adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Quadratics

Mark scheme

4 questions

Exam questions and marking instructions remain credited to their board.

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1 June 2025 - Paper 1 - Q4

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3 October 2021 - Paper 1 - Q12

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Question	Scheme	Marks	AOs
4 (a)	States or uses $a = 4$	B1	1.1a
	Valid method to find $Q(x)$	M1	2.1
	$(x+4)(2x^2-5x+4)$	A1	1.1b
		(3)	
(b)	Attempts to show that their $2x^2-5x+4$ does not have any (real) roots	M1	3.1a
	Correct calculations, reason and conclusion	A1	2.1
		(2)	
			(5 marks)

Notes:

(a)

B1: States or uses $a = 4$ e.g. may be seen in their attempt at dividing algebraically by $x+4$

M1: Attempts to divide f (*x*) by $(x+4)$ to find a three-term quadratic $Q(x)$. There are various methods or ways to present their solution so typically methods

- by inspection look for $2x^3+3x^2-16x+16=(x+4)(2x^2+...x\pm 4)$
- by division look for a quadratic quotient of $2x^2-5x\pm...$

A1: $(x+4)(2x^2-5x+4)$ condone the missing trailing bracket i.e. $(x+4)(2x^2-5x+4$ isw if they attempt to factorise their quadratic factor.

Allow to be scored if seen in (b).

(b)

M1: Attempts to show that their three-term quadratic "$2x^2-5x+4$" does not have any roots:

- **Attempts the discriminant**
  e.g. $b^2-4ac=25-4\times2\times4\ \left(=-7\right)$ (may be embedded in the quadratic formula)
- **Attempts to use the quadratic formula**
  e.g. $\left(x=\right)\dfrac{5\pm\sqrt{\left(-5\right)^2-4\times2\times4}}{4}$ but do not allow directly from a calculator $\dfrac{5\pm\sqrt{7}i}{4}$
- **Attempts to complete the square**
  e.g. $2x^2-5x+4=2\left(x^2-\dfrac{5}{2}x\right)+4=2\left(x-\dfrac{5}{4}\right)^2+...\ \left(=2\left(x-\dfrac{5}{4}\right)^2-\dfrac{25}{8}+4\right)$
- **Uses calculus to find the turning point**
  e.g. $\dfrac{\mathrm{d}(2x^2-5x+4)}{\mathrm{d}x}=4x-5=0\Rightarrow x=\dfrac{5}{4}\Rightarrow y=...$

Note that any attempts using the discriminant or quadratic formula must have the values embedded in the correct places (may be partially evaluated) to score M1

A1: **Dependent on a correct** $Q(x) = 2x^2 - 5x + 4$

Fully correct argument that requires:

- Fully correct work
- A justification depending on strategy and no incorrect reasoning seen
- A conclusion

Examples below – Note we must see working before they proceed to a correct root or minimum value – see M1 for guidance

Strategy	**Correct work examples**	**Justification examples**	**Conclusion examples**
Via discriminant	$b^2 - 4ac = -7$	$-7 < 0$ -7 so no (real) roots but NOT $-7 \neq 0$ so no roots	so "-4 is the only (real) root" / "only one (real) root"
Via using the quadratic formula	$x = \frac{5 \pm \sqrt{7}i}{4}$ or $x = \frac{5 \pm \sqrt{-7}}{4}$	$-7 < 0$ / which is not possible / complex roots o.e. / cannot square root a negative / no (real) roots	
Via completing the square	$2\left(x - \frac{5}{4}\right)^2 + \frac{7}{8}$ or $2\left(x - \frac{5}{4}\right)^2 + \frac{7}{8} = 0 \Rightarrow$ $2\left(x - \frac{5}{4}\right)^2 = -\frac{7}{8}$	which has a minimum value of $\frac{7}{8}$ / minimum (of the positive quadratic) is above the x-axis $-\frac{7}{8} < 0$ / cannot square root a negative / no (real) roots	
Via calculus	$x = \frac{5}{4} \Rightarrow y = \frac{7}{8}$	which has a minimum value of $\frac{7}{8}$ / minimum (of the positive quadratic) is above the x-axis	

Note that it is possible to justify and conclude in one step by using phrases e.g. "**no more** (real) roots" or "**no other** (real) roots"

e.g. $2x^2 - 5x + 4 \Rightarrow b^2 - 4ac = 25 - 32 = -7$ so no more roots which scores M1A1

Condone $25 - 32 < 0$ as a justification that the quadratic has no real roots

Condone the conclusion -4 is the only (real) solution (instead of (real) root).

Note if $(x + 4)$ is described as a root or -4 is described as a factor this scores A0

9(a) Way 1	$H = \pm ax^2 \pm bx \pm c$ $x = 0, H = 2 \Rightarrow c = 2$ **and** either $x = 20, H = 0.8 \Rightarrow 0.8 = 400a + 20b + 2$ **or** $H = ax^2 + bx + c \Rightarrow \frac{dH}{dx} = 2ax + b$ $x = 9, \frac{dH}{dx} = 0 \Rightarrow 18a + b = 0$	M1	3.3
	$H = \pm ax^2 \pm bx \pm c$ $x = 0, H = 2 \Rightarrow c = 2$ **and** $x = 20, H = 0.8 \Rightarrow 0.8 = 20^2 a + 20b + 2$ **and** $H = ax^2 + bx + c \Rightarrow \frac{dH}{dx} = 2ax + b$ $x = 9, \frac{dH}{dx} = 0 \Rightarrow 18a + b = 0$	dM1	3.1b
	$0.8 = 400a + 20b + 2,\ 18a + b = 0 \Rightarrow a = ..., b = ...$	ddM1	1.1b
	$H = -0.03x^2 + 0.54x + 2$	A1	2.2a
		(4)	

(a) Way 1 Notes

Condone use of *y* for *H* for the <u>method</u> marks.

A model of the form $H = x^2 + ax + b$ or $H = -x^2 + ax + b$ **will score no marks.**

Note that it is possible to identify (by symmetry) that the points (–2, 0.8) and (18, 2) also lie on the parabola so you may see valid use of these points.

M1: Uses the equation $H = \pm ax^2 \pm bx \pm c$ to model the path and uses $x = 0$ and $H = 2$ **correctly placed** to establish the value of the constant term and uses $x = 20$ and $H = 0.8$ **or** $x = 9, \frac{dH}{dx} = 0$ to give an equation in '*a*' and '*b*' with $\frac{dH}{dx}$ of the form $...\alpha x + \beta$

An alternative is to recognise that the maximum occurs when $x = -\frac{b}{2a} = 9$ or equivalent

e.g. maximum when $x = 9 \Rightarrow H = a(x-9)^2 + ... = ax^2 - 18ax + ... \Rightarrow b = -18a$

Award for $\pm\frac{b}{2a} = 9$ or equivalent.

They may also use e.g. (–2, 0.8) or (18, 2) to give an equation in *a* and *b*.

dM1: This mark requires:

- uses the equation $H = \pm ax^2 \pm bx \pm c$ to model the path and uses $x = 0$ and $H = 2$ **correctly placed** to establish the value of the constant term
- uses $x = 20$ and $H = 0.8$ **correctly placed and** $x = 9, \frac{dH}{dx} = 0$ to give 2 equations in '*a*' and '*b*' with $\frac{dH}{dx}$ of the form $...ax + b$ or as above using $\pm\frac{b}{2a} = 9$

They may also use e.g. (–2, 0.8) or (18, 2) to give an equation in *a* and *b*.

ddM1: Solves their 2 equations in "*a*" and "*b*" to find their '*a*' and '*b*'.

This may be done on a calculator. You do **not** need to check their method for solving.

A1: Correct **equation**. Must be $H = f(x)$.

(a)	$x = 9 \text{ at max} \Rightarrow H = A \pm B(x-9)^2$		
Way 2	**and** either $x = 0, H = 2 \Rightarrow 2 = A + 81B$ **or** $x = 20, H = 0.8 \Rightarrow 0.8 = A + 121B$	M1	3.3
	$x = 9 \text{ at max} \Rightarrow H = A + B(x-9)^2$ **and** $x = 0, H = 2 \Rightarrow 2 = A + 81B$ **and** $x = 20, H = 0.8 \Rightarrow 0.8 = A + 121B$	dM1	3.1b
	$2 = A + 81B,\ 0.8 = A + 121B \Rightarrow A = 4.43,\ B = -0.03$	ddM1	1.1b
	$H = 4.43 - 0.03(x-9)^2$	A1	2.2a
		(4)	

(a) Way 2 Notes

Condone use of *y* for *H* for the <u>method</u> marks.
A model of the form $H = A \pm (x-9)^2$ will score no marks.

M1: Uses the equation $H = A \pm B(x-9)^2$ or $H = A \pm B(9-x)^2$ to model the path and uses one of the 'end points' **correctly placed** to give an equation in '*A*' and '*B*'
They may also use e.g. (–2, 0.8) or (18, 2) to give an equation in *A* and *B*.

dM1: Uses the equation $H = A + B(x-9)^2$ or $H = A + B(9-x)^2$ to model the path and uses both 'end points' **correctly placed** to give 2 equations in '*A*' and '*B*'
They may also use e.g. (–2, 0.8) or (18, 2) to give an equation in *A* and *B*.

ddM1: Solves their 2 equations in "*A*" and "*B*" to find their '*A*' and '*B*'.
This may be done on a calculator. You do **not** need to check their method for solving.

A1: Correct **equation**. Must be $H = f(x)$.

Note that using $H = A + B(x-9)^2$ followed by the incorrect assumption that $A = 2$ is unlikely to score any marks as they will subsequently not be able to produce 2 equations in "*A*" and "*B*"

<u>Possible alternative 3:</u>

$$H = A\left((x-9)^2 - 81\right) + B$$
$$x = 0, H = 2 \Rightarrow B = 2$$
$$x = 20, H = 0.8 \Rightarrow 0.8 = 40A + B$$
$$B = 2 \Rightarrow A = -0.03$$
$$H = 2 - 0.03\left((x-9)^2 - 81\right)$$

M1: Uses the equation $H = A\left((x-9)^2 - 81\right) + B$ to model the path and uses $H = 2$ when $x = 0$ **correctly placed** to find "*B*"

dM1: Uses the equation $H = A\left((x-9)^2 - 81\right) + B$ to model the path and uses $H = 0.8$ when $x = 20$ **correctly placed.** May also use e.g. (–2, 0.8) or (18, 2)

ddM1: Substitutes their value for "*B*" to find a value for "*A*"

A1: Correct **equation**. Must be $H = f(x)$.

(b)	Examples must focus on why the model may not be appropriate or give situations where the model would break down e.g.: • H is unlikely to be a quadratic function in x • The path is unlikely to be parabolic • Wind may affect the path of the ball • Wind may affect the distance the ball travels • Air resistance has not been considered • The ball is unlikely to travel in a vertical plane (as it may spin) • The ball is not a particle so has dimensions/size • The ground is unlikely to be horizontal • There may be trees (or other hazards) that would affect the path of the ball • The shape of the ball may affect the motion Condone statements (where the link to the model is not completely made) such as • The ball will spin • Ground is not flat • The ball is not a particle Do **not** accept statements that refer to the situation outside the range of the throw e.g. • The model is not valid for all values of x • H will become negative Do **not** accept statements that do not refer to the given model or single word vague answers e.g. • The distances may have been measured incorrectly • The ball is not modelled as a particle • "Friction", "Spin", "Force", "air resistance" • It does not take into account the weight of the ball • It depends how good the thrower is • You cannot throw the ball the same way every time	B1	3.5b
		(1)	
(c)	$x = 16 \Rightarrow H = -0.03(16)^2 + 0.54(16) + 2 = \ldots$	M1	3.4
	$H = 2.96$ So Chandra would not be able to catch the ball	A1	3.2a
		(2)	
			(7 marks)

Notes for (b) and (c)

(b)

B1: Gives a suitable limitation – see scheme

If more than one limitation is given and one is acceptable then award this mark as long as none of the other statements are contradictory (they may be incorrect/inappropriate)

(c)

M1: Substitutes $x = 16$ into their equation modelling the path **to obtain a value** for H.
This may be seen explicitly as above or may be implied by their value (you may need to check). **Must have a quadratic function in x.**

A1: Depends on

- A correct equation
- $H = 2.96$
- Correct conclusion that she cannot catch the ball or equivalent

A minimum for M1A1 could be e.g. $x = 16 \Rightarrow H = 2.96$ "so no"

(c) Alternative:

$$\text{e.g. } 2.5 = 4.43 - 0.03(x-9)^2 \Rightarrow x = 9 + \frac{\sqrt{579}}{3} = 17.02...$$

So Chandra would not be able to catch the ball

M1: Substitutes $H = 2.5$ into their **quadratic** equation modelling the path **to obtain a value** for x. This may be seen explicitly as above or may be implied by their value (you may need to check). **Must have a quadratic function in x.**

A1: Depends on

- A correct equation
- x = awrt 17
- Correct conclusion that she cannot catch the ball or equivalent**.**

A minimum for M1A1 could be e.g. $H = 2.5 \Rightarrow x = 17$ "so no"

Question	Scheme	Marks	AOs

12(a)	$H = ax^2 + bx + c$ and $x = 0,\ H = 3 \Rightarrow H = a x^2 + b x + 3$	M1	3.3
	$H = ax^2 + bx + 3$ and $x = 120, H = 27 \Rightarrow 27 = 14400a + 120b + 3$ **or** $\frac{\mathrm{d}H}{\mathrm{d}x} = 2ax + b = 0$ when $x = 90 \Rightarrow 180a + b = 0$	M1 A1	3.1b 1.1b
	$H = ax^2 + bx + 3$ and $x = 120, H = 27 \Rightarrow 27 = 14400a + 120b + 3$ **and** $\frac{\mathrm{d}H}{\mathrm{d}x} = 2ax + b = 0$ when $x = 90 \Rightarrow 180a + b = 0$ $\Rightarrow a = ..., b = ...$	dM1	3.1b
	$H = -\frac{1}{300}x^2 + \frac{3}{5}x + 3$ o.e.	A1	1.1b
		(5)	
(b)(i)	$x = 90 \Rightarrow H\left(= -\frac{1}{300}(90)^2 + \frac{3}{5}(90) + 3\right) = 30\,\mathrm{m}$	B1	3.4
(b)(ii)	$H = 0 \Rightarrow -\frac{1}{300}x^2 + \frac{3}{5}x + 3 = 0 \Rightarrow x = ...$	M1	3.4
	$x = (-4.868...,)\ 184.868...$ $\Rightarrow x = 185\,(\mathrm{m})$	A1	3.2a
		(3)	
(c)	Examples must focus on why the model may not be appropriate or give values/situations where the model would break down: E.g. • The ground is unlikely to be horizontal • The ball is not a particle so has dimensions/size • The ball is unlikely to travel in a vertical plane (as it will spin) • H is not likely to be a quadratic function in x	B1	3.5b
		(1)	
			(9 marks)

Notes

(a)

M1: Translates the problem into a suitable model and uses $H = 3$ when $x = 0$ to establish $c = 3$
Condone with $a = \pm 1$ so $H = x^2 + b x + 3$ will score M1 but little else

M1: For a correct attempt at **using one of the two other pieces** of information within a quadratic model
Either uses $H = 27$ when $x = 120$ (with $c = 3$) to produce a linear equation connecting a and b for the model **Or** differentiates and uses $\frac{\mathrm{d}H}{\mathrm{d}x} = 0$ when $x = 90$. Alternatives exist here, using the symmetrical nature of the curve, so they could use $x = -\frac{b}{2a}$ at vertex or use point (60, 27) or (180,3).

A1: At least one correct equation connecting a and b. Remember "a" could have been set as negative so an equation such as $27 = -14400a + 120b + 3$ would be correct in these circumstances.

dM1: Fully correct strategy that uses $H = a x^2 + b x + 3$ with the two other pieces of information in order to establish the values of **both** a **and** b for the model

A1: Correct equation, not just the correct values of a, b and c. Award if seen in part (b)

(b)(i)

B1: Correct height including the units. CAO

(b)(ii)

M1: Uses $H = 0$ and attempts to solve for x. Usual rules for quadratics.

A1: Discards the negative solution (may not be seen) and identifies awrt 185 m. Condone lack of units

B1: Candidate should either refer to an issue with one of the four aspects of how the situation has been modelled or give a situation where the model breaks down

- the ball has been modelled as a particle
- there may be trees (or other hazards) in the way that would affect the motion

Condone answers (where the link to the model is not completely made) such as

- the ball will spin
- ground is not flat

Do not accept answers which refer to the situation after it hits the ground (this isn't what was modelled)

- the ball will bounce after hitting the ground
- it gives a negative height for some values for x

Do not accept answers that do not refer to the model in question, or else give single word vague answers

- the height of tee may have been measured incorrectly
- "friction", "spin", "force" etc
- it does not take into account the weight of the ball
- it depends on how good the golfer is
- the shape of the ball will affect the motion
- you cannot hit a ball the same distance each time you hit it

The method using an alternative form of the equation can be scored in a very similar way.
The first M is for the completed square form of the quadratic showing a maximum at $x = 90$

So award M1 for $H = \pm a(x-90)^2 + c$ or $H = \pm a(90-x)^2 + c$. Condone for this mark an equation with

$a = 1 \Rightarrow H = (x-90)^2 + c$ or $c = 3 \Rightarrow H = a(x-90)^2 + 3$ but will score little else

Alt (a)	$H = a(x+b)^2 + c$ and $x = 90$ at $H_{max} \Rightarrow H = a(x-90)^2 + c$	M1	3.3
	$H = 3$ when $x = 0 \Rightarrow 3 = 8100a + c$ **or** $H = 27$ when $x = 120 \Rightarrow 27 = 900a + c$	M1 A1	3.1b 1.1b
	$H = 3$ when $x = 0 \Rightarrow 3 = 8100a + c$ **and** $H = 27$ when $x = 120 \Rightarrow 27 = 900a + c$ $\Rightarrow a = ..., c = ...$	dM1	3.1b
	$H = -\frac{1}{300}(x-90)^2 + 30$ o.e	A1	1.1b
		(5)	
(b)	$x = 90 \Rightarrow H = 0^2 + 30 = 30\,\text{m}$	B1	3.4
		(1)	
	$H = 0 \Rightarrow 0 = -\frac{1}{300}(x-90)^2 + 30 \Rightarrow x = ...$	M1	3.4
	$\Rightarrow x = 185\,(\text{m})$	A1	3.2a
		(2)	

Note that $H = -\frac{1}{300}(x-90)^2 + 30$ is equivalent to $H = -\frac{1}{300}(90-x)^2 + 30$

Other versions using symmetry are also correct so please look carefully at all responses

E.g. Using a starting equation of $H = a(x-60)(x-120) + b$ leads to $H = -\frac{1}{300}(x-60)(x-120) + 27$

8 (a) **Way 1**	$H = Ax(40-x)$ {or $H = Ax(x-40)$}	M1	3.3
	$x=20,\ H=12 \Rightarrow 12 = A(20)(40-20) \Rightarrow A = \frac{3}{100}$	dM1	3.1b
	$H = \frac{3}{100}x(40-x)$ or $H = -\frac{3}{100}x(x-40)$	A1	1.1b
		(3)	
(a) **Way 2**	$H = 12-\lambda(x-20)^2$ {or $H = 12+\lambda(x-20)^2$}	M1	3.3
	$x=40,\ H=0 \Rightarrow 0 = 12-\lambda(40-20)^2 \Rightarrow \lambda = \frac{3}{100}$	dM1	3.1b
	$H = 12-\frac{3}{100}(x-20)^2$	A1	1.1b
		(3)	
(a) **Way 3**	$H = ax^2+bx+c$ (or deduces $H = ax^2+bx$) **Both** $x=0,\ H=0 \Rightarrow 0=0+0+c \Rightarrow c=0$ **and either** $x=40,\ H=0 \Rightarrow 0 = 1600a+40b$ **or** $x=20,\ H=12 \Rightarrow 12 = 400a+20b$ **or** $\frac{-b}{2a} = 20$ $\{\Rightarrow b=-40a\}$	M1	3.3
	$b=-40a \Rightarrow 12 = 400a+20(-40a) \Rightarrow a=-0.03$ so $b=-40(-0.03)=1.2$	dM1	3.1b
	$H = -0.03x^2+1.2x$	A1	1.1b
		(3)	
(b)	{$H=3 \Rightarrow$} $3 = \frac{3}{100}x(40-x) \Rightarrow x^2-40x+100=0$ **or** {$H=3 \Rightarrow$} $3 = 12-\frac{3}{100}(x-20)^2 \Rightarrow (x-20)^2 = 300$	M1	3.4
	e.g. $x = \frac{40 \pm \sqrt{1600-4(1)(100)}}{2(1)}$ or $x = 20 \pm \sqrt{300}$	dM1	1.1b
	{chooses $20+\sqrt{300} \Rightarrow$} greatest distance = awrt 37.3 m	A1	3.2a
		(3)	
(c)	Gives a limitation of the model. Accept e.g. • the ground is horizontal • the ball needs to be kicked from the ground • the ball is modelled as a particle • the horizontal bar needs to be modelled as a line • there is no wind or air resistance on the ball • there is no spin on the ball • no obstacles in the trajectory (or path) of the ball • the trajectory of the ball is a perfect parabola	B1	3.5b
		(1)	
			(7 marks)

Notes for Question 8	
(a)	
M1:	Translates the situation given into a suitable equation for the model. E.g. **Way 1:** {Uses $(0, 0)$ and $(40, 0)$ to write} $H = Ax(40-x)$ o.e. {or $H = Ax(x-40)$ }
	Way 2: {Uses $(20, 12)$ to write} $H = 12-\lambda(x-20)^2$ or $H = 12+\lambda(x-20)^2$
	Way 3: Writes $H = ax^2+bx+c$, **and** uses $(0, 0)$ to deduce $c = 0$ **and** an attempt at using either $(40, 0)$ or $(20, 12)$ **Special Case:** Allow SC M1dM0A0 for not deducing $c = 0$ but attempting to apply both $(40, 0)$ and $(20, 12)$
dM1:	Applies a complete strategy with appropriate constraints to find all constants in their model. **Way 1:** Uses $(20, 12)$ on their model and finds $A = ...$ **Way 2:** Uses either $(40, 0)$ or $(0, 0)$ on their model to find $\lambda = ...$ **Way 3:** Uses $(40, 0)$ and $(20, 12)$ on their model to find $a = ...$ and $b = ...$
A1:	Finds a correct equation linking H to x E.g. $H = \frac{3}{100}x(40-x)$, $H = 12-\frac{3}{100}(x-20)^2$ or $H = -0.03x^2+1.2x$
Note:	Condone writing y in place of H for the M1 and dM1 marks.
Note:	Give final A0 for $y = -0.03x^2+1.2x$
Note:	Give special case M1dM0A0 for writing down any of $H = 12-(x-20)^2$ or $H = x(40-x)$ or $H = x(x-40)$
Note:	Give M1 dM1 for finding $-0.03x^2+1.2x$ or $a = -0.03, b = 1.2, c = 0$ in an implied ax^2+bx or ax^2+bx+c (with no indication of $H = ...$)
(b)	
M1:	Substitutes $H = 3$ into their quadratic equation and proceeds to obtain a 3TQ or a quadratic in the form $(x \pm \alpha)^2 = \beta$; $\alpha, \beta \neq 0$
Note:	E.g. $1.2x-0.03x^2 = 3$ or $40x-x^2 = 100$ are acceptable for the 1st M mark
Note:	Give M0 dM0 A0 for (their A)$x^2 = 3 \Rightarrow x = ...$ or their (their A)x^2 + (their k) $= 3 \Rightarrow x = ...$
dM1:	Correct method of solving their quadratic equation to give ***at least one solution***
A1:	Interprets their solution in the original context by selecting the larger correct value ***and states correct units for their value***. E.g. Accept awrt 37.3 m or $\left(20+\sqrt{300}\right)$ m or $\left(20+10\sqrt{3}\right)$ m
Note:	Condone the use of inequalities for the method marks in part (b)
(c):	
B1:	See scheme
Note:	Give no credit for the following reasons • H (or the height of ball) is negative when $x > 40$ • Bounce of the ball should be considered after hitting the ground • Model will not be true for a different rugby ball • Ball may not be kicked in the same way each time

Question	Scheme	Marks	AOs
	$\frac{d}{dx}(\sin x)=\lim_{h\to 0}\frac{\sin h}{h}\quad\lim_{h\to 0}\frac{\cos h-1}{h}$		