adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Proof

Mark scheme

8 questions

Exam questions and marking instructions remain credited to their board.

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Question	Scheme	Marks	AOs
8 (a)	e.g. The last line should start $25k^2+20k+4$	B1	2.3
		(1)	
(b)	Considers one of the missing calculations $m=5k+3$ and attempts $m^2=\left(5k+3\right)^2=...$ **or** $m=5k+4$ and attempts $m^2=\left(5k+4\right)^2=...$	M1	2.1
	Achieves one correct statement $m^2=\left(5k+3\right)^2=25k^2+30k+9=5\left(5k^2+6k+2\right)-1$ **or** $m^2=\left(5k+4\right)^2=25k^2+40k+16=5\left(5k^2+8k+3\right)+1$	A1	1.1b
	Considers **both** of the missing calculations	dM1	1.1b
	Achieves both correct statements with final concluding remark (see notes)	A1	2.1
		(4)	
			(5 marks)

Notes:

(a)

B1: Corrects the error for the case when $m=5k+2$. The correction may be on the proof in the box or may be described in the main body of the text. May just see the $10k$ crossed out and replaced with $20k$ in the box or described in the main body of the work.

Sight of the quadratic $\left(25k^2\right)+20k\left(+4\right)$ scores B1 and isw e.g. if they make other errors in the expansion.

If B1 is not scored in (a) then allow to score if seen correct in (b) e.g. they may attempt the case $m=5k+2$ as part of their proof in (b).

(b)

Main scheme method uses $m=5k+3$ and $m=5k+4$

You will need to look at both cases and mark the one which is fully correct first.

Allow a different variable to k and may be different letters for the two cases.

If a candidate attempts repeated cases e.g. $m=5k+4$ and $m=5k-1$ then mark both and award the higher mark of the two.

Condone use of *m* as a variable for the first three marks.

There should be no errors in the algebra for the A marks including invisible brackets but do not be concerned with any re-attempt at doing the case $m=5k+2$

distinct cases e.g. $m=5k-2$ **and** $m=5k+4$**, or** $m=5k-1$ **and** $m=5k+3$

but NOT e.g. $m=5k+3$ **and** $m=5k-2$

Note that we are not expecting candidates to state what set of numbers *k* belongs to but we will condone pairs such as $m=-5k-1$ **and** $m=-5k-2$

Typically candidates will show the algebraic steps as in the main scheme but for this particular pair they may justify equivalence using the results in the box for $m=5k+1$ **and** $m=5k+2$ without requiring calculations which is acceptable.

M1: Considers one valid case e.g. $m=5k+3$ and attempts $m^2=\left(5k+3\right)^2$ **or** $m=5k+4$ and attempts $m^2=\left(5k+4\right)^2$

Look for expanding out the brackets and simplifying to a 3TQ. Condone slips.

A1: Achieves one correct statement which includes the case, the quadratic multiplied out and written in the required form

e.g. $m^2=\left(5k+3\right)^2=25k^2+30k+9=5\left(5k^2+6k+2\right)-1$ **or**

e.g. $m^2=\left(5k+4\right)^2=25k^2+40k+16=5\left(5k^2+8k+3\right)+1$

dM1: Considers **both cases** for a valid pair (see first M1 for guidance). It is dependent on the previous method mark. Condone slips.

A1: Full proof with correct statements for both cases for a valid pair. Each must include the case, the quadratic multiplied out and written in the form which is not in terms of *m* (we do not need the "where $n=....$" at the end of the statements - you can ignore these)

Requires a minimal overall conclusion eg. Proven, QED, tick

Condone recovery of interchanging of variables.

m	m^2	$5n\pm1$
$5k+3$	$25k^2+30k+9$	$5\left(5k^2+6k+2\right)-1$
$5k+4$	$25k^2+40k+16$	$5\left(5k^2+8k+3\right)+1$
$5k-1$	$25k^2-10k+1$	$5\left(5k^2-2k\right)+1$
$5k-2$	$25k^2-20k+4$	$5\left(5k^2-4k+1\right)-1$

Ignore any additional cases that are not required to complete the proof

(and ignore replications of the ones given in the box in the question)

Question	Scheme	Marks	AOs
15(i)	$k^2-4k+5=(k-2)^2+...$	M1	2.1
	$k^2-4k+5=(k-2)^2+1$ so $k^2-4k+5 \ldots 1$ so k^2-4k+5 is always positive*	A1*	2.4
		(2)	

Notes

(i) Note: Using e.g. x throughout is acceptable for both marks.

M1: Starts the process of showing the given expression is positive.

Completing the square requires $(k-2)^2 \pm ...$

Differentiation requires them to differentiate to a linear expression in k, set = 0 (which may be implied), solve for k and substitute their k into k^2-4k+5 to reach a value. Ignore e.g. $\frac{dy}{dx}$

Discriminant requires a calculation of $b^2-4ac=(\pm 4)^2-4\times 1\times 5$ {$=-4$} and might be seen embedded in the quadratic formula.

Sketches on their own are insufficient without working to find the minimum point algebraically.

Stating the minimum is (2, 1) without any evidence is M0.

A1*: Completes the proof with no errors and correct reasoning.

Accept e.g. "hence proved" in place of "so k^2-4k+5 is always positive" as long as there is sufficient justification for it being "proved".

As a minimum expect to see e.g.:

- $k^2-4k+5=(k-2)^2+1$ which is always positive as $\underline{(k-2)^2 \ldots 0}$ (or as squares are always positive **or zero**). Must be a correct statement. Do not condone e.g. $(k-2)^2>0$
- $k^2-4k+5=(k-2)^2+1$ so (2, 1) is the **minimum** (point) so k^2-4k+5 is always positive.
- $k^2-4k+5=(k-2)^2+1$ so $\underline{k=2\pm\sqrt{-1}}$ hence k^2-4k+5 has **no real roots** and as k^2 **has a positive coefficient** (condone e.g. positive k^2 or positive quadratic or $a>0$), hence proved.
- $2k-4=0 \Rightarrow k=2 \Rightarrow k^2-4k+5=1$ is the **minimum** value so k^2-4k+5 is always positive.
- $b^2-4ac=16-20\underline{<0}$ so $k^2-4k+5=0$ has **no real roots** and as k^2 **has a positive coefficient** (condone e.g. positive k^2 or positive quadratic or $a>0$), hence proved.

Note that stating $k^2-4k+5=(k-2)^2+1>0$ alone is not sufficient for the A1*

Similarly, stating $k^2-4k+5=(k-2)^2+1>0$ because $(k-2)^2>0$ (or $(k-2)^2$ is positive) is incorrect and scores A0.

SC: Attempts at completing the square for $k=2n$ and/or $k=2n+1$ (or $k=2n-1$) can score M1 provided at least one is attempted as far as $(\ldots)^2+\ldots$ e.g. to $(2n-2)^2+...$ or e.g. $4(n-1)^2+...$

Candidates are unlikely to complete the argument to score A1 (it is **not** sufficient to just consider odd/even) but if you think they might deserve the A1 then send to Review.

$k=2n \rightarrow k^2-4k+5=4n^2-8n+5$ leading to $(2n-2)^2+1$ or $4(n-1)^2+1$

$k=2n+1 \rightarrow k^2-4k+5=4n^2-4n+2$ leading to $(2n-1)^2+1$ or $4\left(n-\frac{1}{2}\right)^2+1$

$k=2n-1 \rightarrow k^2-4k+5=4n^2-12n+10$ leading to $(2n-3)^2+1$ or $4\left(n-\frac{3}{2}\right)^2+1$

Question	Scheme	Marks	AOs
15(ii)	Attempts to solve any 1 pair of the relevant 11 sim. equations. e.g. **one of** $\left.\begin{matrix}3x+2y=28\\2x-5y=1\end{matrix}\right\}\Rightarrow x=...,(y=...)$ **or** … (see notes for all 11)	M1	2.1
	Attempts to solve any 2 pairs of the relevant 11 sim. equations with at least one correct and correctly rejected. e.g. **both** $\left.\begin{matrix}3x+2y=28\\2x-5y=1\end{matrix}\right\}\Rightarrow x=\frac{142}{19},\left(y=\frac{53}{19}\right)$ Not integers **and** $\left.\begin{matrix}3x+2y=7\\2x-5y=4\end{matrix}\right\}\Rightarrow x=...,(y=...)$ **or** … (see notes for all 11 options)	dM1 (A1 on EPEN)	2.2a
	Attempts to solve all 5 pairs of the relevant sim. equations with positive RHS **or** e.g. $\left.\begin{matrix}3x+2y=28\\2x-5y=1\end{matrix}\right\}\Rightarrow x=\frac{142}{19},\left(y=\frac{53}{19}\right)$ Not integers **and** $\left.\begin{matrix}3x+2y=7\\2x-5y=4\end{matrix}\right\}\Rightarrow x=...,(y=...)$ **and** all other cases are not possible as $3x + 2y \neq 5$	ddM1	2.1
	Requires: • All cases considered, with correct values and rejected • Correct reasons given in each case e.g. "not integers" • Concluding statement e.g. "hence proven"	A1	2.4
		(4)	
			(6 marks)

Notes

15(ii)

General Note:

Throughout this question we are condoning if candidates do not reference the following two points which are deemed fairly trivial and acceptable to be omitted at A-level:

- As x and y are integers then both $3x + 2y$ and $2x – 5y$ are integers.
- As x and y are positive then $3x + 2y > 0$

As such, we only *require* candidates to prove that there are no positive integer solutions x and y to simultaneous equations that have a positive RHS.

Note: Any attempt to solve the given simultaneous equations does not score any marks on its own.
Any attempts that use substitutions such as $x = 2n$ etc. are unlikely to score any marks.
There are other methods to eliminate some pairs of simultaneous equations e.g. by showing that the only solution to $3x + 2y = 7$ is (1, 2) which is not on $2x – 5y = 4$. Use Review in such cases.

M1: Attempt to solve any one of the other 11 possible cases (labelled *A*-*K*) below to find a value for x or a value for y. The attempt may be implied by a value for x or y which need not be correct.

dM1: Attempts to solve any two of the other 11 possible cases below to find a value for x or a value for y **with** at least one correct **and** correctly rejected. It is not necessary to find both values of x and y unless the correct value found does not cause a contradiction (see cases *D* and *G*).

ddM1: Attempts to solve all 5 pairs (*A*-*E*) of the relevant simultaneous equations with positive RHS **or** attempts to solve cases *A* and *B* and justifies that these are the only cases that need checking using either:

- $3x+2y \ldots 5$ (because x and y are positive) **or**
- $3x+2y > 2x-5y$ (because x and y are positive) [cases *F*, *G* and *H* do not need checking using this approach because of the general note]

Their values for x and y do not need to be correct for this mark as long as the dM1 is scored.

A1: cso Shows that all necessary cases are impossible with correct values, correct reasons, and a minimal conclusion e.g. "hence proved". There is no need to say "contradiction" or restate the objective (you can also ignore any inaccurate attempt to restate the objective).

All 5 pairs of simultaneous may be rejected in one go if the rejection is sufficiently clear.

The mechanics of solving the simultaneous equations does not need to be shown.

A: $\left.\begin{matrix}3x+2y=28\\2x-5y=1\end{matrix}\right\} \Rightarrow x=\frac{142}{19}, \left(y=\frac{53}{19}\right)$ Not integers

B: $\left.\begin{matrix}3x+2y=7\\2x-5y=4\end{matrix}\right\} \Rightarrow x=\frac{43}{19}, \left(y=\frac{2}{19}\right)$ Not integers

C: $\left.\begin{matrix}3x+2y=4\\2x-5y=7\end{matrix}\right\} \Rightarrow x=\frac{34}{19}, \left(y=-\frac{13}{19}\right)$ Not integers/not positive

D: $\left.\begin{matrix}3x+2y=2\\2x-5y=14\end{matrix}\right\} \Rightarrow (x=2),\ y=-2$ Not positive

E: $\left.\begin{matrix}3x+2y=1\\2x-5y=28\end{matrix}\right\} \Rightarrow x=\frac{61}{19}, \left(y=-\frac{82}{19}\right)$ Not integers/not positive

F: $\left.\begin{matrix}3x+2y=-1\\2x-5y=-28\end{matrix}\right\} \Rightarrow x=-\frac{61}{19}, \left(y=\frac{82}{19}\right)$ Not integers/not positive

G: $\left.\begin{matrix}3x+2y=-2\\2x-5y=-14\end{matrix}\right\} \Rightarrow x=-2, (y=2)$ Not positive

H: $\left.\begin{matrix}3x+2y=-4\\2x-5y=-7\end{matrix}\right\} \Rightarrow x=-\frac{34}{19}, \left(y=\frac{13}{19}\right)$ Not integers/not positive

I: $\left.\begin{matrix}3x+2y=-7\\2x-5y=-4\end{matrix}\right\} \Rightarrow x=-\frac{43}{19}, \left(y=-\frac{2}{19}\right)$ Not integers/not positive

J: $\left.\begin{matrix}3x+2y=-14\\2x-5y=-2\end{matrix}\right\} \Rightarrow x=-\frac{74}{19}, \left(y=-\frac{22}{19}\right)$ Not integers/not positive

K: $\left.\begin{matrix}3x+2y=-28\\2x-5y=-1\end{matrix}\right\} \Rightarrow x=-\frac{142}{19}, \left(y=-\frac{53}{19}\right)$ Not integers/not positiv

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14	When n is even: $(2k+1)^3-(2k)^3=8k^3+12k^2+6k+1-8k^3=6k(2k+1)+1$ $\Rightarrow$ which is odd **or** When n is odd: $(2k+2)^3-(2k+1)^3=8(k^3+3k^2+3k+1)-(8k^3+12k^2+6k+1)=6k(2k+3)+7$ $\Rightarrow$ which is odd	M1 A1	3.1a 2.2a
	When n is even: $(2k+1)^3-(2k)^3=8k^3+12k^2+6k+1-8k^3=6k(2k+1)+1$ $\Rightarrow$ which is odd **and** When n is odd: $(2k+2)^3-(2k+1)^3=8(k^3+3k^2+3k+1)-(8k^3+12k^2+6k+1)=6k(2k+3)+7$ $\Rightarrow$ which is odd	dM1	2.1
	Hence odd for all $n(\in\mathbb{N})$ *	A1*	2.4
			(4 marks)

Notes

General guidance

It is likely that you will see a mixture of methods and approaches within some solutions.
Mark the approach which scores the highest number of marks.

There should be no errors in the algebra but allow e.g. invisible brackets to be "recovered".
Withhold the final mark if n is used instead of k or reference to $n\in\mathbb{R}$ but $n\in\mathbb{Z}^+$ is acceptable.

Main scheme algebraic method using e.g. $n=2k$ **and** $n=2k\pm1$
You will need to look at both cases and mark the one which is fully correct first.
Allow a different variable to k and may be different letters for odd and even

M1: For the key step attempting to find $(n+1)^3-n^3$ when $n=2k$ **or** $n=2k\pm1$ and attempting to multiply out and simplify to achieve a three term quadratic (allow equivalent representation of odd or even e.g. $n=2k+2$ **or** $2n\pm5$)
Condone arithmetical slips and condone the use of e.g. $n=2n$ and $n=2n\pm1$

A1: Complete argument for $n=2k$ **or** $n=2k+1$ (or e.g. $n=2k-1$) showing the result is odd.
Requires:

- Correct simplified quadratic expression e.g. $12k^2+6k+1$ (when $n=2k$), $12k^2+18k+7$ (when $n=2k+1$), $12k^2-6k+1$ (when $n=2k-1$) (may be factorised)
- A reason why the expression is odd e.g. $2k(6k+3)+1$ or may use a divisibility argument e.g. $\frac{12k^2+6k+1}{2}=6k^2+3k+\frac{1}{2}$
- Concludes "odd" o.e. (may be within their final conclusion)
  There should be no errors in the algebra but allow e.g. invisible brackets if they are "recovered"
  Condone the use of e.g. $n=2n$ and $n=2n\pm1$

dM1: Attempts to find $(n+1)^3-n^3$ when $n=2k$ **and** $n=2k\pm1$ and attempts to multiply out and simplify to achieve a three term quadratic (allow equivalent representation of odd or even e.g. $n=2k+2$, $2n\pm5$
Condone arithmetical slips and condone the use of e.g. $n=2n$ and $n=2n\pm1$

A1*: Complete argument for **both** $n=2k$ **and** $n=2k+1$ (or e.g. $n=2k-1$) showing the result is odd for all $n(\in\mathbb{N})$

Requires for both cases:

- Correct simplified expressions for both odd and even (which may be factorised)
- A reason why both of the expressions are odd
- Minimal conclusion (may be within their final conclusion)

An overall conclusion is also required. "Hence odd for all $n(\in\mathbb{N})$" Accept "hence proven", "statement proved", "QED"

The conclusion for when $n=2k$ and $n=2k+1$ may be within the final conclusion rather than separate which is acceptable e.g. "when $n=2k$ and when $n=2k+1$ the expression is odd, hence proven" (following correct simplified expressions and reasons)

	$(n+1)^3$	n^3	$(n+1)^3-n^3$
$n=2k-1$	$8k^3$	$8k^3-12k^2+6k-1$	$12k^2-6k+1$
$n=2k$	$8k^3+12k^2+6k+1$	$8k^3$	$12k^2+6k+1$
$n=2k+1$	$8k^3+24k^2+24k+8$	$8k^3+12k^2+6k+1$	$12k^2+18k+7$

Alternative methods:

Algebraic with logic example

M1: Attempts to multiply out the brackets and simplifies to achieve a three term quadratic. Condone arithmetical slips.

A1: Correct quadratic expression $3n^2+3n+1$

dM1: Attempts to factorise their quadratic such that $n^2+n\rightarrow n(n+1)$ within their expression e.g. $3n(n+1)+1$

A1*: Explains that e.g. $n(n+1)$ is always even as it is the product of two consecutive numbers so $3n(n+1)$ is odd $\times$ even = even so $3n(n+1)+1$ is odd hence odd for all $n(\in\mathbb{N})$

Proof by contradiction example

M1: Attempts to multiply out the brackets and simplifies to achieve a three term quadratic.

A1: Correct quadratic expression $3n^2+3n+1$

dM1: Sets $3n^2+3n+1=2k$ (for some integer k) $\Rightarrow 3n(n+1)=2k-1$

A1*: Explains that $n(n+1)$ is always even as it is the product of two consecutive numbers so $3n(n+1)$ is odd $\times$ even = even but $2k-1$ is odd hence we have a contradiction so $(n+1)^3-n^3$ is odd (for all $n(\in\mathbb{N})$. There must have been a correct opening statement setting up the contradiction e.g. "assume that there exists a value for n for which $(n+1)^3-n^3$ is even"

Solutions via just logic (no algebraic manipulation)

e.g.

If *n* is odd, then $(n+1)^3 - n^3$ is $\text{even}^3 - \text{odd}^3 = \text{even} - \text{odd} = \text{odd}$

If *n* is even, then $(n+1)^3 - n^3$ is $\text{odd}^3 - \text{even}^3 = \text{odd} - \text{even} = \text{odd}$

Both cases must be considered to score any marks and scores SC 1010 if fully correct

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Further Maths method (proof by induction

M1: Assumes true for $n = k$, substitutes $n = k+1$ into $(n+1)^3 - n^3$, multiplies out the brackets and attempts to simplify to a three term quadratic e.g. $3k^2 + 9k + 7$ Condone arithmetical slips

A1: $\left(\text{f}(k+1) = 3k^2 + 3k + 1 + 6(k+1) =\right)\ (k+1)^3 - k^3 + 6(k+1) = \text{f}(k) + 6(k+1)$
which is odd + even = odd

dM1: Attempts to substitute $n = 1 \Rightarrow\ (1+1)^3 - 1^3 = 7$ (which is true) (Condone arithmetical slips evaluating)

A1*: Explains that

- it is true when $n = 1$
- if it is true for $n = k$ then it is true for $n = k+1$
- therefore it is true for all $n\left(\in \mathbb{N}\right)$

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
7 (i)	For setting up the contradiction: There exists integers p and q such that pq is even and both p and q are odd	B1	2.5
	For example, sets $p = 2m+1$ and $q = 2n+1$ and then attempts $pq = (2m+1)(2n+1) = ...$	M1	1.1b
	Obtains $pq = (2m+1)(2n+1) = 4mn + 2m + 2n + 1$ $= 2(2mn + m + n) + 1$ States that this is odd, giving a contradiction so " if pq is even, then at least one of p and q is even" *	A1*	2.1
		(3)	
(ii)			
	$(x+y)^2 < 9x^2 + y^2 \Rightarrow 2xy < 8x^2$	M1	2.2a
	States that as $x < 0 \Rightarrow 2y > 8x$ $\Rightarrow y > 4x$ *	A1*	2.1
		(2)	
			(5 marks)
Notes:			

(i)

B1: For using the "correct"/ allowable language in setting up the contradiction.

Expect to see a minimum of

- "assume" or "let" or "there is " or other similar words
- "pq is even" and "p and q are (both) odd"

M1: Uses a correct algebraic form for p and q and attempting to multiply.

Allow any correct form so $p = 2n+1$ and $q = 2m+3$ would be fine to use

Different variables must be used for p and q, so $p = 2n+1$ and $q = 2n-1$ would be M0

A1*: Full argument .

This requires (1) a correct calculation for their pq

(2) a correct reason and conclusion that it is odd

E.g. $(2m+1)(2n+1) = 4mn+2m+2n+1 = 2(2mn+m+n)+1 = \text{odd}$

E.g. $(2m-1)(2n+1) = 4mn+2m-2n-1 = \text{even} + \text{even} - \text{even} - 1 = \text{odd}$

and (3) a minimal statement implying that they have proven what was required which could be QED, proven etc

Note that B0 M1 A1 is possible

(ii)

M1: For multiplying out and cancelling terms before proceeding to a correct intermediate line such as $2xy < 8x^2$ o.e. such as $2x(4x-y) > 0$

A1*: Full and rigorous proof with reason shown as to why inequality reverses. The point at which it reverses must be correct and a correct reason given

See scheme

Alt: $2xy < 8x^2 \Rightarrow xy - 4x^2 < 0 \Rightarrow x(y-4x) < 0$

as $x < 0$, $(y-4x) > 0 \Rightarrow y > 4x$ scores M1 A1

So, the following should be scored M1 A0 as line 3 is incorrect

$2xy - 8x^2 < 0$

$\Rightarrow 2xy < 8x^2$

$\Rightarrow y < 4x$

$\Rightarrow y > 4x$ as $x < 0$

There should be no incorrect lines in their proof

Question	Scheme	Marks	AOs

11	$n\left(n^2+5\right)$		
	Attempts even **or** odd numbers Sets $n=2k$ **or** $n=2k\pm1$ oe and attempts $n\left(n^2+5\right)$	M1	3.1a
	Achieves $2k\left(4k^2+5\right)$ (for $n=2k$) and states "even" **Or** achieves $(2k+1)\left(4k^2+4k+6\right)=2(2k+1)\left(2k^2+2k+3\right)$ (for $n=2k+1$) and states "even" Or e.g. achieves $(2k-1)\left(4k^2-4k+6\right)=2(2k-1)\left(2k^2-2k+3\right)$ (for $n=2k-1$) and states "even"	A1	2.2a
	Attempts even **and** odd numbers Sets $n=2k$ **and** $n=2k\pm1$ oe and attempts $n\left(n^2+5\right)$	dM1	2.1
	Achieves $2k\left(4k^2+5\right)$ (for $n=2k$) and states "even" **and** achieves $(2k\pm1)\left(4k^2\pm4k+6\right)=2(2k\pm1)\left(2k^2\pm2k+3\right)$ (for $n=2k\pm1$) and states "even" Correct work and states even for both WITH a final conclusion showing that true for all $n(\in\mathbb{N})$ or e.g. true for all even and odd numbers.	A1	2.4
		(4)	
			(4 marks)

Notes:

M1: For the key step attempting to find $n\left(n^2+5\right)$ when $n=2k$ **or** $n=2k\pm1$ or equivalent representation of odd or even e.g. $n=2k+2$ **or** $n=2k\pm7$

Condone the use of e.g. $n=2n$ and $n=2n\pm1$

A1: Achieves $2k\left(4k^2+5\right)$ or e.g. $2\left(4k^3+5k\right)$ and deduces that this is even at the appropriate time.

Or achieves $(2k\pm1)\left(4k^2\pm4k+6\right)=2(2k\pm1)\left(2k^2\pm2k+3\right)$ oe e.g. $2\left(4k^3+6k^2+8k+3\right)$ and deduces that this is even.

Note that if the bracket is expanded to e.g. $8k^3+12k^2+16k+6$ then stating "even" is insufficient – they would need to say e.g. even + even + even + even = even or equivalent

Note it is also acceptable to use a divisibility argument e.g. $\frac{8k^3+10k}{2}=4k^3+5k$ so $8k^3+10k$ must be even.

There should be no errors in the algebra but allow e.g. invisible brackets if they are "recovered".

dM1: Attempts $n\left(n^2+5\right)$ when $n=2k$ **and** $n=2k\pm1$ or equivalent representation of odd or even e.g. $n=2k+2$ **and** $n=2k\pm7$

A1: Correct work and states even for both WITH a final conclusion e.g. so true for all $n(\in\mathbb{N})$.

There should be no errors in the algebra but allow e.g. invisible brackets if they are "recovered".

.....

E.g.

If n is odd, then $n\left(n^2+5\right)$ is odd×(odd + odd) = odd × even = even

If n is even, then $n\left(n^2+5\right)$ is even×(even + odd) = even × odd = even

Both cases must be considered to score any marks and scores SC 1010 if fully correct

OR

E.g. $n\left(n^2+5\right)=n^3+5n$

If n is odd, then n^3 is odd and $5n$ is odd, so n^3+5n is odd + odd = even

If n is even, then n^3 is even and $5n$ is even, so n^3+5n is even + even = even

Both cases must be considered to score any marks and scores SC 1010 if fully correct

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A solution via contradiction.

M1 A1: There exists a number n such that $n\left(n^2+5\right)$ is odd, and so deduces that both n and n^2+5 are odd. Note that M1A0 is not possible via this method.

dM1: Sets $n^2+5=2k+1$ (for some integer k) $\Rightarrow n^2=2k-4=2\left(k-2\right)$ which is even
Must use algebra here for this approach and not a "logic" argument.

A1: States that "this is a contradiction as if n^2 is even, then n is even" and then concludes so "$n\left(n^2+5\right)$ is even for all n. "

Attempts at proof by induction should be sent to review

15(i)	$n=1,\ 2^3=8,\ 3^1=3,\ (8>3)$ $n=2,\ 3^3=27,\ 3^2=9,\ (27>9)$ $n=3,\ 4^3=64,\ 3^3=27,\ (64>27)$ $n=4,\ 5^3=125,\ 3^4=81,\ (125>81)$	M1	2.1
	So if $n \leqslant 4, n \in \mathbb{N}$ then $(n+1)^3 > 3^n$	A1	2.4
		(2)	
(ii)	Begins the proof by negating the statement. "Let m be odd " or "Assume m is not even"	M1	2.4
	Set $m=(2p\pm1)$ and attempt $m^3+5=(2p\pm1)^3+5=...$	M1	2.1
	$=8p^3+12p^2+6p+6$ AND deduces even	A1	2.2a
	Completes proof which requires reason and conclusion • reason for $8p^3+12p^2+6p+6$ being even • acceptable statement such as "this is a contradiction so if m^3+5 is odd then m must be even"	A1	2.4
		(4)	
			(6 marks)

Notes

(i)

M1: A full and rigorous argument that uses all of $n = 1, 2, 3$ and 4 in an attempt to prove the given result. Award for attempts at both $(n+1)^3$ and 3^n for **ALL** values with at least 5 of the 8 values correct.

There is no requirement to compare their sizes, for example state that $27 > 9$

Extra values, say $n = 0$, may be ignored

A1: Completes the proof with no errors and an appropriate/allowable conclusion.

This requires

- all the values for $n = 1, 2, 3$ and 4 correct. Ignore other values
- all pairs compared correctly
- a minimal conclusion. Accept ✓ or hence proven for example

(ii)

M1: Begins the proof by negating the statement. See scheme

This cannot be scored if the candidate attempts m both odd and even

M1: For the key step in setting $m = 2p \pm 1$ and attempting to expand $(2p \pm 1)^3 + 5$

Award for a 4 term cubic expression.

A1: Correctly reaches $(2p+1)^3 + 5 = 8p^3 + 12p^2 + 6p + 6$ and **states** even.

Alternatively reaches $(2p-1)^3 + 5 = 8p^3 - 12p^2 + 6p + 4$ and **states** even.

A1: A full and complete argument that completes the contradiction proof. See scheme.

(1) **A reason** why the expression $8p^3 + 12p^2 + 6p + 6$ or $8p^3 - 12p^2 + 6p + 4$ is even

Acceptable reasons are

- all terms are even
- sight of a factorised expression E.g. $8p^3 - 12p^2 + 6p + 4 = 2(4p^3 - 6p^2 + 3p + 2)$

(2) Acceptable concluding statement

Acceptable concluding statements are

- "this is a contradiction, so if $m^3 + 5$ is odd then m is even"
- "this is contradiction, so proven."
- "So if $m^3 + 5$ is odd them m is even"

S.C If the candidate misinterprets the demand and does not use proof by contradiction but states a counter example to the statement "if $m^3 + 5$ is odd then m must be even" such as when $m = \sqrt[3]{2}$ then they can score special case mark B1

Question	Scheme	Marks	AOs
16	Sets up the contradiction and factorises: There are positive integers p and q such that $(2p+q)(2p-q)=25$	M1	2.1
	If true then $\begin{matrix}2p+q=25\\2p-q=1\end{matrix}$ or $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ **Award for deducing either of the above statements**	M1	2.2a
	Solutions are $p=6.5, q=12$ or $p=2.5, q=0$ Award for one of these	A1	1.1b
	This is a contradiction as there are no integer solutions hence there are no positive integers p and q such that $4p^2-q^2=25$	A1	2.1
		(4)	
			(4 marks)
Notes:			

M1: For the key step in setting up the contradiction and factorising

M1: For deducing that for p and q to be integers then either $\begin{matrix}2p+q=25\\2p-q=1\end{matrix}$ or $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ must be true.

Award for deducing either of the above statements.

You can ignore any reference to $\begin{matrix}2p+q=1\\2p-q=25\end{matrix}$ as this could not occur for positive p and q.

A1: For correctly solving one of the given statements,

For $\begin{matrix}2p+q=25\\2p-q=1\end{matrix}$ candidates only really need to proceed as far as $p=6.5$ to show the contradiction.

For $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ candidates only really need to find either p or q to show the contradiction.

Alt for $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ candidates could state that $2p+q\neq 2p-q$ if p,q are positive integers.

A1: For a complete and rigorous argument with both possibilities and a correct conclusion.

Question	Scheme	Marks	AOs
16 Alt 1	Sets up the contradiction, attempts to make q^2 or $4p^2$ the subject and states that either $4p^2$ is even(*) , or that q^2 (or q) is odd (**) Either There are positive integers p and q such that $4p^2 - q^2 = 25 \Rightarrow q^2 = 4p^2 - 25$ with * or ** Or There are positive integers p and q such that $4p^2 - q^2 = 25 \Rightarrow 4p^2 = q^2 + 25$ with * or **	M1	2.1
	Sets $q = 2n \pm 1$ and expands $(2n \pm 1)^2 = 4p^2 - 25$	M1	2.2a
	Proceeds to an expression such as $4p^2 = 4n^2 + 4n + 26 = 4\left(n^2 + n + 6\right) + 2$ $4p^2 = 4n^2 + 4n + 26 = 4\left(n^2 + n\right) + \frac{13}{2}$ $p^2 = n^2 + n + \frac{13}{2}$	A1	1.1b
	States This is a contradiction as $4p^2$ must be a multiple of 4 Or p^2 must be an integer And concludes there are no positive integers p and q such that $4p^2 - q^2 = 25$	A1	2.1
		(4)	

Alt 2

An approach using odd and even numbers is unlikely to score marks.

To make this consistent with the Alt method, score

M1: Set up the contradiction and start to consider one of the cases below where q is odd, $m \neq n$.

Solutions using the same variable will score no marks.

M1: Set up the contradiction and start to consider BOTH cases below where q is odd, $m \neq n$.

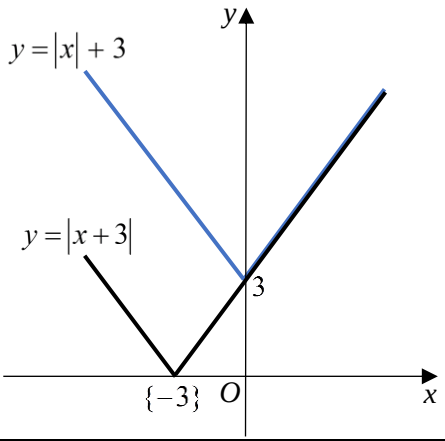
No requirement for evens

A1: Correct work and deduction for one of the two scenarios where q is odd

A1: Correct work and deductions for both scenarios where q is odd with a final conclusion

Options	Example of Calculation	Deduction
p (even) q (odd)	$4p^2 - q^2 = 4 \times (2m)^2 - (2n+1)^2 = 16m^2 - 4n^2 - 4n - 1$	One less than a multiple of 4 so cannot equal 25
p (odd) q (odd)	$4p^2 - q^2 = 4 \times (2m+1)^2 - (2n+1)^2 = 16m^2 + 16m - 4n^2 - 4n + 3$	Three more than a multiple of 4 so cannot equal 25

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3	Statement: "If m and n are irrational numbers, where $m \neq n$, then mn is also irrational."		
(a)	E.g. $m = \sqrt{3}, n = \sqrt{12}$	M1	1.1b
	$\{mn =\}\ \ (\sqrt{3})(\sqrt{12}) = 6$ $\Rightarrow$ statement untrue **or** 6 is not irrational or 6 is rational	A1	2.4
		(2)	
(b)(i), (ii) Way 1	 V shaped graph {reasonably} symmetrical about the y-axis with vertical interpret (0, 3) or 3 stated or marked on the positive y-axis	B1	1.1b
	Superimposes the graph of $y = \|x+3\|$ on top of the graph of $y = \|x\| + 3$	M1	3.1a
	the graph of $y = \|x\| + 3$ is either the same or above the graph of $y = \|x+3\|$ {for corresponding values of x} **or** when $x \geq 0$, both graphs are equal (or the same) when $x < 0$, the graph of $y = \|x\| + 3$ is above the graph of $y = \|x+3\|$	A1	2.4
		(3)	
(b)(ii) Way 2	**Reason 1** When $x \geq 0$, $\|x\|+3 = \|x+3\|$ **Reason 2** When $x < 0$, $\|x\|+3 > \|x+3\|$	Any one of Reason 1 or Reason 2 — M1 3.1a Both Reason 1 and Reason 2 — A1 2.4	
			(5 marks)

Notes for Question 3

(a)	
M1:	States or uses any pair of ***different*** numbers that will disprove the statement. E.g. $\sqrt{3}, \sqrt{12}$; $\sqrt{2}, \sqrt{8}$; $\sqrt{5}, -\sqrt{5}$; $\frac{1}{\pi}, 2\pi$; $3\text{e}, \frac{4}{5\text{e}}$;
A1:	Uses correct reasoning to disprove the given statement, with a correct conclusion
Note:	Writing $(3\text{e})\left(\frac{4}{5\text{e}}\right) = \frac{12}{5} \Rightarrow$ untrue is sufficient for M1A1
(b)(i)	
B1:	See scheme
(b)(ii)	
M1:	For constructing a method of comparing $\|x\|+3$ with $\|x+3\|$. See scheme.
A1:	Explains fully why $\|x\|+3 \geq \|x+3\|$. See scheme.
Note:	Do not allow either $x > 0$, $\|x\|+3 \geq \|x+3\|$ or $x \geq 0$, $\|x\|+3 \geq \|x+3\|$ as a valid reason
Note	$x = 0$ (or where necessary $x = -3$) need to be considered in their solutions for A1
Note:	Do not allow an incorrect statement such as $x \leq 0$, $\|x\|+3 > \|x+3\|$ for A1

Notes for Question 3 Continued	
(b)(ii)	
Note:	Allow M1A1 for $x>0,\ \lvert x\rvert+3=\lvert x+3\rvert$ and for $x\leq 0,\ \lvert x\rvert+3\geqslant\lvert x+3\rvert$
Note:	Allow M1 for any of • x is positive, $\lvert x\rvert+3=\lvert x+3\rvert$ • x is negative, $\lvert x\rvert+3>\lvert x+3\rvert$ • $x>0,\ \lvert x\rvert+3=\lvert x+3\rvert$ • $x\leq 0,\ \lvert x\rvert+3\geq\lvert x+3\rvert$ • $x>0,\ \lvert x\rvert+3$ and $\lvert x+3\rvert$ are equal • $x\geq 0,\ \lvert x\rvert+3$ and $\lvert x+3\rvert$ are equal • when $x\geq 0,$ both graphs are equal • for positive values $\lvert x\rvert+3$ and $\lvert x+3\rvert$ are the same Condone for M1 • $x\leq 0,\ \lvert x\rvert+3>\lvert x+3\rvert$ • $x<0,\ \lvert x\rvert+3\geq\lvert x+3\rvert$

(b)(ii) Way 3	• For $x>0,\ \lvert x\rvert+3=\lvert x+3\rvert$ • For $-3<x<0,$ as $\lvert x\rvert+3>3$ and $\{0<\}\ \lvert x+3\rvert<3,$ then $\lvert x\rvert+3>\lvert x+3\rvert$ • For $x\leq -3,$ as $\lvert x\rvert+3=-x+3$ and $\lvert x+3\rvert=-x-3,$ then $\lvert x\rvert+3>\lvert x+3\rvert$	M1 A1	3.1a 2.4

Question	Scheme	Marks	AOs