adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Stationary points

Mark scheme

8 questions

Exam questions and marking instructions remain credited to their board.

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Question	Scheme	Marks	AOs
7 (a)	Either $x \leq -1$ or $2 \leq x \leq 5$	M1	2.2a
	Both $\{x : x \in \mathbb{R}, x \leq -1\} \cup \{x : x \in \mathbb{R}, 2 \leq x \leq 5\}$ o.e.	A1	2.5
		(2)	
(b)	States $(y =)\ \alpha(x+1)^2(x-5)^2$ or $(f(x) =)\ \alpha(x+1)^2(x-5)^2$	M1	1.1b
	Substitutes $(0,-75)$ into $y = \alpha(x+1)^2(x-5)^2$ and attempts to find the value for α	dM1	3.1a
	$y = -3(x+1)^2(x-5)^2$ o.e.	A1	2.1
		(3)	
(c)	Substitutes $x = 2$ into their $y = -3(x+1)^2(x-5)^2 \Rightarrow y = (-243)$	M1	2.1
	$0 < k < 243$	A1ft	1.1b
		(2)	
			(7 marks)

Notes:

(a)

M1: Either

- $x \leq -1$ o.e. e.g. $-1 \geq x$
- $2 \leq x \leq 5$ o.e.

but condone use of strict inequalities anywhere for this mark.
e.g. $2 < x < 5$ or $2 < x \leq 5$ or $2 \leq x < 5$ May also write e.g. $x < 5$ and $x > 2$ which scores M1 but not "$x < 5$ or $x > 2$"

Allow interval notation such as e.g. $[2,5]$ or $(-\infty,-1]$ or condone e.g. $(2,5)$

Ignore incorrect inequality statements not related to the one which is valid.
e.g. "$2 \leqslant x < 5$ and $x > -1$" which scores M1 for the first inequality.

A1: Requires $\{\ \}$ and $\cup$

$\{x : x \leq -1\} \cup \{x : 2 \leq x \leq 5\}$ or $\{x \mid x \leq -1\} \cup \{x \mid 2 \leq x \leq 5\}$ either way round

but condone $\{x \leq -1\} \cup \{2 \leq x \leq 5\}$, $\{x \leq -1 \cup 2 \leq x \leq 5\}$.

Allow e.g. $\{x : x \leq -1\} \cup \{x : 2 \leq x \cap x \leq 5\}$

Use of $\cap$ to join the two separate regions is A0
It is acceptable (but not required) to mention $\mathbb{R}$
e.g. $\{x : x \in \mathbb{R}, x \leq -1\} \cup \{x : x \in \mathbb{R}, 2 \leq x \leq 5\}$

Condone use of a lower limit written as e.g. $\{x : -\infty \leq x \leq -1\} \cup \{x : 2 \leq x \leq 5\}$

A correct expression but missing e.g. $y = ...$ **or** $f(x) = ...$ **scores M1dM1A0**

M1: Forms the equation of the form $(y =)\ \alpha(x+1)^2(x-5)^2$. Condone $\alpha = 1$

Award for sight of $\alpha(x+1)^2(x-5)^2$ even with $\alpha = 1$ i.e. $(x+1)^2(x-5)^2$

dM1: Substitutes $(0,-75)$ into the form $y = \alpha(x+1)^2(x-5)^2$ and attempts to find the value for α. It is dependent on the previous method mark.

A1: $y = -3(x+1)^2(x-5)^2$ o.e. (e.g. $y = -3x^4 + 24x^3 - 18x^2 - 120x - 75$)

isw after a correct answer. Condone $f(x) = -3(x+1)^2(x-5)^2$ but not $C = -3(x+1)^2(x-5)^2$

A correct equation scores all 3 marks. Allow if seen in (c)

isw if they attempt to multiply out.

Alternative I part (b):

Using the form $y = ax^4 + bx^3 + cx^2 + dx + e$ **, then setting up and solving simultaneous equations.**

There are various versions of this but can be marked similarly.

M1: Sets *e* equal to -75 (may just be seen in their equation) **and** forms **three correct** different equations in *a*, *b*, *c* and *d* which may be unsimplified.

Note that the form $y = ax^4 + bx^3 + cx^2 + dx + e$ **is M0 until *e* is set equal to** -75

There are 5 equations that can be formed, only 3 are necessary for this mark.

Do not condone slips.

Using $(-1,0)$ $\Rightarrow 0 = a - b + c - d - 75$ o.e.

Using $(5,0)$ $\Rightarrow 0 = 625a + 125b + 25c + 5d - 75$ o.e.

Using $\frac{dy}{dx} = 0$ at $x = 2$ $\Rightarrow 0 = 32a + 12b + 4c + d$ o.e.

Using $\frac{dy}{dx} = 0$ at $x = -1$ $\Rightarrow 0 = -4a + 3b - 2c + d$ o.e.

Using $\frac{dy}{dx} = 0$ at $x = 5$ $\Rightarrow 0 = 500a + 75b + 10c + d$ o.e.

dM1: Forms **four correct** different equations and solves to find values for *a*, *b*, *c* and *d*. You do not need to be concerned by the process of solving. A calculator can be used to solve the equations.

A1: $y = -3x^4 + 24x^3 - 18x^2 - 120x - 75$ o.e. isw if they attempt to factorise but withhold this mark if they e.g. divide all terms by 3.

Condone $f(x) = ...$ but not $C = ...$

A correct equation scores all 3 marks. Allow if seen in (c)

Alternative II part (b): Uses the form $y = (x+1)(x-5)(ax^2+bx+c)$

M1: Substitutes $x=0,\ y=-75$ $-75=-5c \Rightarrow c=15$, multiplies out, differentiates

$\Rightarrow \dfrac{dy}{dx} = (2x-4)(ax^2+bx+15)+(x^2-4x-5)(2ax+b)$

and forms **a correct equation** in a and b which may be unsimplified.

Using $\dfrac{dy}{dx}=0$ at $x=2$ $\Rightarrow 0=4a+b$ o.e.

Using $\dfrac{dy}{dx}=0$ at $x=-1$ $\Rightarrow 0=a-b+15$ o.e.

Using $\dfrac{dy}{dx}=0$ at $x=5$ $\Rightarrow 0=5a+b+3=0$ o.e.

dM1: Forms **two correct** different equations and solves to find values for a and b. You do not need to be concerned by the process of solving. A calculator can be used to solve the equations.

A1: $y=(x+1)(x-5)(-3x^2+12x+15)$ o.e. isw if they attempt to multiply out or factorise

Condone $f(x)=...$ but not $C=...$ but withhold this mark if they e.g. divide all terms by 3. A correct equation scores all 3 marks. Allow if seen in (c)

Alternative III part (b): Uses $\dfrac{dy}{dx}=\beta(x+1)(x-2)(x-5)$ (β may be 1) and integrates.

M1: Integrates $\left(\dfrac{dy}{dx}=\right)\beta(x+1)(x-2)(x-5)$ to $(y=)\ \beta\left(\dfrac{1}{4}x^4-2x^3+\dfrac{3}{2}x^2+10x+k\right)$ and forms **one correct** equation using either $(0,-75)$: $-75=\beta k$ (allow $-75=k$)

$(-1,0)$: $0=\beta\left(\dfrac{1}{4}+2+\dfrac{3}{2}-10+k\right)$ $(5,0)$: $0=\beta\left(\dfrac{625}{4}-250+\dfrac{75}{2}+50+k\right)$

dM1: Forms a different equation using one of $(0,-75)$, $(-1,0)$, $(5,0)$ and solves to find values for β and k. You do not need to be concerned by the process of solving . A calculator can be used to solve the equations.

A1: $y=-12\left(\dfrac{1}{4}x^4-2x^3+\dfrac{3}{2}x^2+10x+\dfrac{25}{4}\right)$ o.e. isw if they attempt to multiply out or factorise

Condone $f(x)=...$ but not $C=...$ but withhold this mark if they e.g. divide all terms by 3. A correct equation scores all 3 marks. Allow if seen in (c)

(c)

M1: Substitutes $x=2$ into their $y=-3(x+1)^2(x-5)^2$ (must be a quartic in any form) and proceeds to find a value for y. Sight of their $\pm y$ (or ± 243) scores M1.

You may need to check this on your calculator if only a value is seen.

A1ft: $0<k<243$ o.e. ft on their negative y value at $x = 2$.

Allow use of set notation, interval notation and allow e.g. $k<243,\ k>0$ but do not allow OR or $\cup$. Do not accept $0\le k\le 243$ o.e.

If there are multiple attempts at describing the region, mark what appears to be their final answer.

This mark can only be scored if they have a negative quartic graph function

i.e. $\alpha<0$ for their $y=\alpha(x+1)^2(x-5)^2$ or $a<0$ for their $y=ax^4+bx^3+cx^2+dx+e$

5(a)	$2(3)^3-9(3)^2+5(3)+k=0\Rightarrow k=...$	**M1**	1.1b
	$54-81+15+k=0\Rightarrow k=12$* or $-12+k=0\Rightarrow k=12$*	**A1***	1.1b
		(2)	
	(a) Alternative by verification:		
	$2(3)^3-9(3)^2+5(3)+12=0$	**M1**	1.1b
	$54-81+15+12=0$ Hence $k = 12$ *	**A1***	1.1b
		(2)	
(b)	$\int\left(2x^3-9x^2+5x+12\right)dx...x^4\pm...x^3\pm...x^2\pm...x\pm...$	**M1**	3.1a
	$\frac{1}{2}(3)^4-3(3)^3+\frac{5}{2}(3)^2+12(3)+c=-10\Rightarrow c=...$	**dM1**	1.1b
	$(0,\ -28)$	**A1**	2.2a
		(3)	
			(5 marks)

Notes

(a) **Mark (a) and (b) together**

M1: Substitutes $x = 3$ completely into the given derivative, sets = 0 and solves for k.

e.g., $2(3)^3-9(3)^2+5(3)+k=0\Rightarrow k=...$

May be implied by e.g. $54-81+15+k=0\Rightarrow k=...$ with at least 2 correctly evaluated powers.

A1*: Obtains $k = 12$ with no errors seen and sufficient working shown. As a minimum you would need to see e.g., $2(3)^3-9(3)^2+5(3)+k=0\Rightarrow -12+k=0\Rightarrow k=12$ **or** $54-81+15+k=0\Rightarrow k=12$ **or** $2\times27-9\times9+5(3)+k=0\Rightarrow k=12$

But $2(3)^3-9(3)^2+5(3)+k=0\Rightarrow k=12$ scores M1A0 for lack of working

Note that some are just writing the expression for $\frac{dy}{dx}$, then write "sub in $x = 3$" but don't actually show 3 substituted in and then go on to write $-12+k=0$ leading to $k=12$ scores M0A0*.

Alternative:

M1: Substitutes $x = 3$ and $k = 12$ into the given derivative and attempts to evaluate

A1*: Correct work to obtain an answer of 0 with a (minimal) conclusion e.g., tick, hence proven etc.

As a minimum you would need to see e.g., $2(3)^3-9(3)^2+5(3)+12=54-81+15+12=0$ ✓

(b)

M1: Attempts to integrate. Evidence can be taken for integrating to obtain at least 2 from:

$2x^3\rightarrow...x^4$ or $-9x^2\rightarrow...x^3$ or $5x\rightarrow...x^2$ or $12\rightarrow...x$ where … are constants

dM1: Substitutes $x = 3$ into their integrated expression that includes a constant of integration, sets this equal to ±10 and proceeds to find their constant. **Depends on the previous mark.**

If the substitution is not shown this mark may be implied by their value for c or by their equation e.g., $18+c=\pm10$

A1: $(0, -28)$ Condone -28 or $y = -28$ but not just $c = -28$. There must be no other values or points.

Condone $(-28,\ 0)$ following $y = -28$

<u>Beware of circular arguments which avoid doing part (a) e.g.</u>

Integration is used on the given derivative to give y in terms of x, k and c

$(3, -10)$ is substituted to give $3k + c = 8$

Part (b) is then done first using $k = 12$ to find $c = -28$

This is then substituted into $3k + c = 8$ to give $k = 12$
This scores (a) M0A0 (b) M1dM1A1 (if -28 is identified as the intercept)

Alternative for part (a) using algebraic division:

$$
\begin{array}{r}
2x^2 - 3x \;\; - 4 \\
x-3\overline{)\,2x^3 - 9x^2 + 5x + k} \\
\underline{2x^3 - 6x^2} \\
\quad -3x^2 + 5x \\
\quad \underline{-3x^2 + 9x} \\
\qquad -4x + k \\
\qquad \underline{-4x + 12} \\
\qquad\quad k - 12 \text{ (or } 0)
\end{array}
$$

leading to $k - 12 = 0$ and then $k = 12$.

M1: Attempts to divide the given cubic by $(x - 3)$ and proceeds as far as a remainder set = 0. Requires at least $2x^2 \pm 3x$.

A1*: Obtains $k = 12$ with no errors seen and sufficient working. Their algebraic division needs to be correct but allow them to have either $k - 12$ or 0 as their "remainder". If their remainder is given in their working as 0 they may proceed directly to $k = 12$.

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
6 (a)	$2 < x < 6$	B1	1.1b
		(1)	
(b)	States either $k > 8$ or $k < 0$	M1	3.1a
	States e.g. $\{k : k > 8\} \cup \{k : k < 0\}$	A1	2.5
		(2)	
(c)	**Please see notes for alternatives**		
	States $y = ax(x-6)^2$ or $f(x) = ax(x-6)^2$	M1	1.1b
	Substitutes $(2,8)$ into $y = ax(x-6)^2$ and attempts to find a	dM1	3.1a
	$y = \frac{1}{4}x(x-6)^2$ or $f(x) = \frac{1}{4}x(x-6)^2$ o.e	A1	2.1
		(3)	
			(6 marks)
Notes: Watch for answers written by the question. If they are beside the question and in the answer space, the one in the answer space takes precedence			

(a)

B1: Deduces $2 < x < 6$ o.e. such as $x > 2, x < 6$ $x > 2$ and $x < 6$ $\{x : x > 2\} \cap \{x : x < 6\}$ $x \in (2,6)$

Condone attempts in which set notation is incorrectly attempted but correct values can be seen or implied E.g. $\{x > 2\} \cap \{x < 6\}$ $\{x > 2, x < 6\}$. Allow just the open interval $(2,6)$

Do not allow for incorrect inequalities such as e.g. $x > 2$ or $x < 6$, $\{x : x > 2\} \cup \{x : x < 6\}$ $x \in [2,6]$

(b)

M1: Establishes a correct method by finding one of the (correct) inequalities

States either $k > 8$ (condone $k \geqslant 8$) or $k < 0$ (condone $k \leqslant 0$)

Condone for this mark $y \leftrightarrow k$ or $f(x) \leftrightarrow k$ and $8 < k < 0$

A1: Fully correct solution in the form $\{k : k > 8\} \cup \{k : k < 0\}$ or $\{k | k > 8\} \cup \{k | k < 0\}$ either way around but condone $\{k < 0\} \cup \{k > 8\}$, $\{k : k < 0 \cup k > 8\}$, $\{k < 0 \cup k > 8\}$. It is not necessary to mention $\mathbb{R}$, e.g. $\{k : k \in \mathbb{R}, k > 8\} \cup \{k : k \in \mathbb{R}, k < 0\}$ Look for $\{\ \}$ and $\cup$

Do not allow solutions not in set notation such as $k < 0$ or $k > 8$.

(c)

M1: Realises that the equation of C is of the form $y = ax(x-6)^2$. Condone with $a = 1$ for this mark.

So award for sight of $ax(x-6)^2$ even with $a = 1$

dM1: Substitutes $(2,8)$ into the form $y = ax(x-6)^2$ and attempts to find the value for a.

It is dependent upon having an equation, which the (y =..) may be implied, of the correct form.

A1: Uses all of the information to form a correct **equation** for C $y = \frac{1}{4}x(x-6)^2$ o.e.

ISW after a correct answer. Condone $f(x) = \frac{1}{4}x(x-6)^2$ but not $C = \frac{1}{4}x(x-6)^2$.

Allow this to be written down for all 3 marks

Examples of alternative methods

.....

Alternative I part (c):

Using the form $y = ax^3 + bx^2 + cx$ and setting up then solving simultaneous equations.

There are various versions of this but can be marked similarly

M1: Realises that the equation of *C* is of the form $y = ax^3 + bx^2 + cx$ and forms two equations in *a*, *b* and *c*. Condone with $a = 1$ for this mark.

Note that the form $y = ax^3 + bx^2 + cx + d$ is M0 until *d* is set equal to 0.

There are four equations that could be formed, only two are necessary for this mark.

Condone slips

Using (6, 0) $\Rightarrow 216a+36b+6c=0$

Using (2, 8) $\Rightarrow 8a+4b+2c=8$

Using $\frac{dy}{dx}=0$ at $x = 2$ $\Rightarrow 12a+4b+c=0$

Using $\frac{dy}{dx}=0$ at $x = 6 \Rightarrow 108a+12b+c=0$

dM1: Forms and solves three different equations, one of which must be using (2, 8) to find values for *a*, *b* and *c*. A calculator can be used to solve the equations

A1: Uses all of the information to form a correct equation for *C* $y = \frac{1}{4}x^3 - 3x^2 + 9x$ o.e.

ISW after a correct answer. Condone $f(x) = \frac{1}{4}x^3 - 3x^2 + 9x$

.....

Alternative II part (c)

Using the gradient and integrating

M1: Realises that the gradient of *C* is zero at 2 and 6 so sets $f'(x) = k(x-2)(x-6)$ oe **and** attempts to integrate. Condone with $k = 1$

dM1: Substitutes $x = 2, y = 8$ into $f(x) = k\left(...x^3 + ...x + ...\right)$ and finds a value for *k*

A1: Uses all of the information to form a correct equation for *C* $y = \frac{3}{4}\left(\frac{1}{3}x^3 - 4x^2 + 12x\right)$ o.e.

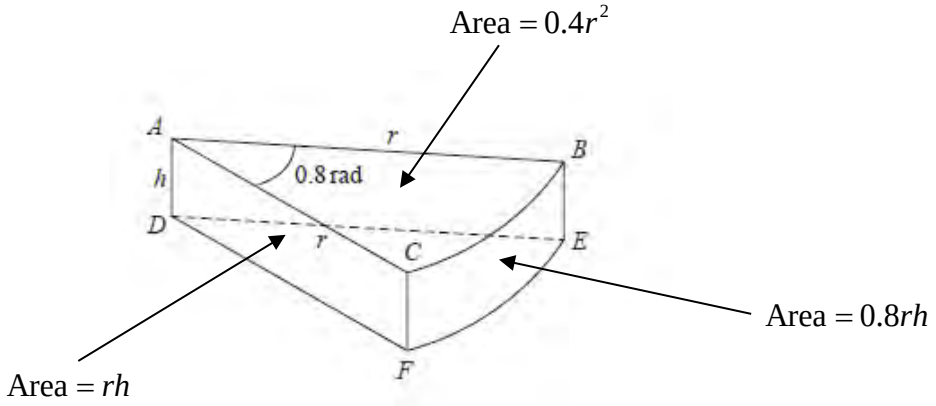
ISW after a correct answer. Condone $f(x) = \frac{3}{4}\left(\frac{1}{3}x^3 - 4x^2 + 12x\right)$

.....

Question	Scheme	Marks	AOs

15 (a)	Sets up an allowable equation using volume = 240 E.g. $\frac{1}{2}r^2 \times 0.8h = 240 \Rightarrow h = \frac{600}{r^2}$ o.e.	M1 A1	3.4 1.1b
	Attempts to substitute their $h = \frac{600}{r^2}$ into $(S =)\frac{1}{2}r^2 \times 0.8 + \frac{1}{2}r^2 \times 0.8 + 2rh + 0.8rh$	dM1	3.4
	$S = 0.8r^2 + 2.8rh = 0.8r^2 + 2.8 \times \frac{600}{r} = 0.8r^2 + \frac{1680}{r}$ *	A1*	2.1
		(4)	
(b)	$\left(\frac{dS}{dr}\right) = 1.6r - \frac{1680}{r^2}$	M1 A1	3.1a 1.1b
	Sets $\frac{dS}{dr} = 0 \Rightarrow r^3 = 1050$ $r =$ awrt 10.2	dM1 A1	2.1 1.1b
		(4)	
(c)	Attempts to substitute their positive r into $\left(\frac{d^2S}{dr^2}\right) = 1.6 + \frac{3360}{r^3}$ and considers its value or sign	M1	1.1b
	E.g. Correct $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3}$ with $\frac{d^2S}{dr^2}_{r=10.2} = 5 > 0$ proving a minimum value of S	A1	1.1b
		(2)	
			(10 marks)
Notes:			

Volume $= 0.4r^2h$



Total surface area = $2rh + 0.8r^2 + 0.8rh$

(a)

Sight of $\frac{1}{2}r^2\times 0.8\times h=240$ leading to $h=...$ or $rh=...$ scores this mark

But condone an equation of the correct form so allow for $kr^2h=240\Rightarrow h=...$ or $rh=...$

A1: A correct expression for $h=\frac{600}{r^2}$ or $rh=\frac{600}{r}$ which may be left unsimplified.

This may be implied when you see an expression for S or part of S E.g $2rh=2r\times\frac{600}{r^2}$

dM1: Attempts to substitute their $h=\frac{a}{r^2}$ o.e. such as $hr=\frac{a}{r}$ into a **correct** expression for S

Sight of $\frac{1}{2}r^2\times 0.8+\frac{1}{2}r^2\times 0.8+rh+rh+0.8rh$ with an appropriate substitution

Simplified versions such as $0.8r^2+2rh+0.8rh$ used with an appropriate substitution is fine.

A1*: Correct work leading to the given result.

$S=$, $SA=$ or surface area = must be seen at least once in the correct place

The method must be made clear so expect to see evidence. For example

$S=0.8r^2+2rh+0.8rh\Rightarrow S=0.8r^2+2r\times\frac{600}{r^2}+0.8r\times\frac{600}{r^2}\Rightarrow S=0.8r^2+\frac{1680}{r}$ would be fine.

(b) There is no requirement to see $\frac{dS}{dr}$ in part (b). It may even be called $\frac{dy}{dx}$.

M1: Achieves a derivative of the form $pr\pm\frac{q}{r^2}$ where p and q are non- zero constants

A1: Achieves $\left(\frac{dS}{dr}\right)=1.6r-\frac{1680}{r^2}$

dM1: Sets or implies that their $\frac{dS}{dr}=0$ and proceeds to $mr^3=n,\quad m\times n>0$. It is dependent upon a correct attempt at differentiation. This mark may be implied by a correct answer to their $pr-\frac{q}{r^2}=0$

A1: $r=$ awrt 10.2 or $\sqrt[3]{1050}$

(c)

M1: Attempts to substitute their positive r (found in (b)) into $\left(\frac{d^2S}{dr^2}=\right)e\pm\frac{f}{r^3}$ where e and f are non zero and finds its value or sign.

Alternatively considers the sign of $\left(\frac{d^2S}{dr^2}=\right)e\pm\frac{f}{r^3}$ (at their positive r found in (b))

Condone the $\frac{d^2S}{dr^2}$ to be $\frac{d^2y}{dx^2}$ or being absent, but only for this mark.

A1: States that $\frac{d^2S}{dr^2}$ or $S''=1.6+\frac{3360}{r^3}=$ awrt $5>0$ proving a minimum value of S

This is dependent upon having achieved $r=$ awrt 10 and a correct $\frac{d^2S}{dr^2}=1.6+\frac{3360}{r^3}$

It can be argued without finding the value of $\frac{d^2S}{dr^2}$. E.g. $\frac{d^2S}{dr^2}=1.6+\frac{3360}{r^3}>0$ as $r>0$, so minimum value of S. For consistency it is also dependent upon having achieved $r=$ awrt 10

Do **NOT** allow $\frac{d^2y}{dx^2}$ for this mark

Question	Scheme	Marks	AOs

12(a)	$f(x)=\dfrac{e^{3x}}{4x^2+k} \Rightarrow f'(x)=\dfrac{\left(4x^2+k\right)3e^{3x}-8xe^{3x}}{\left(4x^2+k\right)^2}$ or $f(x)=e^{3x}\left(4x^2+k\right)^{-1} \Rightarrow f'(x)=3e^{3x}\left(4x^2+k\right)^{-1}-8xe^{3x}\left(4x^2+k\right)^{-2}$	M1 A1	1.1b 1.1b
	$f'(x)=\dfrac{\left(12x^2-8x+3k\right)e^{3x}}{\left(4x^2+k\right)^2}$	A1	2.1
		(3)	
(b)	If $y=f(x)$ has at least one stationary point then $12x^2-8x+3k=0$ has at least one root	B1	2.2a
	Applies $b^2-4ac\,(\geqslant)\,0$ with $a=12, b=-8, c=3k$	M1	2.1
	$0<k\leqslant\dfrac{4}{9}$	A1	1.1b
		(3)	
			(6 marks)

Notes:

(a)

M1: Attempts the quotient rule to obtain an expression of the form $\dfrac{\alpha\left(4x^2+k\right)e^{3x}-\beta xe^{3x}}{\left(4x^2+k\right)^2}$, $\alpha\beta>0$

condoning bracketing errors/omissions as long as the intention is clear.
If the quotient rule formula is quoted it must be correct.

Condone e.g. $f'(x)=\dfrac{\left(4x^2+k\right)3e^{3x}-8xe^{3x}}{\left(4x^2+k\right)}$ provided an incorrect formula is not quoted.

May also see product rule applied to $e^{3x}\left(4x^2+k\right)^{-1}$ to obtain an expression of the form

$\alpha e^{3x}\left(4x^2+k\right)^{-1}+\beta xe^{3x}\left(4x^2+k\right)^{-2}$ $\alpha,\beta\neq0$ condoning bracketing errors/omissions as long as the intention is clear. If the product rule formula is quoted it must be correct.

A1: Correct differentiation in any form with correct bracketing which may be implied by subsequent work.

A1: Obtains $f'(x)=\left(12x^2-8x+3k\right)g(x)$ where $g(x)=\dfrac{e^{3x}}{\left(4x^2+k\right)^2}$ **or equivalent**

e.g. $g(x)=e^{3x}\left(4x^2+k\right)^{-2}$

Allow recovery from "invisible" brackets earlier and apply isw here once a correct answer is seen.
Note that the complete form of the answer is not given so allow candidates to go from e.g.

$\dfrac{\left(4x^2+k\right)3e^{3x}-8xe^{3x}}{\left(4x^2+k\right)^2}$ or $3e^{3x}\left(4x^2+k\right)^{-1}-8xe^{3x}\left(4x^2+k\right)^{-2}$ to $\dfrac{\left(12x^2-8x+3k\right)e^{3x}}{\left(4x^2+k\right)^2}$ for the final mark.

(b) Note that B0M1A1 is not possible in (b)

B1: Deduces that if $y = \mathrm{f}(x)$ has at least one stationary point then $12x^2 - 8x + 3k = 0$ has at least one root. There is no requirement to formally state $\dfrac{\mathrm{e}^{3x}}{\left(4x^2 + k\right)^2} > 0$

This may be implied by an attempt at $b^2 - 4ac \geqslant 0$ or $b^2 - 4ac > 0$ condoning slips.

M1: Attempts $b^2 - 4ac \ldots 0$ with $a = 12,\ b = -8,\ c = 3k$ where … is e.g. "=", <, >, etc.

Alternatively attempts to complete the square and sets rhs ...0

E.g. $12x^2 - 8x + 3k = 0 \Rightarrow x^2 - \dfrac{2}{3}x + \dfrac{1}{4}k = 0 \Rightarrow \left(x - \dfrac{1}{3}\right)^2 = \dfrac{1}{9} - \dfrac{1}{4}k$ leading to $\dfrac{1}{9} - \dfrac{1}{4}k \geqslant 0$

A1: $0 < k \leqslant \dfrac{4}{9}$ but condone $k \leqslant \dfrac{4}{9}$ and condone $0 \leqslant k \leqslant \dfrac{4}{9}$

Must be in terms of *k* not *x* so do not allow e.g. $0 < x \leqslant \dfrac{4}{9}$ but condone $\left(0,\ \dfrac{4}{9}\right]$ or $\left[0,\ \dfrac{4}{9}\right]$

Question	Scheme	Marks	AOs

2(a)	$f(x)=(x-2)^2\pm...$	M1	1.2
	$f(x)=(x-2)^2+1$	A1	1.1b
		(2)	
(b)(i)	$P=(0,\ 5)$	B1	1.1b
(b)(ii)	$Q=(2,\ 1)$	B1ft	1.1b
		(2)	
			(4 marks)
Notes			

(a)

M1: Achieves $(x-2)^2\pm...$ or states $a = -2$

A1: Correct expression $(x-2)^2+1$ ISW after sight of this

Condone $a = -2$ and $b = 1$. Condone $(x-2)^2+1=0$

(b)

(i) B1: Correct coordinates for P. Allow to be expressed $x=0,\ y=5$

(ii) B1ft: Correct coordinates for Q. Allow to be expressed $x=2,\ y=1$ (Score for the correct answer or follow through their part (a) so allow $(-a, b)$ where a and b are numeric)

Score in any order if they state $P=(0,\ 5)$ and $Q=(2,\ 1)$

.....

Allow part (b) to be awarded from a sketch. So award

First B1 from a sketch crossing the y-axis at 5

Second B1 from a sketch with minimum at $(2,\ 1)$

.....

Question	Scheme	Marks	AOs

5(a)(i)	$\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$	M1 A1	1.1b 1.1b
(ii)	$\frac{d^2y}{dx^2} = 60x^2 - 144x + 84$	A1ft	1.1b
		(3)	
(b)(i)	$x = 1 \Rightarrow \frac{dy}{dx} = 20 - 72 + 84 - 32$	M1	1.1b
	$\frac{dy}{dx} = 0$ so there is a stationary point at $x = 1$	A1	2.1
	Alternative for (b)(i)		
	$20x^3 - 72x^2 + 84x - 32 = 4(x-1)^2(5x-8) = 0 \Rightarrow x = ...$	M1	1.1b
	When $x = 1$, $\frac{dy}{dx} = 0$ so there is a stationary point	A1	2.1
(b)(ii)	**Note that in (b)(ii) there are no marks for <u>just</u> evaluating** $\left(\frac{d^2y}{dx^2}\right)_{x=1}$		
	E.g. $\left(\frac{d^2y}{dx^2}\right)_{x=0.8} = ...$ $\left(\frac{d^2y}{dx^2}\right)_{x=1.2} = ...$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=0.8} > 0$, $\left(\frac{d^2y}{dx^2}\right)_{x=1.2} < 0$ Hence point of inflection	A1	2.2a
		(4)	
	Alternative 1 for (b)(ii)		
	$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 60x^2 - 144x + 84 = 0$ (is inconclusive) $\left(\frac{d^3y}{dx^3}\right) = 120x - 144 \Rightarrow \left(\frac{d^3y}{dx^3}\right)_{x=1} = ...$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 0$ and $\left(\frac{d^3y}{dx^3}\right)_{x=1} \neq 0$ Hence point of inflection	A1	2.2a
	Alternative 2 for (b)(ii)		
	E.g. $\left(\frac{dy}{dx}\right)_{x=0.8} = ...$ $\left(\frac{dy}{dx}\right)_{x=1.2} = ...$	M1	2.1
	$\left(\frac{dy}{dx}\right)_{x=0.8} < 0$, $\left(\frac{dy}{dx}\right)_{x=1.2} < 0$ Hence point of inflection	A1	2.2a
			(7 marks)

Notes

(a)(i)

M1: $x^n \rightarrow x^{n-1}$ for at least one power of x

A1: $\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$

(a)(ii)

A1ft: Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$

(b)(i)

M1: Substitutes $x = 1$ into their $\frac{dy}{dx}$

A1: Obtains $\frac{dy}{dx} = 0$ following a correct derivative and makes a conclusion which can be minimal e.g. tick, QED etc. which may be in a preamble e.g. stationary point when $\frac{dy}{dx} = 0$ and then shows $\frac{dy}{dx} = 0$

Alternative:

M1: Attempts to solve $\frac{dy}{dx} = 0$ by factorisation. This may be by using the factor of $(x - 1)$ or possibly using a calculator to find the roots and showing the factorisation. Note that they may divide by 4 before factorising which is acceptable. Need to either see either $4(x-1)^2(5x-8)$ or $(x-1)^2(5x-8)$ for the factorisation or $x = \frac{8}{5}$ and $x = 1$ seen as the roots.

A1: Obtains $x = 1$ and makes a conclusion as above

(b)(ii)

M1: Considers the value of the second derivative either side of $x = 1$. Do not be too concerned with the interval for the method mark.

(NB $\frac{d^2y}{dx^2} = (x-1)(60x-84)$ so may use this factorised form when considering $x < 1$, $x > 1$ for sign change of second derivative)

A1: Fully correct work including a correct $\frac{d^2y}{dx^2}$ with a reasoned conclusion indicating that the stationary point is a point of inflection. Sufficient reason is e.g. "sign change"/ "> 0, < 0". If values are given they should be correct (but be generous with accuracy) but also just allow "> 0" and "< 0" provided they are correctly paired. The interval must be where $x < 1.4$

Alternative 1 for (b)(ii)

M1: Shows that second derivative at $x = 1$ is zero and **then finds the third derivative at $x = 1$**

A1: Fully correct work including a correct $\frac{d^2y}{dx^2}$ with a reasoned conclusion indicating that stationary point is a point of inflection. Sufficient reason is "$\neq 0$" but must follow a correct third derivative and a correct value if evaluated. For reference $\left(\frac{d^3y}{dx^3}\right)_{x=1} = -24$

Alternative 2 for (b)(ii)

M1: Considers the value of the first derivative either side of $x = 1$. Do not be too concerned with the interval for the method mark.

A1: Fully correct work with a reasoned conclusion indicating that stationary point is a point of inflection. Sufficient reason is e.g. "same sign"/"both negative"/"< 0, < 0". If values are given they should be correct (but be generous with accuracy). The interval must be where $x < 1.4$

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
f′(x)	-32	-24.3	-17.92	-12.74	-8.64	-5.5	-3.2	-1.62	-0.64	-0.14	0
f″(x)	84	70.2	57.6	46.2	36	27	19.2	12.6	7.2	3	0

x	1.1	1.2	1.3	1.4	1.5	1.6	1.7
f′(x)	-0.1	-0.32	-0.54	-0.64	-0.5	0	0.98
f″(x)	-1.8	-2.4	-1.8	0	3	7.2	12.6

2(a)	(i) $\frac{dy}{dx} = 2x - 2 - 12x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	(ii) $\frac{d^2y}{dx^2} = 2 + 6x^{-\frac{3}{2}}$	B1ft	1.1b
		(3)	
(b)	Substitutes $x = 4$ into their $\frac{dy}{dx} = 2\times4 - 2 - 12\times4^{-\frac{1}{2}} = ...$	M1	1.1b
	Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point" oe	A1	2.1
		(2)	
(c)	Substitutes $x = 4$ into their $\frac{d^2y}{dx^2} = 2 + 6\times4^{-\frac{3}{2}} = (2.75)$	M1	1.1b
	$\frac{d^2y}{dx^2} = 2.75 > 0$ and states "hence minimum"	A1ft	2.2a
		(2)	
			(7 marks)

(a)(i)

M1: Differentiates to $\frac{dy}{dx} = Ax + B + Cx^{-\frac{1}{2}}$ **A1:** $\frac{dy}{dx} = 2x - 2 - 12x^{-\frac{1}{2}}$ (Coefficients may be unsimplified)

(a)(ii)

B1ft: Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx}$ (Their $\frac{dy}{dx}$ must have a negative or fractional index)

(b)

M1: Substitutes $x = 4$ into their $\frac{dy}{dx}$ and attempts to evaluate. There must be evidence $\left.\frac{dy}{dx}\right|_{x=4} = ...$

Alternatively substitutes $x = 4$ into an equation resulting from $\frac{dy}{dx} = 0$ Eg. $\frac{36}{x} = (x-1)^2$ and equates

A1: There must be a reason and a minimal conclusion. Allow ✓ , QED for a minimal conclusion

Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point" oe

Alt Shows that $x = 4$ is a root of the resulting equation and states "hence there is a stationary point"

All aspects of the proof must be correct including a conclusion

(c)

M1: Substitutes $x = 4$ into their $\frac{d^2y}{dx^2}$ and calculates its value, or implies its sign by a statement such as

when $x = 4 \Rightarrow \frac{d^2y}{dx^2} > 0$. This must be seen in (c) and not labelled (b). Alternatively calculates the gradient of C either side of $x = 4$ or calculates the value of y either side of $x = 4$.

A1ft: For **a correct calculation**, **a valid reason** and **a correct conclusion**. Ignore additional work where candidate finds $\frac{d^2y}{dx^2}$ left and right of $x = 4$. Follow through on an incorrect $\frac{d^2y}{dx^2}$ but it is dependent upon having a negative or fractional index. Ignore any references to the word convex. The nature of the turning point is "minimum".

Using the gradient look for correct calculations, a valid reason…. goes from negative to positive, and a correct conclusion …minimum.

Question	Scheme	Marks	AOs