adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Root location and iteration

Mark scheme

9 questions

Exam questions and marking instructions remain credited to their board.

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Question	Scheme	Marks	AOs
9 (a)	$0 = 15T - T\,\mathrm{e}^{0.2T} \Rightarrow \mathrm{e}^{0.2T} = 15$	M1	3.4
	$\left(T =\right)$ awrt 13.5	A1	1.1b
		(2)	
(b)	Attempts to differentiate using the product rule $\left(\frac{\mathrm{d}}{\mathrm{d}t}\left(t\,\mathrm{e}^{0.2t}\right) =\right)\ 0.2t\,\mathrm{e}^{0.2t} + \mathrm{e}^{0.2t}$	M1	1.1b
	$v = 15t - t\,\mathrm{e}^{0.2t} \Rightarrow \left(\frac{\mathrm{d}v}{\mathrm{d}t} =\right) 15 - \left(0.2t\,\mathrm{e}^{0.2t} + \mathrm{e}^{0.2t}\right)$	A1	1.1b
	Sets their $\frac{\mathrm{d}v}{\mathrm{d}t} = 0 \Rightarrow \mathrm{e}^{0.2t}\left(0.2t + 1\right) = 15 \Rightarrow \mathrm{e}^{0.2t} = \frac{15}{0.2t+1}$	dM1	3.1b
	$\Rightarrow t = 5\ln\left(\frac{75}{t+5}\right)$ *	A1*	2.1
		(4)	
(c)	(i) Attempts $t_2 = 5\ln\left(\frac{75}{8+5}\right)$	M1	1.1b
	awrt 8.478	A1	1.1b
	(ii) awrt 8.55 seconds **(including units)**	A1	3.2a
		(3)	
			(9 marks)

Notes:

(a) May use t or another variable which is acceptable.

M1: Uses the model with $v = 0$ and proceeds to $\mathrm{e}^{0.2T} = 15$ Do not be concerned by the use of an inequality sign instead of an equals.
May be implied by awrt 13.5

A1: $\left(T =\right)$ awrt 13.5 units are not required but if given they must be seconds (or e.g. secs or s)

M1: Attempts to use the product rule to differentiate $t\,\mathrm{e}^{0.2t}$ achieving the form $A\mathrm{e}^{0.2t}+Bt\mathrm{e}^{0.2t}$ (A and B both non zero but may be 1) which may be unsimplified. It is likely to be part of an expression.

A1: $\left(\frac{\mathrm{d}v}{\mathrm{d}t}=\right)15-\left(0.2t\,\mathrm{e}^{0.2t}+\mathrm{e}^{0.2t}\right)$ o.e. which may be unsimplified. (Condone a missing trailing bracket)

Do not allow recovery of signs to score this mark if they initially write e.g. $15-0.2t\,\mathrm{e}^{0.2t}+\mathrm{e}^{0.2t}=0$ and on a later line correct this. e.g. $15-0.2t\,\mathrm{e}^{0.2t}-\mathrm{e}^{0.2t}=0$

dM1: Sets $15\pm A\mathrm{e}^{0.2t}\pm Bt\mathrm{e}^{0.2t}=0$ (the =0 may be implied), attempts to make $\mathrm{e}^{\pm0.2t}$ (or $C\mathrm{e}^{\pm0.2t}$) the subject **and** proceeds to the form

$C\mathrm{e}^{0.2t}=\frac{D}{E+Ft}$ or $C\mathrm{e}^{-0.2t}=\frac{E+Ft}{D}$ (where C can be 1 and $A, B, C, D, E, F\neq0$)

It is dependent on the previous method mark.

May see $15-"0.2t\,\mathrm{e}^{0.2t}-\mathrm{e}^{0.2t}"=0\Rightarrow\mathrm{e}^{0.2t}=\frac{15}{0.2t+1}$ which scores dM1

They must take out a factor of $\mathrm{e}^{\pm0.2t}$ (or $C\mathrm{e}^{\pm0.2t}$) and divide by their bracket. Condone sign slips in their rearrangement, however, if they take logs of both sides first, the rearrangement must be correct (with no sign slips).

Allow invisible brackets to be implied by further work which is not the given answer.

A1*: Achieves the given answer with **no errors seen including use of invisible brackets** (but condone a missing trailing bracket) All previous marks in (b) must have been scored. The **= 0** must have been seen somewhere in their solution. Do not allow this mark to be scored for proceeding directly from

$\mathrm{e}^{0.2t}(0.2t+1)=15\Rightarrow t=5\ln\left(\frac{75}{t+5}\right)$ which is A0*

We must see either $\mathrm{e}^{0.2t}=\frac{15}{0.2t+1}$ o.e. or an unsimplified expression for t

e.g. $t=5\ln\left(\frac{15}{0.2t+1}\right)$ before achieving the given answer. Condone $t=5\ln\frac{75}{t+5}$

(c) (i) Check by the question. If there is a contradiction between answers, the answer in the main body of the script takes precedence.

M1: Attempts to use the iteration formula at least once. $t_2=5\ln\left(\frac{75}{8+5}\right)$. May be implied by awrt 8.76 or awrt 8.48 or awrt 8.58. It is not implied by awrt 8.55

A1: awrt 8.478 (on its own can score M1A1)

(c)(ii) This mark can only be scored provided in (c)(i) M1 has been scored so M0A0A1 is not a possible mark profile.

A1: awrt 8.55 **seconds** (e.g. **s** or **secs**) **Requires units.**

If the candidate lists their iterations but does not select an answer then take the final value, which still requires units to be stated (which in many cases is likely to be omitted)

Note that awrt 8.55 does not imply M1**.**

Question	Scheme	Marks	AOs
3(a)	$\{f(3.6)=\}\ 3.6+\tan\left(\frac{1}{2}(3.6)\right)=-0.686\ldots<0$ **and** $\{f(3.7)=\}\ 3.7+\tan\left(\frac{1}{2}(3.7)\right)=0.211\ldots>0$	M1	1.1b
	Change of sign and function is continuous in the interval $\Rightarrow$ conclusion e.g. "there is a root in [3.6, 3.7]" *	A1*	2.4
		(2)	
(b)	Use of $\tan\left(\frac{1}{2}x\right)\rightarrow\ldots\sec^2\left(\frac{1}{2}x\right)$	M1	1.1b
	$\{f'(x)=\}\ 1+\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$	A1	1.1b
		(2)	
(c)	Attempts $3.7-\frac{3.7+\tan\left(\frac{1}{2}(3.7)\right)}{"1+\frac{1}{2}\sec^2\left(\frac{1}{2}(3.7)\right)"}=\ldots$ (N.B. $f(3.7)=0.211\ldots$ and $f'(3.7)=7.58\ldots$)	M1	1.1b
	α = awrt 3.672	A1	1.1b
		(2)	
			(6 marks)

Notes

(a)

M1: Attempts both f(3.6) and f(3.7) or a narrower interval that contains the root 3.672…
(which may be implied by sight of f(3.6) = … and f(3.7) = … with at least one correct)
and obtains at least one correct to 1 significant figure (rounded or truncated) for their interval
and considers their signs. Use of degrees is M0.
Some examples for consideration of sign (which are also sufficient for the change of sign part of reasoning for the A1):

- f(3.6) = –0.6 < 0 and f(3.7) = 0.2 > 0
- $f(3.6)\times f(3.7)<0$
- f(3.6) = –0.7, f(3.7) = 0.2 "change in sign"

For reference f(3.6) = –0.68626… and f(3.7) = 0.21194…

A1*: This mark requires:

- both f(3.6) and f(3.7) correct to 1 significant figure (rounded or truncated) (or their values correct to 1 significant figure if using a narrower interval)
- a reference to sign change
- a reference to continuity {of f(x)}
- a (minimal) conclusion, e.g. "hence root", "proved", ✓, #, QED, $3.6<\alpha<3.7$

Accept as a minimum, "change of sign, continuous, root".
Do not condone "change in sign therefore continuous" or other incorrect statements such as "x is continuous", "the interval is continuous" – these score A0.
Condone "the graph is continuous".
Condone reference to x in place of α in their conclusion, e.g. "hence x lies in the interval".
Condone statements such as "there is at least one root" in place of their conclusion.

(b) **Note:** Their answer to (b) may be seen in part (c) provided that they have not clearly attempted part (b) incorrectly, e.g., an attempt at $f^{-1}(x)$ in (b).

M1: For $\tan\left(\frac{1}{2}x\right) \rightarrow ...\sec^2\left(\frac{1}{2}x\right)$ o.e. The brackets are not required. You may see attempts at the quotient rule but the method should be correct and they should reach something equivalent to $...\sec^2\left(\frac{1}{2}x\right)$.

e.g. $\tan\left(\frac{1}{2}x\right) = \frac{\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)} \rightarrow \frac{k\cos\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right) - -k\sin\left(\frac{1}{2}x\right)\sin\left(\frac{1}{2}x\right)}{\cos^2\left(\frac{1}{2}x\right)}$ where k is a **positive** constant scores M1. If the formula is seen it must be correct.

A1: $\{f'(x) =\}\ 1 + \frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$ o.e. which may be unsimplified and apply isw.

The brackets are not required. There is no need for $f'(x) =$ just look for the expression.

Note that $\{f'(x) =\}\ \frac{3}{2} + \frac{1}{2}\tan^2\left(\frac{1}{2}x\right)$ is correct and appears occasionally.

$\{f'(x) =\}\ 1 + \frac{1}{2}\sec\frac{1}{2}x^2$ is condoned for M1A0 only but $1 + \frac{1}{2}\left(\sec\frac{1}{2}x\right)^2$ scores M1A1.

(c)

M1: Attempts $3.7 - \frac{f(3.7)}{f'(3.7)}$ and obtains a value following through on their f′(*x*) as long as it is a "changed" function in terms of *x*.

Just stating $3.7 - \frac{f(3.7)}{f'(3.7)} = ...$ without evidence of use of 3.7 in f(*x*) (note that this evidence might come from part (a)) **and** in their f′(*x*) is M0 unless implied by a correct value for both f(*x*) **and** f′(*x*) **or** by their final answer.

Must be a correct N-R formula used – you may need to check their values – accuracy of at least 3s.f. rounded or truncated required.

Allow if attempted in degrees. For reference in degrees f (3.7) = 3.73… and f′(3.7) = 1.50… and gives $\alpha = 1.21…$

Note that the full N-R accuracy is 3.672051617.

For reference, the value of α is approximately 3.673194406… and scores M0A0 without other valid work.

A1: For awrt 3.672 Ignore any subsequent iterations.

Question	Scheme	Marks	AOs

15(a)	$...xe^x + ...e^x$	M1	1.1b
	$k\left(xe^x + e^x\right)$	A1	1.1b
	$\frac{d}{dx}\left(\sqrt{e^{3x}-2}\right) = \frac{1}{2}\times 3e^{3x}\left(e^{3x}-2\right)^{-\frac{1}{2}}$	B1	1.1b
	$\left(f'(x) =\right) \frac{(e^{3x}-2)^{\frac{1}{2}}("7"xe^x + "7"e^x) - "\frac{3}{2}"e^{3x}(e^{3x}-2)^{-\frac{1}{2}}\times "7"xe^x}{e^{3x}-2}$	dM1	2.1
	$f'(x) = \frac{7e^x\left(e^{3x}(2-x)-4x-4\right)}{2(e^{3x}-2)^{\frac{3}{2}}}$	A1	1.1b
		(5)	
(b)	$e^{3x}(2-x)-4x-4=0 \Rightarrow x(...e^{3x}\pm...) = ...e^{3x}\pm...$	M1	1.1b
	$\Rightarrow x = \frac{2e^{3x}-4}{e^{3x}+4}$ *	A1*	2.1
		(2)	
(c)	Draws a vertical line $x=1$ up to the curve then across to the line $y=x$ then up to the curve finishing at the root (need to see a minimum of 2 vertical and horizontal lines tending to the root)	B1	2.1
		(1)	
(d)(i)	$x_2 = \frac{2e^3-4}{e^3+4} = 1.5017756...$	M1	1.1b
	$x_2 =$ awrt 1.502	A1	1.1b
(ii)	$\beta = 1.968$	dB1	2.2b
		(3)	
(e)	$h(x) = \frac{2e^{3x}-4}{e^{3x}+4} - x$ $h(0.4315) = -0.000297...$ $\quad$ $h(0.4325) = 0.000947...$	M1	3.1a
	Both calculations correct and e.g. states: • There is a change of sign • e.g $f'(x)$ is continuous • $\alpha = 0.432$ (to 3dp)	A1cao	2.4
		(2)	
			(13 marks)

Notes

(a)

M1: Attempts the product rule on xe^x (or may be $7xe^x$) achieving an expression of the form $...xe^x \pm ...e^x$. If it is clear that the quotient rule has been applied instead which may be quoted then M0.

A1: $k\left(xe^x + e^x\right)$ (e.g. $7(xe^x + e^x)$) or equivalent which may be unsimplified (may be implied by further work)

B1: $\left(\frac{d}{dx}\left(\sqrt{e^{3x}-2}\right) =\right) \frac{1}{2}\times 3e^{3x}\left(e^{3x}-2\right)^{-\frac{1}{2}}$ (simplified or unsimplified)

dM1: Attempts to use the quotient rule. It is dependent on the previous method mark.
Score for achieving an expression of the form

$$\left(\mathrm{f}'(x)=\right)\ \frac{(\mathrm{e}^{3x}-2)^{\frac{1}{2}}("7"x\mathrm{e}^{x}+"7"\mathrm{e}^{x})-"\frac{3}{2}"\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}\times"7"x\mathrm{e}^{x}}{\mathrm{e}^{3x}-2}$$ or equivalent (do not be concerned by the constants for their "7" or their "$\frac{3}{2}$" which may be both 1)

If it is clear that the quotient rule has been applied the wrong way round then score M0.
Alternatively, applies the product rule. Score for achieving an expression of the form

$$\left(\mathrm{f}'(x)=\right)\ (\mathrm{e}^{3x}-2)^{-\frac{1}{2}}("7"x\mathrm{e}^{x}+"7"\mathrm{e}^{x})-"\frac{3}{2}"\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{3}{2}}\times"7"x\mathrm{e}^{x}$$ or equivalent (do not be concerned by the constants for their "7" or their "$\frac{3}{2}$" which may be both 1)

Do not condone invisible brackets.

A1: $\left(\mathrm{f}'(x)=\right)\ \frac{7\mathrm{e}^{x}\left(\mathrm{e}^{3x}(2-x)-4x-4\right)}{2(\mathrm{e}^{3x}-2)^{\frac{3}{2}}}$ following a fully correct differentiated expression.

You may need to check to see if (a) is continued after other parts for evidence of this.

Condone the lack of $\mathrm{f}'(x)=$ on the left hand side or allow the use of $\frac{\mathrm{d}y}{\mathrm{d}x}$ or y' instead.

Alternative (a) attempt using the triple product rule

e.g. $\frac{\mathrm{d}}{\mathrm{d}x}\left(7x\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}\right)=7\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}+7x\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}+7x\mathrm{e}^{x}\times\left(-\frac{1}{2}\right)\times3\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{3}{2}}$

$$\Rightarrow\frac{\left(7\mathrm{e}^{x}+7x\mathrm{e}^{x}\right)(\mathrm{e}^{3x}-2)+7x\mathrm{e}^{x}\times\left(-\frac{1}{2}\right)\times3\mathrm{e}^{3x}}{(\mathrm{e}^{3x}-2)^{\frac{3}{2}}}=\frac{7\mathrm{e}^{x}\left(\mathrm{e}^{3x}-2+x\mathrm{e}^{3x}-2x-\frac{3}{2}x\mathrm{e}^{3x}\right)}{(\mathrm{e}^{3x}-2)^{\frac{3}{2}}}\Rightarrow\frac{7\mathrm{e}^{x}\left(\mathrm{e}^{3x}(2-x)-4x-4\right)}{2(\mathrm{e}^{3x}-2)^{\frac{3}{2}}}$$

M1: Attempts the product rule on $x\mathrm{e}^{x}\rightarrow\ldots x\mathrm{e}^{x}\pm\ldots\mathrm{e}^{x}$ which may be seen within the expression $\ldots\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}\pm\ldots x\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}+\ldots$ simplified or unsimplified.

A1: $k\left(x\mathrm{e}^{x}+\mathrm{e}^{x}\right)$ which may be seen within the expression $k\left(\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}+x\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}\right)+\ldots$ simplified or unsimplified.

B1: $\left(-\frac{1}{2}\right)\times3\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{3}{2}}$ which may be seen within the expression $\ldots.+k\left(x\mathrm{e}^{x}\times\left(-\frac{1}{2}\right)\times3\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{3}{2}}\right)$ simplified or unsimplified.

dM1: A complete method using all three products (which may appear all on one line). Do not condone invisible brackets.

A1: As above in main scheme notes.

(b) **Note that if they do not have values $A=-4,\ B=-4$ in (a) (which may be seen later) then maximum score is M1A0***

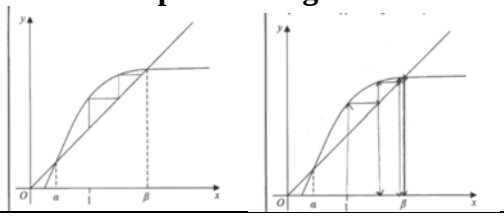
M1: Sets their $\mathrm{e}^{3x}(2-x)-4x-4$ equal to zero, collects terms in x on one side of the equation and non x terms on the other and attempts to factorise the side with x as a common factor. Condone sign slips only for this mark. Allow A and B to be used instead of $"-4"$ and $"-4"$

A1*: Achieves the given answer with no errors including invisible brackets. If they do not reach the printed answer then it is A0. If they subsequently write $x_{n+1} = \dfrac{2e^{3x_n} - 4}{e^{3x_n} + 4}$ then isw

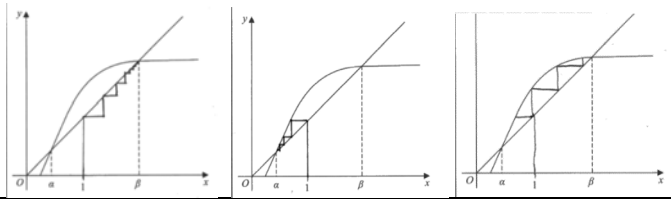
(c)

B1: Starting at $x_1 = 1$ look for at least 2 sets of vertical and horizontal lines drawn (may be dashes) tending to β. Condone a lack of arrows on the lines but the sequence of lines should finish at the point of intersection where the root is. Condone the initial vertical line not starting from the *x*-axis. Mark the intention to draw horizontal and vertical lines. If they have any lines to the left of $x = 1$ this is B0. If they use both diagrams and do not indicate which one they want marking, then the "copy of Diagram 1" should be marked.

Examples scoring B1:



Examples scoring B0:



(d)(i)

M1: Substitutes 1 into the iterative formula. The values embedded in the formula is sufficient for this mark. May be implied by awrt 1.50

A1: awrt 1.502 isw

(d)(ii)

dB1: 1.968 cao (which can only be scored if M1 is scored in (d)(i))

SC: If (d)(i) is rounded to 1.50 then allow 1.97 in (d)(ii) to score M1A0dB1 for (d)

(e)

M1: Attempts to substitute $x = 0.4315$ and 0.4325 into a suitable function and gets one value correct (rounded or truncated to 1sf). It is allowable to use a tighter interval that contains the root 0.4317388728…

If no function is stated then may be implied by their answers to e.g. $f'(0.4315)$, $f'(0.4325)$

You will need to check their calculation is correct.

Other possible functions include:

- $h(x) = x - \dfrac{2e^{3x} - 4}{e^{3x} + 4}$ (other way round to MS) $h(0.4315) = 0.0002974...$, $h(0.4325) = -0.0009479...$
- their $f'(x) = \pm\left(\dfrac{7e^{x}\left(e^{3x}(2-x) - 4x - 4\right)}{2(e^{3x} - 2)^{\frac{3}{2}}}\right)$

(If correct *A* and *B* then $f'(0.4315) = \mp 0.005789...$, $f'(0.4325) = \pm 0.01831...$)

- their $g(x) = \pm\left(e^{3x}(2-x) - 4x - 4\right)$

(If correct *A* and *B* then $g(0.4315) = \mp 0.002275...$, $g(0.4325) = \pm 0.007261...$)

A1: Requires

- Both calculations correct (rounded or truncated to 1sf)
- A statement that there is a change in sign and that their **function** is continuous (must refer to the function used for the substitution (which is not $f(x)$)

  Accept equivalent statements for $f'(0.4315) < 0$, $f'(0.4325) > 0$ e.g. $f'(0.4315) \times f'(0.4325) < 0$, "one negative one positive". A minimum is "change of sign and continuous" but do **not** allow this mark if the comment about continuity is clearly incorrect e.g. "because *x* is continuous" or "because the interval is continuous"
- A minimal conclusion e.g. "hence $\alpha = 0.432$", "so rounds to 0.432". Do not allow "hence root"

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Question	Scheme	Marks	AOs
8 (a)	25	B1	3.4
		(1)	
(b)	Attempts to differentiate using the product rule $\frac{dv}{dt} = \ln(t+1)\times -0.4 + \frac{(10-0.4t)}{t+1}$	M1 A1	3.1b 1.1b
	Sets their $\frac{dv}{dt} = 0 \Rightarrow \frac{(10-0.4\boldsymbol{t})}{(\boldsymbol{t}+1)} = 0.4\ln(t+1)$ **and** then makes progress towards making "$\boldsymbol{t}$" the subject (See notes for this)	dM1	1.1b
	$t = \frac{25-\ln(t+1)}{1+\ln(t+1)}$ $t = \frac{26}{1+\ln(t+1)} - 1$ *	A1*	2.1
		(4)	
(c)	(i) Attempts $t_2 = \frac{26}{1+\ln 8} - 1$	M1	1.1b
	awrt 7.298	A1	1.1b
	(ii) awrt 7.33 seconds	A1	3.2a
		(3)	
			(8 marks)
Notes:			

(a)

B1: 25 but condone 25 seconds. If another value is given (apart from 0) it is B0

(b)

M1: Attempts to use the product rule in an attempt to differentiate $v = (10-0.4t)\ln(t+1)$

Look for $(10-0.4t)\times\frac{1}{(t+1)} \pm k\ln(t+1)$, where k is a constant, condoning slips.

If you see direct evidence of an incorrect rule used e.g. $vu'-uv'$ it is M0

You will see attempts from $v = 10\ln(t+1) - 0.4t\ln(t+1)$ which can be similarly marked.

In this case look for $\frac{a}{t+1} \pm \frac{bt}{t+1} \pm c\ln(t+1)$

A1: Correct differentiation. Condone a missing left hand or it seen as v', $\frac{dy}{dx}$ or even = 0

$\left(\frac{dv}{dt}\right) = \ln(t+1)\times -0.4 + \frac{(10-0.4t)}{t+1}$ or equivalent such as $\left(\frac{dv}{dt}\right) = \frac{10}{t+1} - \frac{0.4t}{(t+1)} - 0.4\ln(t+1)$

dM1: Score for setting their dV/dt = 0 (which must be in an appropriate form) and proceeding to an equation where the variable t occurs only once – ignoring $\ln(t + 1)$.

See two examples of how this can be achieved below. It is dependent upon the previous M.

Look for the following steps

- An allowable derivative set (or implied) = 0 E.g. $\ln(t+1)\times 0.4 = \dfrac{(10-0.4t)}{t+1}$
- Cross multiplication (or division) and rearrangement to form an equation where the variable t only occurs once.

E.g.1.

$$\ln(t+1)\times 0.4 = \frac{(10-0.4t)}{t+1}$$
$$\Rightarrow \ln(t+1) = \frac{25-t}{t+1}$$
$$\Rightarrow \ln(t+1) = -1+\frac{26}{t+1}$$

E.g 2

$$\ln(t+1)\times 0.4 = \frac{(10-0.4t)}{t+1}$$
$$\Rightarrow 0.4t\ln(t+1)+0.4\ln(t+1) = 10-0.4t$$
$$\Rightarrow 0.4t\left(1+\ln(t+1)\right) = 10-0.4\ln(t+1)$$

A1*: Correctly proceeds to the given answer of $t = \dfrac{26}{1+\ln(t+1)} - 1$ showing all key steps.

The key steps must include

- use of $\dfrac{\mathrm{d}v}{\mathrm{d}t}$ or v' which must be correct
- a correct line preceding the given answer, usually $t = \dfrac{25-\ln(t+1)}{1+\ln(t+1)}$ or $\dfrac{26}{t+1} - 1 = \ln(t+1)$

(c) (i)

M1: Attempts to use the iteration formula at least once.

Usually to find $t_2 = \dfrac{26}{1+\ln 8} - 1$ which may be implied by awrt 7.44

A1: awrt 7.298. This alone will score both marks as iteration is implied. ISW after sight of this value. As t_3 is the only value that rounds to 7.298 just score the rhs, it does not need to be labelled t_3

(c)(ii)

A1: Uses repeated iteration until value established as awrt 7.33 **seconds**. Allow awrt 7.33 **s**

Requires units. It also requires some evidence of iteration which will be usually be awarded from the award of the M

6(a)	$\left(f'(x)=\right)4\cos\left(\frac{1}{2}x\right)-3$	M1 A1	1.1b 1.1b
	Sets $f'(x)=4\cos\left(\frac{1}{2}x\right)-3=0\Rightarrow x=$	dM1	3.1a
	$x=14.0$ Cao	A1	3.2a
		(4)	
(b)	Explains that $f(4)>0$, $f(5)<0$ **and** the function is continuous	B1	2.4
		(1)	
(c)	Attempts $x_1=5-\frac{8\sin 2.5-15+9}{"4\cos 2.5-3"}$ (NB $f(5)=-1.212\ldots$ and $f'(5)=-6.204\ldots$)	M1	1.1b
	$x_1=$ awrt 4.80	A1	1.1b
		(2)	
			(7 marks)

Notes:

(a)

M1: Differentiates to obtain $k\cos\left(\frac{1}{2}x\right)\pm\alpha$ where α is a constant which may be zero and no other terms. The brackets are not required.

A1: Correct derivative $f'(x)=4\cos\left(\frac{1}{2}x\right)-3$. Allow unsimplified e.g. $f'(x)=\frac{1}{2}\times 8\cos\left(\frac{1}{2}x\right)-3x^0$

There is no need for $f'(x)=\ldots$ or $\frac{dy}{dx}=\ldots$ just look for the expression and the brackets are not required.

dM1: For the complete strategy of proceeding to a value for *x*.

Look for

- $f'(x)=a\cos\left(\frac{1}{2}x\right)+b=0,\quad a,b\neq 0$
- Correct method of finding a valid solution to $a\cos\left(\frac{1}{2}x\right)+b=0$

Allow for $a\cos\left(\frac{1}{2}x\right)+b=0\Rightarrow\cos\left(\frac{1}{2}x\right)=\pm k\Rightarrow x=2\cos^{-1}\left(\pm k\right)$ where $|k|<1$

If this working is not shown then you may need to check their value(s).

For example $4\cos\left(\frac{1}{2}x\right)-3=0\Rightarrow x=1.4\ldots$ or 11.1… (or 82.8… or 637…. or 803 in degrees) would indicate this method.

A1: Selects the correct turning point $x=14.0$ and not just 14 or unrounded e.g. 14.011…
Must be this value only and no other values unless they are clearly rejected or 14.0 clearly selected. Ignore any attempts to find the *y* coordinate.
Correct answer with no working scores no marks.

(b)

B1: See scheme. Must be a full reason, (e.g. change of sign and continuous)
Accept equivalent statements for $f(4)>0$, $f(5)<0$ e.g. $f(4)\times f(5)<0$, "there is a change of sign", "one negative one positive". A minimum is "change of sign and continuous" but do **not** allow this mark if the comment about continuity is clearly incorrect e.g. "because *x* is

(c)

M1: Attempts $x_1 = 5 - \frac{f(5)}{f'(5)}$ to obtain a value following through on their $f'(x)$ as long as it is a "changed" function.

Must be a correct N-R formula used – may need to check their values.

Allow if attempted in degrees. For reference in degrees f (5) = –5.65… and f ′(5) = 0.996… and gives $x_1 = 10.67...$

There must be clear evidence that $5 - \frac{f(5)}{f'(5)}$ is being attempted.

so e.g. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \Rightarrow x_1 = 4.80$ scores M0 as does e.g. $x_1 = x - \frac{8\sin\left(\frac{1}{2}x\right) - 3x + 9}{4\cos\left(\frac{1}{2}x\right) - 3} = 4.80$

BUT evidence may be provided by the accuracy of their answer. Note that the full N-R accuracy is 4.804624337 so e.g. 4.805 or 4.804 (truncated) with no evidence of incorrect work may imply the method.

A1: $x_1 =$ awrt 4.80 not awrt 4.8 but isw if awrt 4.80 is seen. Ignore any subsequent iterations.

Note that work for part (a) cannot be recovered in part (c)

Note also:

$5 - \frac{f(5)}{f'(5)} =$ awrt 4.80 following a correct derivative scores M1A1

$5 - \frac{f(5)}{f'(5)} \neq$ awrt 4.80 with no evidence that $5 - \frac{f(5)}{f'(5)}$ was attempted scores M0

4(a)	$f'(x) = 2x + \dfrac{4x-4}{2x^2-4x+5}$	M1 A1	1.1b 1.1b
	$2x + \dfrac{4x-4}{2x^2-4x+5} = 0 \Rightarrow 2x\left(2x^2-4x+5\right)+4x-4=0$	dM1	1.1b
	$2x^3-4x^2+7x-2=0$*	A1*	2.1
		(4)	
(b)	(i) $x_2 = \dfrac{1}{7}\left(2+4(0.3)^2-2(0.3)^3\right)$	M1	1.1b
	$x_2 = 0.3294$	A1	1.1b
	(ii) $x_4 = 0.3398$	A1	1.1b
		(3)	
(c)	$h(x) = 2x^3-4x^2+7x-2$ $h(0.3415) = 0.00366...$ $h(0.3405) = -0.00130...$	M1	3.1a
	States: • there is a change of sign • $f'(x)$ is continuous • α = 0.341 to 3dp	A1	2.4
		(2)	
			(9 marks)
	Notes		

(a)

M1: Differentiates $\ln\left(2x^2-4x+5\right)$ to obtain $\frac{g(x)}{2x^2-4x+5}$ where g(x) could be 1

A1: For $f'(x)=2x+\frac{4x-4}{2x^2-4x+5}$

dM1: Sets their f′(x) $=ax+\frac{g(x)}{2x^2-4x+5}$ = 0 and uses "**correct**" algebra, condoning slips, to obtain a cubic equation. E.g Look for $ax\left(2x^2-4x+5\right)\pm g(x)=0$ o.e. , condoning slips, followed by some attempt to simplify

A1*: Achieves $2x^3-4x^2+7x-2=0$ with no errors. (The dM1 mark must have been awarded)

(b)(i)

M1: Attempts to use the iterative formula with $x_1=0.3$. If no method is shown award for $x_2=$ awrt 0.33

A1: $x_2=\text{awrt } 0.3294$ Note that $\frac{1153}{3500}$ is correct

Condone an incorrect suffix if it is clear that a correct value has been found

(b)(ii)

A1: $x_4=\text{awrt } 0.3398$ Condone an incorrect suffix if it is clear that a correct value has been found

(c)

M1: Attempts to substitute x = 0.3415 and x = 0.3405 into a suitable function and gets one value correct (rounded or truncated to 1 sf). It is allowable to use a tighter interval that contains the root 0.340762654

Examples of suitable functions are $2x^3-4x^2+7x-2$, $x-\frac{1}{7}\left(4x^2-2x^3+2\right)$ and f '(x) as this has been found in part (a) with f '(0.3405)= - 0.00067.., f '(0.3415)= (+) 0.0018

There must be sufficient evidence for the function, which would be for example, a statement such as $h(x)=2x^3-4x^2+7x-2$ or sight of embedded values that imply the function, not just a value or values even if both are correct. Condone $h(x)$ being mislabelled as f

$h(0.3415)=2\times0.3415^3-4\times0.3415^2+7\times0.3415-2$

A1: Requires

- both calculations correct (rounded or truncated to 1sf)
- a statement that there is a change in sign and that the function is continuous
- a minimal conclusion e.g. ✓, proven, α = 0.341, root

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
11 (a) Way 1	$\{y = x^x \Rightarrow\}\quad \ln y = x\ln x$	B1	1.1a
	$\dfrac{1}{y}\dfrac{dy}{dx} = 1 + \ln x$	M1	1.1b
		A1	2.1
	$\left\{\dfrac{dy}{dx} = 0 \Rightarrow\right\}\quad \dfrac{x}{x} + \ln x = 0$ or $1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = ...$	M1	1.1b
	$x = e^{-1}$ or awrt 0.368	A1	1.1b
	Note: $k \neq 0$	**(5)**	
(a) Way 2	$\{y = x^x \Rightarrow\}\quad y = e^{x\ln x}$	B1	1.1a
	$\dfrac{dy}{dx} = \left(\dfrac{x}{x} + \ln x\right)e^{x\ln x}$	M1	1.1b
		A1	2.1
	$\left\{\dfrac{dy}{dx} = 0 \Rightarrow\right\}\quad \dfrac{x}{x} + \ln x = 0$ or $1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = ...$	M1	1.1b
	$x = e^{-1}$ or awrt 0.368	A1	1.1b
	Note: $k \neq 0$	**(5)**	
(b) Way 1	Attempts both $1.5^{1.5} = 1.8...$ and $1.6^{1.6} = 2.1...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$1.8... < 2$ and $2.1... > 2$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
(c)	Attempts $x_{n+1} = 2x_n^{1-x_n}$ at least once with $x_1 = 1.5$ Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63	M1	1.1b
	$\{x_4 = 1.67313... \Rightarrow\}\ x_4 = 1.673$ (3 dp) **cao**	A1	1.1b
		(2)	
(d)	Give 1st B1 for any of • oscillates • periodic • non-convergent • divergent • fluctuates • goes up and down • 1, 2, 1, 2, 1, 2 • alternates (condone)	Give B1 B1 for any of • periodic {sequence} with period 2 • oscillates between 1 and 2	B1	2.5
		Condone B1 B1 for any of • fluctuates between 1 and 2 • keep getting 1, 2 • alternates between 1 and 2 • goes up and down between 1 and 2 • 1, 2, 1, 2, 1, 2, …	B1	2.5
			(2)	
				(11 marks)

Note	**A common solution**
	A maximum of 3 marks (i.e. B1 1st M1 and 2nd M1) can be given for the solution $\log y = x\log x \Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = 1 + \log x$ $\left\{\dfrac{dy}{dx} = 0 \Rightarrow\right\}\ 1 + \log x = 0 \Rightarrow x = 10^{-1}$
	• 1st B1 for $\log y = x\log x$ • 1st M1 for $\log y \to \lambda\dfrac{1}{y}\dfrac{dy}{dx};\ \lambda \neq 0$ or $x\log x \to 1 + \log x$ or $\dfrac{x}{x} + \log x$ • 2nd M1 can be given for $1 + \log x = 0 \Rightarrow \log x = k \Rightarrow x = ...;\quad k \neq 0$

Question	Scheme	Marks	AOs
11 (b) Way 2	For $x^x - 2$, attempts both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$-0.16... < 0$ and $0.12... > 0$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
11 (b) Way 3	For $\ln y = x \ln x$, attempts both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$0.608... < 0.69...$ and $0.752... > 0.69...$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
11 (b) Way 4	For $\log y = x \log x$, attempts both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ and at least one result is correct to awrt 2 dp	M1	1.1b
	$0.264... < 0.301...$ and $0.326... > 0.301...$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	

Notes for Question 11

(a)	**Way 1**
B1:	$\ln y = x \ln x$. Condone $\log_x y = x \log_x x$ or $\log_x y = x$
M1:	For either $\ln y \to \frac{1}{y}\frac{dy}{dx}$ or $x \ln x \to 1 + \ln x$ or $\frac{x}{x} + \ln x$
A1:	Correct differentiated equation. i.e. $\frac{1}{y}\frac{dy}{dx} = 1 + \ln x$ or $\frac{1}{y}\frac{dy}{dx} = \frac{x}{x} + \ln x$ or $\frac{dy}{dx} = y(1 + \ln x)$ or $\frac{dy}{dx} = x^x(1 + \ln x)$
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = ...$; k is a constant and $k \neq 0$
A1:	$x = e^{-1}$ or awrt 0.368 only (with no other solutions for x)
Note:	Give no marks for no working leading to 0.368
Note:	Give M0 A0 M0 A0 for $\ln y = x \ln x \to x = 0.368$ with no intermediate working
(a)	**Way 2**
B1:	$y = e^{x \ln x}$
M1:	For either $y = e^{x \ln x} \Rightarrow \frac{dy}{dx} = f(\ln x)e^{x \ln x}$ or $x \ln x \to 1 + \ln x$ or $\frac{x}{x} + \ln x$
A1:	Correct differentiated equation. i.e. $\frac{dy}{dx} = \left(\frac{x}{x} + \ln x\right)e^{x \ln x}$ or $\frac{dy}{dx} = (1 + \ln x)e^{x \ln x}$ or $\frac{dy}{dx} = x^x(1 + \ln x)$
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = ...$; k is a constant and $k \neq 0$
A1:	$x = e^{-1}$ or awrt 0.368 only (with no other solutions for x)
Note:	Give B1 M1 A0 M1 A1 for the following solution: $\{y = x^x \Rightarrow\}$ $\ln y = x \ln x \Rightarrow \frac{dy}{dx} = 1 + \ln x \Rightarrow 1 + \ln x = 0 \Rightarrow x = e^{-1}$ or awrt 0.368

Notes for Question 11 Continued	
(b)	**Way 1**
M1:	Attempts both $1.5^{1.5} = 1.8...$ and $1.6^{1.6} = 2.1...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1.5} =$ awrt $1.8...$ and $1.6^{1.6} =$ awrt $2.1...$, reason (e.g. $1.8... < 2$ and $2.1... > 2$ or states C cuts through $y = 2$), C continuous and conclusion
(b)	**Way 2**
M1:	Attempts both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ correct to awrt 1 dp, reason (e.g. $-0.16... < 0$ and $0.12... > 0$, sign change or states C cuts through $y = 0$), C continuous and conclusion
(b)	**Way 3**
M1:	Attempts both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ correct to awrt 1 dp, reason (e.g. $0.608... < 0.69...$ and $0.752... > 0.69...$ or states they are either side of $\ln 2$), C continuous and conclusion.
(b)	**Way 4**
M1:	Attempts both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ and at least one result is correct to awrt 2 dp
A1:	Both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ correct to awrt 2 dp, reason (e.g. $0.264... < 0.301...$ and $0.326... > 0.301...$ or states they are either side of $\log 2$), C continuous and conclusion.
(c)	
M1:	An attempt to use the given or their formula once. Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63
A1:	States $x_4 = 1.673$ **cao** (to 3 dp)
Note:	Give M1 A1 for stating $x_4 = 1.673$
Note:	M1 can be implied by stating their final answer $x_4 =$ awrt 1.673
Note:	$x_2 = 1.63299...$, $x_3 = 1.46626...$, $x_4 = 1.67313...$
(d)	
B1:	see scheme
B1:	see scheme
Note:	Only marks of B1B0 or B1B1 are possible in (d)
Note:	Give B0 B0 for "Converges in a cob-web pattern" or "Converges up and down to α"

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
4 (a)	Attempts $f(3) =$ and $f(4) =$ where $f(x) = \pm(2\ln(8-x)-x)$	M1	2.1
	$f(3) = (2\ln(5)-x) = (+)0.22$ and $f(4) = (2\ln(4)-4) = -1.23$ Change of sign and function continuous in interval $[3,4] \Rightarrow$ Root *	A1*	2.4
		(2)	
(b)	For annotating the graph by drawing a cobweb diagram starting at $x_1 = 4$ It should have at least two spirals	M1	2.4
	Deduces that the iteration formula **can be used** to find an approximation for α because **the cobweb spirals inwards** for the cobweb diagram	A1	2.2a
		(2)	
			(4 marks)

Notes:

(a)

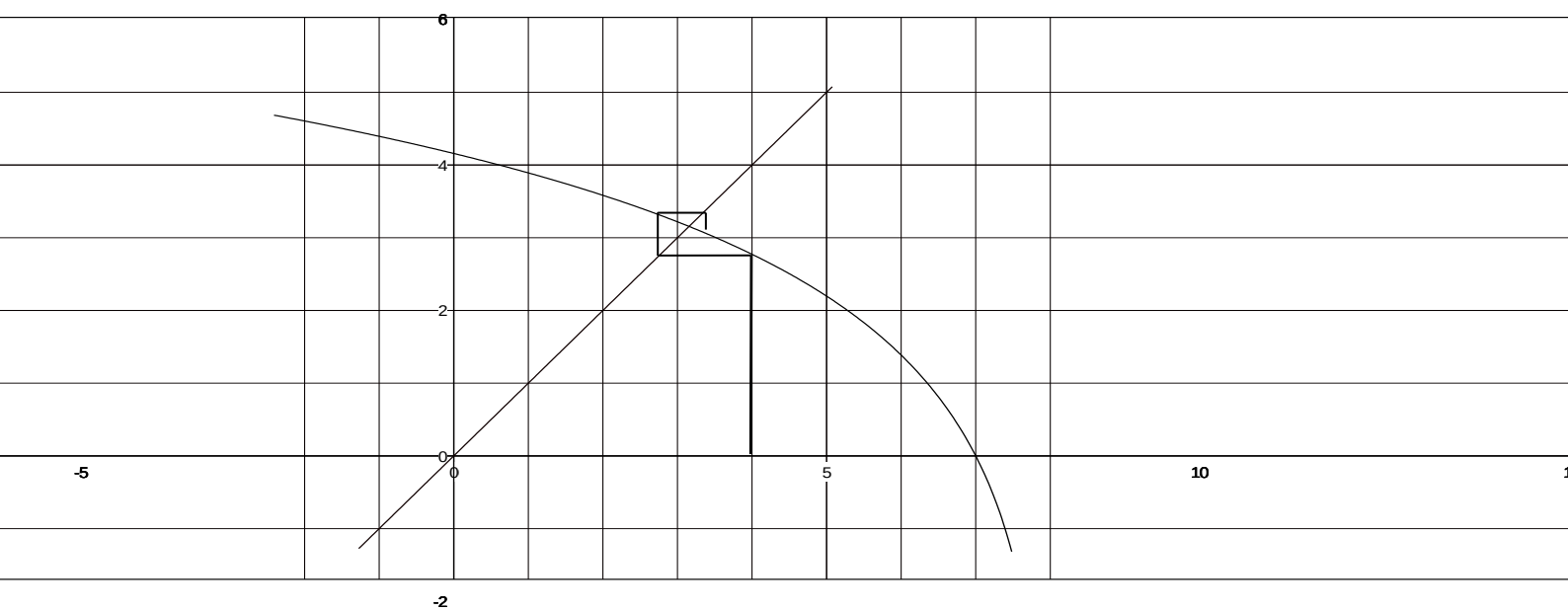
M1: Attempts $f(3) =$ and $f(4) =$ where $f(x) = \pm(2\ln(8-x)-x)$ or alternatively **compares** $2\ln 5$ to 3 and $2\ln 4$ to 4. This is not routine and cannot be scored by substituting 3 and 4 in both functions

A1: Both values (calculations) correct to at least 1 sf with correct explanation and conclusion. (See underlined statements)
When comparing terms, allow reasons to be $2ln8 = 3.21 > 3$, $2\ln 4 = 2.77 < 4$ or similar

(b)

M1: For an attempt at using a cobweb diagram. Look for 5 or more correct straight lines. It may not start at 4 but it must show an understanding of the method. **If there is no graph then it is M0 A0**

A1: For a correct attempt starting at 4 and deducing that the iteration **can be used** as the iterations **converge to the root**. You must statement that it can be used with a suitable reason. Suitable reasons could be " it spirals inwards", it gets closer to the root", it converges "



Question	Scheme	Marks	AOs

5	The equation $2x^3+x^2-1=0$ has exactly one real root		
(a)	$\left\{f(x)=2x^3+x^2-1\Rightarrow\right\}\ f'(x)=6x^2+2x$	B1	1.1b
	$\left\{x_{n+1}=x_n-\dfrac{f(x_n)}{f'(x_n)}\Rightarrow\right\}\ \left\{x_{n+1}\right\}=x_n-\dfrac{2x_n^{\ 3}+x_n^{\ 2}-1}{6x_n^{\ 2}+2x_n}$	M1	1.1b
	$=\dfrac{x_n(6x_n^{\ 2}+2x_n)-(2x_n^{\ 3}+x_n^{\ 2}-1)}{6x_n^{\ 2}+2x_n}\Rightarrow x_{n+1}=\dfrac{4x_n^{\ 3}+x_n^{\ 2}+1}{6x_n^{\ 2}+2x_n}$ *	A1*	2.1
		(3)	
(b)	$\left\{x_1=1\Rightarrow\right\}\ x_2=\dfrac{4(1)^3+(1)^2+1}{6(1)^2+2(1)}$ or $x_2=1-\dfrac{2(1)^3+(1)^2-1}{6(1)^2+2(1)}$	M1	1.1b
	$\Rightarrow x_2=\dfrac{3}{4},\ x_3=\dfrac{2}{3}$	A1	1.1b
		(2)	
(c)	Accept any reasons why the Newton-Raphson **method** cannot be used with $x_1=0$ which refer or ***allude*** to either the stationary point or the tangent. E.g. • There is a stationary point at $x=0$ • Tangent to the curve (or $y=2x^3+x^2-1$) would not meet the x-axis • Tangent to the curve (or $y=2x^3+x^2-1$) is horizontal	B1	2.3
		(1)	
			(6 marks)

Notes for Question 5

(a)	
B1:	States that $f'(x)=6x^2+2x$ or states that $f'(x_n)=6x_n^{\ 2}+2x_n$ (Condone $\dfrac{dy}{dx}=6x^2+2x$)
M1:	Substitutes $f(x_n)=2x_n^{\ 3}+x_n^{\ 2}-1$ and their $f'(x_n)$ into $x_{n+1}=x_n-\dfrac{f(x_n)}{f'(x_n)}$
A1*:	A correct intermediate step of making a common denominator which leads to the given answer
Note:	Allow B1 if $f'(x)=6x^2+2x$ is applied as $f'(x_n)$ (or $f'(x)$) in the NR formula $\left\{x_{n+1}\right\}=x_n-\dfrac{f(x_n)}{f'(x_n)}$
Note:	Allow M1A1 for • $x_{n+1}=x-\dfrac{2x^3+x^2-1}{6x^2+2x}=\dfrac{x(6x^2+2x)-(2x^3+x^2-1)}{6x^2+2x}\Rightarrow x_{n+1}=\dfrac{4x_n^{\ 3}+x_n^{\ 2}+1}{6x_n^{\ 2}+2x_n}$
Note	Condone $x=x-\dfrac{2x^3+x^2-1}{"6x^2+2x"}$ for M1
Note	Condone $x_n-\dfrac{2x_n^{\ 3}+x_n^{\ 2}-1}{"6x_n^{\ 2}+2x_n"}$ or $x-\dfrac{2x^3+x^2-1}{"6x^2+2x"}$ (i.e. no $x_{n+1}=...$) for M1
Note:	Give M0 for $x_{n+1}=x_n-\dfrac{f(x_n)}{f'(x_n)}$ followed by $x_{n+1}=2x_n^{\ 3}+x_n^{\ 2}-1-\dfrac{2x_n^{\ 3}+x_n^{\ 2}-1}{6x_n^{\ 2}+2x_n}$
Note:	Correct notation, i.e. x_{n+1} and x_n must be seen in their final answer for A1*

Notes for Question 5 Continued	
(b)	
M1:	An attempt to use the given or their formula once. Can be implied by $\frac{4(1)^3+(1)^2+1}{6(1)^2+2(1)}$ or 0.75 o.e.
Note:	Allow one slip in substituting $x_1=1$
A1:	$x_2=\frac{3}{4}$ and $x_3=\frac{2}{3}$
Note:	Condone $x_2=\frac{3}{4}$ and $x_3=$ awrt 0.667 for A1
Note:	Condone $\frac{3}{4},\frac{2}{3}$ listed in a correct order ignoring subscripts
(c)	
B1:	See scheme
Note:	Give B0 for the following isolated reasons: e.g. • You cannot divide by 0 • The fraction (or the NR formula) is undefined at $x=0$ • At $x=0$, $f'(x_1)=0$ • x_1 cannot be 0 • $6x^2+2x$ cannot be 0 • the denominator is 0 which cannot happen • if $x_1=0$, $6x^2+2x=0$

Question	Scheme	Marks	AOs