adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Stationary points

Question paper

8 questions

Exam questions and marking instructions remain credited to their board.

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6 October 2021 - Paper 1 - Q2

7 October 2021 - Paper 2 - Q5

8 June 2018 - Paper 1 - Q2

7.

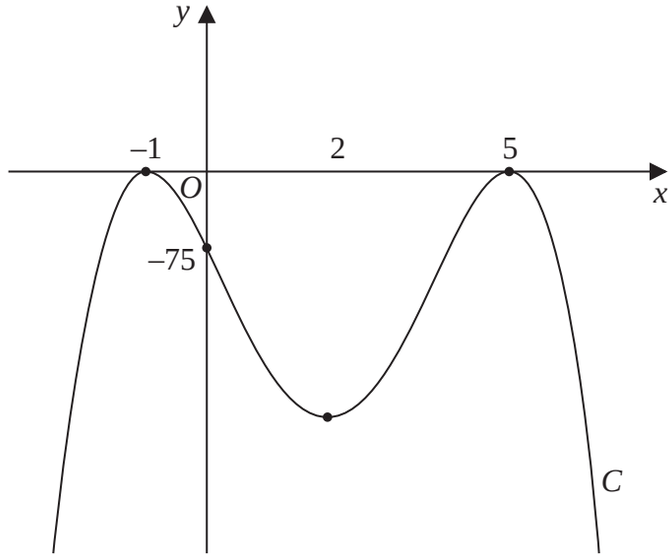


Figure 1

Figure 1 shows a sketch of a curve C with equation $y = f(x)$, where $f(x)$ is a quartic expression in x.

The curve

- has maximum turning points at $(-1, 0)$ and $(5, 0)$
- crosses the y-axis at $(0, -75)$
- has a minimum turning point at $x = 2$

(a) Find the set of values of x for which

$$f'(x) \geqslant 0$$

writing your answer in set notation.

(2)

(b) Find the equation of C. You may leave your answer in factorised form.

(3)

The curve C_1 has equation $y = f(x) + k$, where k is a constant.

Given that the graph of C_1 intersects the x-axis at exactly four places,

(c) find the range of possible values for k.

(2)

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 7 marks)

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5. The curve C has equation $y = f(x)$

The curve

- passes through the point $P(3, -10)$
- has a turning point at P

Given that

$$\frac{dy}{dx} = 2x^3 - 9x^2 + 5x + k$$

where k is a constant,

(a) show that $k = 12$

(2)

(b) Hence find the coordinates of the point where C crosses the y-axis.

(3)

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Question 5 continued

(Total for Question 5 is 5 marks)

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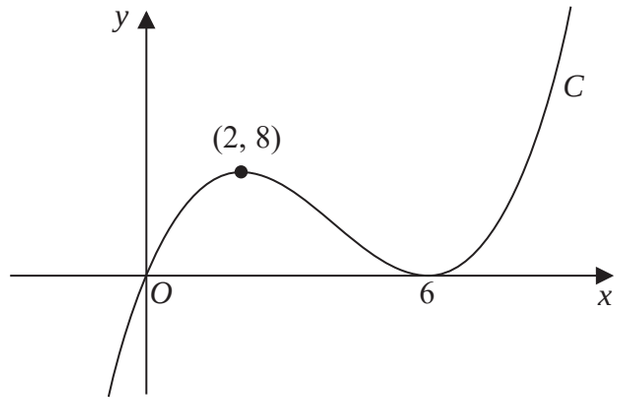


Figure 1

Figure 1 shows a sketch of a curve C with equation $y = f(x)$ where $f(x)$ is a cubic expression in x.

The curve

- passes through the origin
- has a maximum turning point at $(2, 8)$
- has a minimum turning point at $(6, 0)$

(a) Write down the set of values of x for which

$$f'(x) < 0$$

(1)

The line with equation $y = k$, where k is a constant, intersects C at only one point.

(b) Find the set of values of k, giving your answer in set notation.

(2)

(c) Find the equation of C. You may leave your answer in factorised form.

(3)

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Question 6 continued

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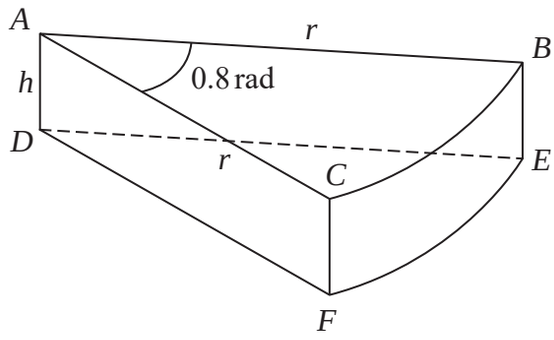


Figure 5

A company makes toys for children.

Figure 5 shows the design for a solid toy that looks like a piece of cheese.

The toy is modelled so that

- face ABC is a sector of a circle with radius r cm and centre A
- angle $BAC = 0.8$ radians
- faces ABC and DEF are congruent
- edges AD, CF and BE are perpendicular to faces ABC and DEF
- edges AD, CF and BE have length h cm

Given that the volume of the toy is $240\,\text{cm}^3$

(a) show that the surface area of the toy, $S\,\text{cm}^2$, is given by

$$S = 0.8r^2 + \frac{1680}{r}$$

making your method clear.

(4)

Using algebraic differentiation,

(b) find the value of r for which S has a stationary point.

(4)

(c) Prove, by further differentiation, that this value of r gives the minimum surface area of the toy.

(2)

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Question 15 continued

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Question 15 continued

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12. The function f is defined by

$$f(x) = \frac{e^{3x}}{4x^2 + k}$$

where k is a positive constant.

(a) Show that

$$f'(x) = (12x^2 - 8x + 3k)\,g(x)$$

where $g(x)$ is a function to be found.

(3)

Given that the curve with equation $y = f(x)$ has at least one stationary point,

(b) find the range of possible values of k.

(3)

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Question 12 continued

(Total for Question 12 is 6 marks)

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2. Given that

$$f(x) = x^2 - 4x + 5 \qquad x \in \mathbb{R}$$

(a) express $f(x)$ in the form $(x + a)^2 + b$ where a and b are integers to be found.

(2)

The curve with equation $y = f(x)$

- meets the y-axis at the point P
- has a minimum turning point at the point Q

(b) Write down

(i) the coordinates of P

(ii) the coordinates of Q

(2)

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Question 2 continued

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5. The curve C has equation

$$y = 5x^4 - 24x^3 + 42x^2 - 32x + 11 \qquad x \in \mathbb{R}$$

(a) Find

(i) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(ii) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$

(3)

(b) (i) Verify that C has a stationary point at $x = 1$

(ii) Show that this stationary point is a point of inflection, giving reasons for your answer.

(4)

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Question 5 continued

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2. A curve C has equation

$$y = x^2 - 2x - 24\sqrt{x}, \qquad x > 0$$

(a) Find (i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 4$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

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Question 2 continued

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