adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Proof

Question paper

8 questions

Exam questions and marking instructions remain credited to their board.

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1 June 2025 - Paper 1 - Q8

2 June 2024 - Paper 1 - Q15

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7 October 2020 - Paper 1 - Q16

8 June 2018 - Paper 2 - Q3

8. A student was asked to prove that the square of any number can be expressed in the form $5n$ or $5n \pm 1$ where $n \in \mathbb{N}$

The start of the student's proof is shown in the box below.

Let $m = 5k$

Consider $m^2 = (5k)^2 = 25k^2 = 5 \times 5k^2 = 5n$ where $n = 5k^2$ ✓

Let $m = 5k + 1$

Consider $m^2 = (5k + 1)^2$

$= 25k^2 + 10k + 1 = 5(5k^2 + 2k) + 1 = 5n + 1$ where $n = 5k^2 + 2k$ ✓

Let $m = 5k + 2$

Consider $m^2 = (5k + 2)^2$

$= 25k^2 + 10k + 4 = 5(5k^2 + 2k + 1) - 1 = 5n - 1$ where $n = 5k^2 + 2k + 1$ ✓

(a) Identify and correct an algebraic error in the box above.

(1)

(b) Show the calculations and statements that are required to complete the proof.

(4)

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Question 8 continued

(Total for Question 8 is 5 marks)

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15. (i) Show that $k^2 - 4k + 5$ is positive for all real values of k.

(2)

(ii) A student was asked to prove by contradiction that

"There are no positive integers x and y such that $(3x + 2y)(2x - 5y) = 28$"

The start of the student's proof is shown below.

Assume that positive integers x and y exist such that
$(3x + 2y)(2x - 5y) = 28$

If $3x + 2y = 14$ and $2x - 5y = 2$

$$\left.\begin{aligned} 3x + 2y &= 14 \\ 2x - 5y &= 2 \end{aligned}\right\} \Rightarrow x = \frac{74}{19},\ y = \frac{22}{19} \quad \text{Not integers}$$

Show the calculations and statements needed to complete the proof.

(4)

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Question 15 continued

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Question 15 continued

(Total for Question 15 is 6 marks)

TOTAL FOR PAPER IS 100 MARKS

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14. Prove, using algebra, that

$$(n + 1)^3 - n^3$$

is odd for all $n \in \mathbb{N}$

(4)

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Question 14 continued

(Total for Question 14 is 4 marks)

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7. (i) Given that p and q are integers such that

$$pq \text{ is even}$$

use algebra to prove by contradiction that at least one of p or q is even.

(3)

(ii) Given that x and y are integers such that

- $x < 0$
- $(x + y)^2 < 9x^2 + y^2$

show that $y > 4x$

(2)

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Question 7 continued

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11. Prove, using algebra, that

$$n(n^2 + 5)$$

is even for all $n \in \mathbb{N}$.

(4)

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Question 11 continued

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15. (i) Use proof by exhaustion to show that for $n \in \mathbb{N}$, $n \leqslant 4$

$$(n + 1)^3 > 3^n$$

(2)

(ii) Given that $m^3 + 5$ is odd, use proof by contradiction to show, using algebra, that m is even.

(4)

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Question 15 continued

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Question 15 continued

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Question 15 continued

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Question 15 continued

(Total for Question 15 is 6 marks)

TOTAL FOR PAPER IS 100 MARKS

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16. Prove by contradiction that there are no positive integers p and q such that

$$4p^2 - q^2 = 25$$

(4)

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Question 16 continued

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Question 16 continued

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3. (a) "If m and n are irrational numbers, where $m \neq n$, then mn is also irrational."

Disprove this statement by means of a counter example.

(2)

(b) (i) Sketch the graph of $y = |x| + 3$

(ii) Explain why $|x| + 3 \geqslant |x + 3|$ for all real values of x.

(3)

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Question 3 continued

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