of this equation.

Beware If $p = 0.2$ comes from incorrect working, we've seen $p = \frac{0.6}{0.3} = 0.2$, score M0A0

A1 for $p = 0.2$ (or exact equivalent)
B1ft for $q = 0.07$ (or exact equivalent) ft their p i.e. $q = 0.27 -$ "0.2" where 0 „ p „ 0.27

(c) M1 for a correct ratio of probability expressions or a correct ratio of probabilities ft their values of p and q (provided both probabilities) or letters p and q
A1 for 0.325 or exact equivalent. Correct answer only will score 2/2
NB on epen this is labelled M1 but treat it as A1

Qu 2	Scheme	Marks	AO
(a)	Comment in context about either **independence** or **random** packing e.g. "prizes must be placed in packets at random/independently of each other" **or** about **constant probability** e.g. "the probability of a packet containing a prize is constant/ the same/fixed"	B1 **(1)**	3.5b
(b)(i)	[P($T = 6$) =] 0.17273… awrt **0.173**	B1	1.1b
(ii)	[P($T < 3$) = P($T \leq 2$) =] 0.061587… awrt **0.0616**	B1 **(2)**	1.1b
(c)	[K= no. of boxes with fewer than 3 packets containing a prize]		
	$K \sim$ B(5, "0.0616")	M1	1.1b
	P($K = 2$) = 0.031344… in the range **[0.0313~0.0314]**	A1 **(2)**	1.1b
(d)	$H_0 : p = \frac{1}{7}$ $H_1 : p < \frac{1}{7}$	B1	2.5
	[X = no of packets containing a prize] X~B(110, $\frac{1}{7}$)	M1	3.3
	[P($X \leq 9$)] = 0.038292…	A1	3.4
	[Significant result or reject H_0] E.g. there is evidence to support Kamil's claim	A1 **(4)**	2.2b
		(9 marks)	

	Notes
(a)	B1 **May use idea of independent events:** a suitable reason, **in context**, covering idea of random packing or packets filled independently. Should mention key words/ideas of: prizes in packets **or** packets in boxes **May use idea of constant probability**. Must see key words underlined in scheme. Idea of probability with "independence" or "not affected by other packets" is B0 B0 for: **Idea of only 2 cases**. E.g. Packet contains a prize or not or **Idea of a fixed number of trials**. E.g. Need a fixed number of packets in each box
(b)(i)	B1 for awrt 0.173
(ii)	B1 for awrt 0.0616
(c)	M1 for sight of B(5, "0.0616") or ${}^5C_2 ("0.0616")^2 (1-"0.0616")^3$ ft their answer to (b)(ii). A1 for an answer in the range [0.0313 to 0.0314] Use of 0.0616 gives 0.031356..ans only 2/2
(d)	B1 for both hypotheses correct in terms of p or π M1 for selecting an appropriate model, may be implied by 1st A1 or P($X = 9$) = 0.0199(2…) 1st A1 for 0.038 or better or allow 0.04 with sight of P($X \leq 9$)
ALT	**Critical Region.** Allow CR of $X \leq 9$ (or $X < 10$) provided a supporting probability is seen e.g. A1 for correct CR plus P($X \leq 10$) = 0.0718… (accept 2sf or 1sf if prob statement seen) 2nd A1 (dep on 1st A1 but indep of hyp's) for a suitable conclusion in context that suggests support for (Kamil's) claim **or** states that there is evidence that proportion /probability/chance of packets containing a prize is less than $\frac{1}{7}$ Do not award 2nd A1 for contradictory statements e.g. "not significant" so "supports claim"
Normal	Sight of $N\left(\frac{110}{7}, \frac{660}{49}\right)$ or awrt 13.5 or probability of 0.045(20..) or 0.033(66..) scores M1

Qu 5	Scheme	Marks	AO
(a)	$P(S \cap \{X = 50\}) = P(S \cap \{X = 80\})$[= a constant, V] $\Rightarrow b \times \frac{k}{50} = c \times \frac{k}{80}$	M1	3.1a
	May see: $\frac{k}{50} = \frac{V}{b}$ and $\frac{k}{80} = \frac{V}{c}$ (condone any letter for V even S)		
	So $\underline{c = \frac{8}{5}b}$ *	A1cso*	1.1b
		(2)	
(b)	$d = 2b$ **or** $a = \frac{2}{5}b$ **or** $c = 4a$ **or** $d = 5a$ **or** $d = \frac{5}{4}c$	M1 A1	2.1 3.3
	$\frac{2}{5}b + b + \frac{8}{5}b + 2b = 1$	M1	2.1
	$\Rightarrow 5b = 1$ so $b = \frac{1}{5}$ (o.e.)	A1	1.1b
	$\underline{\underline{a = \frac{2}{25} \quad b = \frac{1}{5} \quad c = \frac{8}{25} \quad d = \frac{2}{5}}}$	A1	3.2a
		(5)	
(c)	[Experiment suggests for Nav] $P(S \mid \{X = 100\}) = 0.3 \Rightarrow k = 30$		
	or $0.3 = \frac{V}{0.4} \Rightarrow V = 0.12$	B1	2.4
	So model won't work since		
	$P(S \mid X = 20) = \frac{30}{20}$ or $\frac{0.12}{0.08}$ and so would be greater than 1		
		(1)	
		(8 marks)	

	Notes
(a)	M1 for use of $P(S \mid X = x) \times P(X = x)$ for $x = 50$ and $x = 80$ (Must see k or their V) Any expression or equation MUST be based on the probability statements in qu.
*	A1cso for rearranging to required result, no incorrect work seen, condone poor notation
NB	Use of values e.g. $b = \frac{50}{20 + 50 + 80 + 100}$ to prove (a) is M0A0 but scores 2nd M1A1 in (b)
	Marks for (b) may be awarded for work seen in (a)
(b)	1st M1 for at least one other relationship (either probability the subject) from the list. 1st A1 for a second different relationship (either probability the subject) from the list. or Allow for: $\frac{ak}{20} = \frac{bk}{50} = \frac{ck}{80} = \frac{dk}{100}$ for 1st M1 1st A1 2nd M1 for using or stating sum of prob's = 1 May be implied by one correct probability. 2nd A1 for one correct probability e.g. $b = \frac{1}{5}$ or exact equivalent such as 0.2 3rd A1 for all correct probabilities. Allow exact equivalents e.g. $c = 0.32$ **Sight of correct distribution or list of probs with no obvious incorrect working is 5/5**
(c)	B1 for deducing $k = 30$ **and** giving a suitable example to show model breaks down

Qu 1	Scheme	Marks	AO
(a)	A, C or D, B or D, C	B1 **(1)**	1.2
(b)	[$p = 0.4 - 0.07 - 0.24 =$] **0.09**	B1 **(1)**	1.1b
(c)	A and B independent implies		1.1b
	$\mathrm{P}(A) \times 0.4 = 0.24$ or $(q + 0.16 + 0.24) \times 0.4 = 0.24$	M1	
	so $\mathrm{P}(A) = 0.6$ and $q =$ **0.20**	A1cso **(2)**	1.1b
(d)(i)	$\mathrm{P}(B' \mid C) = 0.64$ gives $\frac{r}{r+p} = 0.64$ or $\frac{r}{r+"0.09"} = 0.64$	M1	3.1a
	$r = 0.64r + 0.64$ "p" so $0.36r = 0.0576$ so $r =$ **0.16**	A1	1.1b
(ii)	Using sum of probabilities = 1 e.g. "0.6" + 0.07 + "0.25" + s =1	M1	1.1b
	so $s =$ **0.08**	A1 **(4)**	1.1b
		(8 marks)	

	Notes
(a)	B1 for one correct pair. If more than one pair they must all be correct. Condone in a correct probability statement such as $\mathrm{P}(A \cap C) = 0$ or correct use of set notation e.g. $A \cap C = \varnothing$ BUT e.g. "P(A) and P(C) are mutually exclusive" alone is B0
(b)	B1 for $p = 0.09$ (Maybe stated in Venn Diagram [VD]) [If values in VD and text conflict, take text or a value used in a later part]
(c)	M1 for a correct equation in one variable for P(A) or q using independence or for seeing **both** $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$ and $0.24 = 0.6 \times 0.4$ A1cso for $q = 0.20$ or exact equivalent (dep on correct use of independence)
Beware	Use of P(A) = 1 – P(B) = 0.6 leading to $q = 0.2$ scores M0A0
(d)(i)	1st M1 for use of $\mathrm{P}(B' \mid C) = 0.64$ leading to a correct equation in r and possibly p. Can ft their p provided $0 < p < 1$ 1st A1 for $r = 0.16$ or exact equivalent
(ii)	2nd M1 for use of total probability = 1 to form a linear equation in s. Allow p, q, r etc Can follow through their values provided each of p, q, r are in [0, 1) 2nd A1 for $s = 0.08$ or exact equivalent

Qu 4	Scheme	Marks	AO
(a)	$\frac{k}{10}+\frac{k}{20}+\frac{k}{30}+\frac{k}{40}+\frac{k}{50}=1$ or $\frac{1}{600}\left(60k+30k+20k+15k+12k\right)=1$	M1	1.1b
	So $k=\frac{600}{137}$ (*)	A1cso	1.1b
		(2)	
(b)	(Cases are:) $D_1=30, D_2=50$ and $D_1=50, D_2=30$ and $D_1=40, D_2=40$	M1	2.1
	$\mathrm{P}\left(D_1+D_2=80\right)=\frac{k}{50}\times\frac{k}{30}\times 2 + \left(\frac{k}{40}\right)^2$	M1	3.4
	= 0.0375619… awrt **0.0376**	A1	1.1b
		(3)	
(c)	Angles are: $a,\quad a+d,\quad a+2d,\quad a+3d$	M1	3.1a
	$S_4 = a + (a + d) + (a + 2d) + (a + 3d) = 360$	M1	2.1
	$2a + 3d = 180$ (o.e.)	A1	2.2a
	Smallest angle is $a > 50$ consider cases:	M1	3.1b
	$d = 10$ so $a = 75$ or $d = 20$ so $a = 60$ [$d = 30$ gives $a = 45$ no good]		
	$\mathrm{P}(D = 10 \text{ or } 20) = \frac{3k}{20} = \frac{90}{137}$	A1	1.1b
		(5)	
		(10 marks)	

Notes

(a)

M1 for clear use of sum of probabilities = 1 (all terms seen)

A1 cso (*) M1 scored and no incorrect working seen.

Verify **(Assume $k = \frac{600}{137}$)** to score the final A1 they must have a final comment "$\therefore k = \frac{600}{137}$"

(b)

1st M1 for selecting at least 2 of the relevant cases (may be implied by their correct probs)
e.g. allow 30, 50 and 50,30 i.e. D_1 and D_2 labels not required

2nd M1 for using the model to obtain a correct expression for two different probabilities.
May use letter k or their value for k.

Allow for $\frac{k}{50}\times\frac{k}{30} + \left(\frac{k}{40}\right)^2$ or $2\times\left(\frac{k}{50}\times\frac{k}{30} + \left(\frac{k}{40}\right)^2\right)$

A1 for awrt 0.0376 (exact fraction is $\frac{705}{18769}$)

(c)

1st M1 for recognising the 4 angles and finding expressions in terms of d and their a

2nd M1 for using property of quad with these 4 angles (equation can be un-simplified)
Allow these two marks for use of a (possible) value of d
e.g. $a + a + 10 + a + 20 + a + 30 = 360$ (If at least 3 cases seen allow A1 for e.g. $4a = 300$)
or allow M1M1 for a set of 4 angles with sum 360 and possible value of d (3 cases for A1)
e.g. (for $d = 20$) 60, 80, 100, 120

1st A1 for $2a + 3d = 180$ condition (o.e.) [Must be in the form $pa + qd = N$]

3rd M1 for examining cases and getting $d = 10$ and $d = 20$ only

2nd A1 for $\frac{90}{137}$ or exact equivalent

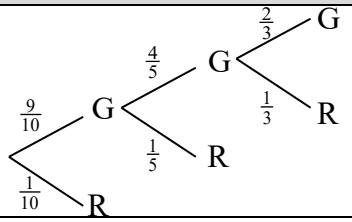
The correct answer and no obviously incorrect working will score 5/5

A final answer of awrt 0.657 (0.65693…) with no obviously incorrect working scores 4/5

Qu 5	Scheme	Marks	AO
(a)	{Let X = time spent, P(X > 15) = } 0.105649… awrt **0.106**	B1 **(1)**	1.1b
(b)	$H_0 : \mu = 10$ $H_1 : \mu > 10$	B1	2.5
	$\bar{X} \sim N\left(10, \left(\frac{4}{\sqrt{20}}\right)^2\right)$; $P(\bar{X} > 11.5)$ = 0.046766… [Condone 0.9532…]	M1;A1	3.3;3.4
	[This is significant (< 5%) so] there is evidence to support the complaint	A1 **(4)**	2.2b
(c)(i)	[P(T < 2) =] 0.1956… awrt **0.196**	B1 **(1)**	1.1b
(ii)	Require $\frac{P(0 < T < 2)}{P(T > 0)} = \frac{0.119119...}{0.923436...}$; = 0.1289955… awrt **0.129**	M1 A1;A1 **(3)**	3.4 1.1bx2
(iii)	The current model suggests **non-negligible** probability of T values < 0 which is impossible	B1 **(1)**	3.5b
(d)	Require t such that P($T > t \mid T > 2$) = 0.5 or P($T < t \mid T > 2$) = 0.5	M1	3.1b
	e.g. $\frac{P(T > t)}{P(T > 2)} = 0.5$; so P($T > t$) = 0.5×[1 – (c)(i)] or P($T > t$) = 0.5×0.8043..	M1; A1ft	1.1b 3.4
	[i.e. P($T > t$) = 0.40… implies] $\frac{t-5}{3.5} = 0.2533$ or P($T < t$) = "0.5978.."	M1	1.1b
	t = 5.886… or from calculator 5.867… so awrt **5.9**	A1 **(5)**	1.1b
		(15 marks)	

	Notes
(a)	B1 for awrt 0.106 (from calculator) [Allow 10.6%]
(b)	B1 for both hypotheses correct in terms of μ. M1 for selection of a correct model (sight or use of correct normal- may not have label $\bar{X}$) 1st A1 for use of this model to get probability allow 0.046~0.047 [Condone awrt 0.953]
ALT	**OR** test statistic z = 1.677… (awrt 1.68) and cv of 1.64 (or better) **or** CR $\bar{X}$ > 11.47.. 2nd A1 (dep on 1st A1 or at least $P(\bar{X} > 11.5) < 0.05$ (o.e.))
SC	for a correct conclusion in context -must mention **complaint**/claim or **time**/mins is > 10 **(M0 for $\bar{X}$ ~N(11.5, …)** for correct probability **and** conclusion (score M0A0A1 on epen)
(c)(i)	B1 for awrt 0.196 (from calculator) [Allow 19.6%]
(ii)	M1 for a correct probability ratio expression (may be implied by 1st A1 scored) 1st A1 for a correct ratio of probabilities (both correct or truncated to 2 dp) 2nd A1 for awrt 0.129
(iii)	B1 for a suitable explanation of why model is not suitable based on negative T values Must say that a **significant** proportion of values < 0 (o.e.) e.g. P(T > 0) should be **closer** to 1 or Difference between P(T < 2| T > 0) and P(T < 2) is **too big** (o.e.)
(d)	1st M1 for a correct conditional probability statement to start the problem or 0.5×P(T > 2) 2nd M1 for correct ratio of probability expressions [Must have P($T > t$) or P(2 < T < t)] 1st A1ft for a correct equation for P($T > t$) (o.e.) ft their answer to part (c)[May be in a diagram] 3rd M1 for attempt to find t (standardising and sight of 0.2533) or prepare to use calc (ft) Arriving at P(T < median) = 1 – 0.5×"their 0.8043" will score 1st 4 marks 2nd A1 for awrt 5.9 Sight of awrt 5.9 and at least one M mark scores 5/5 [Answer only send to review]

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1(a)		B1 dB1	1.1b 1.1b
		(2)	
(b)	$\frac{9}{10}\times\frac{4}{5}\times\frac{2}{3}$	M1	1.1b
	$=\frac{12}{25}(=0.48)$	A1	1.1b
		(2)	
(c)	$\frac{9}{10}\times\frac{1}{5}+\frac{9}{10}\times\frac{4}{5}\times\frac{1}{3}$ or $1-\left(\frac{1}{10}+\frac{9}{10}\times\frac{4}{5}\times\frac{2}{3}\right)$	M1	3.1b
	$=\frac{21}{50}$ $(=0.42)$	A1	1.1b
		(2)	
(d)	[P(Red from B|Red selected)] $=\frac{\frac{9}{10}\times\frac{1}{5}}{\frac{1}{10}+\frac{9}{10}\times\frac{1}{5}+\frac{9}{10}\times\frac{4}{5}\times\frac{1}{3}}\left[=\frac{\frac{9}{50}}{\frac{13}{25}}\right]$	M1	3.1b
	$=\frac{9}{26}$	A1	1.1b
		(2)	
			(8 marks)

Notes

	Allow decimals or percentages throughout this question.
(a)	**B1:** for correct shape (3 pairs) and at least **one** label on **at least two** pairs G(reen) and R(ed) allow G and G' **or** R and R' as labels, etc. condone 'extra' pairs if they are labelled with a probability of 0 **dB1:** (dep on previous B1) all correct i.e. for all 6 correct probabilities on the correct branches with at least one label on **each** pair
(b)	**M1:** Multiplication of 3 correct probabilities (allow ft from their tree diagram) **A1:** $\frac{12}{25}$ oe
(c)	**M1:** Either addition of only two correct products (product of two probs + product of three probs) which may ft from their tree diagram or for $1-('\frac{1}{10}'+'(b)')$ **A1:** $\frac{21}{50}$ oe
(d)	**M1:** Correct ratio of probabilities **or** correct ft ratio of probabilities e.g. $\frac{'\frac{9}{10}'\times'\frac{1}{5}'}{1-'(b)'}$ or $\frac{'\frac{9}{10}'\times'\frac{1}{5}'}{'\frac{1}{10}'+'(c)'}$ with num < den **A1:** $\frac{9}{26}$ (allow awrt 0.346)

Question	Scheme	Marks	AOs

Qu 1	Scheme	Marks	AO
(a)	c: 0 1 2 3 4 5 6 7 8 P($C = c$): $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$	B1 B1ft (2)	1.2 1.2
(b)	P($C < 4$) = $\frac{4}{9}$ (accept 0.444 or better)	B1 (1)	3.4
(c)	Probability lower than expected suggests model is <u>not</u> good	B1ft (1)	3.5a
(d)	e.g. Cloud cover will vary from month to month and place to place So e.g. use a non-uniform distribution	B1 (1)	3.5c
		(5 marks)	

Notes

(a) 1st B1 for a correct set of values for c. Allow $\left\{\frac{1}{8}, \frac{2}{8}, \ldots \frac{8}{8}\right\}$

2nd B1ft for correct probs from their values for c, consistent with discrete uniform distrib'n
Maybe as a prob. function. Allow P($X = x$) = $\frac{1}{9}$ for $0 \leqslant x \leqslant 8$ provided x = {0, 1, 2, …, 8} is clearly defined somewhere.

(b) B1 for using correct model to get $\frac{4}{9}$ (o.e.)

SC **Sample space {1, …, 8}** If scored B0B1 in (a) for this allow P($C < 4$) = $\frac{3}{8}$ to score B1 in (b)

(c) B1ft for comment that states that the model proposed is or is not a good one based on their model in part (a) and their probability in (b)
|(b) – 0.315| > 0.05 Allow e.g. "it is not suitable"; "it is not accurate" etc
|(b) – 0.315| ⩽ 0.05 Allow a comment that suggests it <u>is</u> suitable
No prob in (b) Allow a comparison that mentions 50% or 0.5 and rejects the model
No prob in (b) and no 50% or 0.5 or **(b) > 1** scores B0
Ignore any comments about location or weather patterns.

(d) B1 for a sensible refinement considering variations in month or location
Just saying "not uniform" is B0
Context & "non-uniform" Allow mention of different locations, months <u>and</u> non-uniform
<u>or</u> use more locations to form a new distribution with probabilities based on frequencies
Context & "binomial" Allow mention of different locations, months <u>and</u> binomial
Just refined model Model must be outlined and discrete and non-uniform
e.g. higher probabilities for more cloud cover <u>or</u> lower probabilities for less cloud cover
Continuous model Any model that is based on a continuous distribution. e.g. normal is B0

Qu 3	Scheme	Marks	AO
(a)	The <u>probability</u> of a dart hitting the target is <u>constant</u> (from child to child and for each throw by each child) (o.e.)	B1	1.2
	The <u>throws</u> of each of the darts are <u>independent</u> (o.e.)	B1 (2)	1.2
(b)	[P($H \geqslant 4$) = 1 – P($H \leqslant 3$) = 1 – 0.9872 = 0.012795.. =] awrt **<u>0.0128</u>**	B1 (1)	1.1b
(c)	P($F = 5$) = $0.9^4 \times 0.1$, = 0.06561 = awrt **<u>0.0656</u>**	M1, A1 (2)	3.4 1.1b
(d)	n | 1 | 2 | … | 10 P($F = n$) | 0.01 | $0.01 + \alpha$ | … | $0.01+9\alpha$	M1	3.1b
	Sum of probs = 1 $\Rightarrow \frac{10}{2}[2\times 0.01+9\alpha]=1$	M1A1	3.1a 1.1b
	[i.e. $5(0.02 + 9\alpha) = 1$ or $0.1 + 45\alpha = 1$] so α = **<u>0.02</u>**	A1 (4)	1.1b
(e)	P($F = 5$| Thomas' model) = **<u>0.09</u>**	B1ft (1)	3.4
(f)	<u>Peta's</u> model assumes the <u>probability</u> of hitting target is <u>constant</u> (o.e.) **and** <u>Thomas</u>' model assumes this <u>probability increases</u> with each attempt(o.e.)	B1 (1)	3.5a
		(11 marks)	

Notes

(a) 1st B1 for stating that the <u>probability</u> (or possibility or chance) is <u>constant</u> (or fixed or same)
2nd B1 for stating that <u>throws</u> are <u>independent</u> ["trials" are independent is B0]

(b) B1 for awrt 0.0128 (found on calculator)

(c) M1 for a probability expression of the form $(1-p)^4 \times p$ where $0 < p < 1$
A1 for awrt 0.0656
SC Allow M1A0 for answer only of 0.066

(d) 1st M1 for setting up the distribution of F with at least 3 correct values of n and P($F = n$) in terms of α. (Can be implied by 2nd M1 or 1st A1)
2nd M1 for use of sum of probs = 1 **and** clear summation or use of arithmetic series formula (allow 1 error or missing term). (Can be implied by 1st A1)
1st A1 for a correct equation for α
2nd A1 for α = 0.02 (must be exact and come from correct working)

(e) B1ft for value resulting from $0.01 + 4\times$"their α" (provided α and the answer are probs)
Beware If their answer is the same as their (c) (or a rounded version of their (c)) score B0

(f) B1 for a suitable comment about the <u>probability</u> of hitting the target
ALT Allow idea that Peta's model suggests the dart may never hit the target but Thomas' says that it will hit at least once (in the first 10 throws).