adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Statistical distributions

Mark scheme

8 questions

Exam questions and marking instructions remain credited to their board.

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Question	Scheme	Marks	AOs
6(a)	Use sum of probs = 1: $1=\frac{1}{k}(2+4+...+2n)$ Allow $1=\frac{S_r}{k}$ where $S_r=\sum_{r=1}^{n}2r$	M1	3.1a
	Sum e.g. $\frac{n}{2}[4+2(n-1)]$ or $\frac{n}{2}[2+2n]$ or use $\sum_{1}^{n}r=\frac{n(n+1)}{2}$	M1	2.1
	Use sum formula: e.g. $1=\frac{1}{k}\times\frac{n}{2}[4+2(n-1)]$ or $1=\frac{n}{2}\left[\frac{2}{k}+\frac{2n}{k}\right]$	A1	1.1b
	$k = n(n + 1)$ *	A1*cso	1.1b
		(4)	
6(b)	Cases: R = 16, 18, 20, 22 and 24	M1	3.4
	Probability = $\left[\frac{16+18+20+22+24}{20\times21}\right]=\frac{5}{21}$	A1	1.1b
		(2)	
6(c)	[No real roots $\Rightarrow\Delta<0$] so $g^2-4(3g-5)[<0]$	M1	3.1a
	$[g^2-12g+20<0\Rightarrow]$ $(g-10)(g-2)[<0]$	M1	2.1
	So require $2 < g < 10$ or $4\le g\le 8$	M1	1.1b
	So require g = 4, 6, 8 only	dM1	1.1b
	Probability = $\left[\frac{4+6+8}{20\times21}\right]=\frac{18}{420}=\frac{3}{70}$	A1	3.2a
		(5)	
			(11 marks)

Notes:

(a)	M1	for clear attempt/intention to use sum of probs = 1 for n terms. Must see k and "= 1" Condone missing … $\sum_{2}^{2n}r$ is M0 since not n terms
	M1	for using an arithmetic series or Σr to find sum to n terms. No need for k
	A1	(dep on M1M1) for a correct working leading to a correct equation in k and n
	A1*	cso for correct solution with both Ms clearly scored and no incorrect working seen
	SC	Start with $1=\frac{n}{2}\left[\frac{2}{k}+\frac{2n}{k}\right]$ or $1=\frac{1}{k}\times\frac{n}{2}[4+2(n-1)]$ and no mention of a and l or a and d Score **SC B2** (M1M0A1A0) 1st B1 for the start and 2nd B1cso for completing to printed answer.
(b)	M1	for identifying the correct values of R
	A1	for a correct probability (any exact equivalent) Allow $0.\dot{2}38095\dot{2}$ Correct answer with no working scores 2 marks
(c)	M1	for attempt to use discriminant [< 0 not needed here]. Condone g^2-12g
	M1	for attempting to factorise or find critical values for their 3TQ (ignore their < & allow = 0)
	M1	for choosing the correct (inside) region, ft their critical values from a 3TQ with < 0
	dM1	(dep on 3rd M1) for choosing/identifying the appropriate values of g (ft their critical values from 3TQ with < 0 used)
	A1	for an exact probability in any form. Allow $0.0\dot{4}28571\dot{4}$ A correct answer with no incorrect working will score full marks
Acc	**If scored A0 for an answer of awrt 0.238 in (b), allow A1 in (c) for awrt 0.0429**	

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Qu 1	Scheme	Marks	AO
(a)	X~B(10, $\frac{1}{6}$) [Allow 0.167 or better for $\frac{1}{6}$]	M1	3.3
(i)	[P(X = 3) =] 0.155045… **awrt <u>0.155</u>**	A1	1.1b
(ii)	[P(X < 3) = P(X „ 2) =] 0.775226… **awrt <u>0.775</u>**	A1	1.1b
		(3)	
(b)	[Let D = no. of days when X = 3] D~B(60, "0.155")	M1	3.3
	P(D …12) = 1 – P(D „ 11) [Allow 1 – P(D < 12)]	M1	3.4
	= 1 – 0.78819… **awrt <u>0.212</u>**	A1	1.1b
		(3)	
(c)	[n = 600, p = $\frac{1}{6}$] estimate = **<u>100</u>**	B1	3.4
		(1)	
(d)	[S = total no. of sixes over 60 days.] $S \approx T \sim N\left("100", \sqrt{\frac{5}{6}\times 100}^{2}\right)$	M1A1	3.3,1.1b
	P(S > 95) ≈ P([T >]95.5) <u>or</u> $P\left([Z>]\frac{95.5-"100"}{"9.128..."}\right)$ <u>or</u> P([Z >] – 0.49..)	M1	3.4
	= 0.688976… **awrt <u>0.689</u>**	A1	1.1b
		(4)	
		(11 marks)	

	Notes
	If you see any attempt using an *n*-sided die with *n* not equal to 6 please send to review.
(a)	M1 for sight or use of the correct distribution. <u>Must</u> have B, or Bin or Bpd or Bcd **and** the correct value for n and p, just n = 10, p = $\frac{1}{6}$ is M0
	Implied by one answer correct to 2dp <u>or</u> by sight of $\binom{10}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^7$ or one of :
	[P(X = 0) =] **0.16**(1…),[P(X = 1) =] **0.32**(3…), [P(X = 2) =] **0.29**(0…), [P(X „ 3) =] **0.93**(0…)
(i)	1st A1 for awrt 0.155
(ii)	2nd A1 for awrt 0.775
(b)	1st M1 for selecting a correct model. Sight or use of correct binomial, ft their (a)(i)
	May be implied by sight of [P(D „ 11) =] 0.78… or 0.79 <u>or</u> [P(D „ 12) =] 0.87…
	2nd M1 for correct interpretation of "at least 12" and writing or using 1 – P(D „ 11)
	We are <u>not</u> attempting to ft their incorrect 0.155 on our calculators here.
	A1 for awrt 0.212 [Answer only 3/3]
(c)	B1 for 100 but must be seen in part (c) i.e. between (b) and (d)
(d)	1st M1 for attempting normal **with mean = 100** <u>or</u> ft their answer to (c)
	May be implied by the correct mean and a correctly labelled s.d. (σ) or var (σ^2)
	1st A1 for correctly labelled standard deviation allow $\sqrt{\frac{250}{3}} = \sqrt{83.3...}$ = 9.1(28….) <u>or</u>
	correctly labelled variance. Implied by N($\mu, \frac{250}{3}$) <u>or</u> correct answer.
	2nd M1 for attempt at continuity correction i.e. sight of 95 <u>+</u> 0.5
	2nd A1 for awrt 0.689 [Answer only 4/4]
NB	**If they don't state the model for 1st M1 but just give probabilities with probability statements (*Y* is any letter):**
$\sigma = \frac{250}{3} =$	1st M1 implied by: P(Y > 94.5) = 0.52(63…), P(Y > 95) = 0.52(39…), P(Y > 95.5) = 0.52(15..)
No cc	1st M1 1st A1 implied by: P(T > 95) = 0.70(805…)
	1st M1 1st A1 2nd M1 implied by: P(T > 94.5) = 0.72(657…)
	Exact binomial gives 0.68567… and will likely score 0/4

Qu 5	Scheme	Marks	AO
(a)	$P(S \cap \{X = 50\}) = P(S \cap \{X = 80\})$[= a constant, V] $\Rightarrow b \times \frac{k}{50} = c \times \frac{k}{80}$	M1	3.1a
	May see: $\frac{k}{50} = \frac{V}{b}$ and $\frac{k}{80} = \frac{V}{c}$ (condone any letter for V even S)		
	So $\underline{c = \frac{8}{5}b}$ *	A1cso*	1.1b
		(2)	
(b)	$d = 2b$ **or** $a = \frac{2}{5}b$ **or** $c = 4a$ **or** $d = 5a$ **or** $d = \frac{5}{4}c$	M1 A1	2.1 3.3
	$\frac{2}{5}b + b + \frac{8}{5}b + 2b = 1$	M1	2.1
	$\Rightarrow 5b = 1$ so $b = \frac{1}{5}$ (o.e.)	A1	1.1b
	$\underline{\underline{a = \frac{2}{25} \quad b = \frac{1}{5} \quad c = \frac{8}{25} \quad d = \frac{2}{5}}}$	A1	3.2a
		(5)	
(c)	[Experiment suggests for Nav] $P(S \mid \{X = 100\}) = 0.3 \Rightarrow k = 30$		
	or $0.3 = \frac{V}{0.4} \Rightarrow V = 0.12$	B1	2.4
	So model won't work since		
	$P(S \mid X = 20) = \frac{30}{20}$ or $\frac{0.12}{0.08}$ and so would be greater than 1		
		(1)	
		(8 marks)	

	Notes
(a)	M1 for use of $P(S \mid X = x) \times P(X = x)$ for $x = 50$ and $x = 80$ (Must see k or their V) Any expression or equation MUST be based on the probability statements in qu.
*	A1cso for rearranging to required result, no incorrect work seen, condone poor notation
NB	Use of values e.g. $b = \frac{50}{20 + 50 + 80 + 100}$ to prove (a) is M0A0 but scores 2nd M1A1 in (b)
	Marks for (b) may be awarded for work seen in (a)
(b)	1st M1 for at least one other relationship (either probability the subject) from the list. 1st A1 for a second different relationship (either probability the subject) from the list. or Allow for: $\frac{ak}{20} = \frac{bk}{50} = \frac{ck}{80} = \frac{dk}{100}$ for 1st M1 1st A1 2nd M1 for using or stating sum of prob's = 1 May be implied by one correct probability. 2nd A1 for one correct probability e.g. $b = \frac{1}{5}$ or exact equivalent such as 0.2 3rd A1 for all correct probabilities. Allow exact equivalents e.g. $c = 0.32$ **Sight of correct distribution or list of probs with no obvious incorrect working is 5/5**
(c)	B1 for deducing $k = 30$ **and** giving a suitable example to show model breaks down

Qu 6	Scheme	Marks	AO
(a)	[Sum of probs = 1 implies] $\log_{36} a + \log_{36} b + \log_{36} c = 1$	M1	3.1a
	$\Rightarrow \log_{36}(abc) = 1$ so $abc = 36$	A1	3.4
	All probabilities greater than 0 implies each of a, b and $c > 1$	B1	2.2a
	$36 = 2^2 \times 3^2$ (or 3 numbers that multiply to give 36 e.g. 2, 2, 9 etc)	dM1	2.1
	Since a, b and c are distinct must be **2, 3, 6** **($a = 2$, $b = 3$, $c = 6$)**	A1	3.2a
		(5)	
(b)	$(\log_{36} a)^2 + (\log_{36} b)^2 + (\log_{36} c)^2$	M1	3.4
	[= 0.0374137…+ 0.09398737…+0.25]		
	= 0.38140… awrt **0.381**	A1	1.1b
		(2)	
		(7 marks)	

	Notes
(a)	1st M1 for a start to the problem using sum of probabilities leading to eq'n in a, b and c
	1st A1 for reducing to the equation $abc = 36$ [Must follow from their equation.]
NB	Can go straight from $abc = 36$ to the answer for full marks for part (a).
	B1 for deducing that each value > 1 (may be implied by 3 integers all > 1 in the next line)
	2nd dM1 (dep on M1A1) for writing 36 as a product of prime factors or 3 values with product = 36 and none = 1
	2nd A1 for 2, 3 and 6 as a list or $a = 2$, $b = 3$ and $c = 6$
SC	**M0M0** If no method marks scored but a correct answer given score: M0A0B1M0A1 (2/5)
Ans only	This gets the SC score of 2/5 [Question says show your working clearly]
(b)	M1 for a correct expression in terms of a, b and c or values; ft their integers a, b and c Condone invisible brackets if the answer implies they are used.
	A1 for awrt 0.381

Qu 4	Scheme	Marks	AO
(a)	$\frac{k}{10}+\frac{k}{20}+\frac{k}{30}+\frac{k}{40}+\frac{k}{50}=1$ or $\frac{1}{600}\left(60k+30k+20k+15k+12k\right)=1$	M1	1.1b
	So $k=\frac{600}{137}$ (*)	A1cso	1.1b
		(2)	
(b)	(Cases are:) $D_1=30, D_2=50$ and $D_1=50, D_2=30$ and $D_1=40, D_2=40$	M1	2.1
	$\mathrm{P}\left(D_1+D_2=80\right)=\frac{k}{50}\times\frac{k}{30}\times 2 + \left(\frac{k}{40}\right)^2$	M1	3.4
	= 0.0375619… awrt **0.0376**	A1	1.1b
		(3)	
(c)	Angles are: $a,\quad a+d,\quad a+2d,\quad a+3d$	M1	3.1a
	$S_4 = a + (a + d) + (a + 2d) + (a + 3d) = 360$	M1	2.1
	$2a + 3d = 180$ (o.e.)	A1	2.2a
	Smallest angle is $a > 50$ consider cases:	M1	3.1b
	$d = 10$ so $a = 75$ or $d = 20$ so $a = 60$ [$d = 30$ gives $a = 45$ no good]		
	$\mathrm{P}(D = 10 \text{ or } 20) = \frac{3k}{20} = \frac{90}{137}$	A1	1.1b
		(5)	
		(10 marks)	

Notes

(a)

M1 for clear use of sum of probabilities = 1 (all terms seen)

A1 cso (*) M1 scored and no incorrect working seen.

Verify **(Assume $k = \frac{600}{137}$)** to score the final A1 they must have a final comment "$\therefore k = \frac{600}{137}$"

(b)

1st M1 for selecting at least 2 of the relevant cases (may be implied by their correct probs)
e.g. allow 30, 50 and 50,30 i.e. D_1 and D_2 labels not required

2nd M1 for using the model to obtain a correct expression for two different probabilities.
May use letter k or their value for k.

Allow for $\frac{k}{50}\times\frac{k}{30} + \left(\frac{k}{40}\right)^2$ or $2\times\left(\frac{k}{50}\times\frac{k}{30} + \left(\frac{k}{40}\right)^2\right)$

A1 for awrt 0.0376 (exact fraction is $\frac{705}{18769}$)

(c)

1st M1 for recognising the 4 angles and finding expressions in terms of d and their a

2nd M1 for using property of quad with these 4 angles (equation can be un-simplified)
Allow these two marks for use of a (possible) value of d
e.g. $a + a + 10 + a + 20 + a + 30 = 360$ (If at least 3 cases seen allow A1 for e.g. $4a = 300$)
or allow M1M1 for a set of 4 angles with sum 360 and possible value of d (3 cases for A1)
e.g. (for $d = 20$) 60, 80, 100, 120

1st A1 for $2a + 3d = 180$ condition (o.e.) [Must be in the form $pa + qd = N$]

3rd M1 for examining cases and getting $d = 10$ and $d = 20$ only

2nd A1 for $\frac{90}{137}$ or exact equivalent

The correct answer and no obviously incorrect working will score 5/5

A final answer of awrt 0.657 (0.65693…) with no obviously incorrect working scores 4/5

Question	Scheme	Marks	AOs
4 (a)	$\frac{132}{184} = 0.71739...$ awrt **0.717**	B1	1.1b
		(1)	
(b)(i)	$P(X \geq 6) = 1 - P(X \leq 5)$ or $P([X =]6) + P([X =]7) + P([X =]8)$	M1	3.4
	$= 1 - 0.296722...$ awrt **0.703**	A1	1.1b
		(2)	
(b)(ii)	$184 \times P(X = 7)$ $[= 184 \times 0.2811\ldots]$	M1	1.1b
	$= 51.7385...$ awrt **51.7**	A1	1.1b
		(2)	
(c)	Part (a) and part (b)(i) are similar **and** the expected number of 7s (51.7 or 0.281) matches with the number of 7s found in the data set (52 or 0.283) so Magali's model is supported.	B1ft	3.5a
		(1)	
(d)	$\frac{23}{28} = 0.82142...$ awrt **0.821**	B1	1.1b
		(1)	
(e)	Any one of… • Part (d)/'0.821' differs from part (a)/(b)(i)/(0.7…) • there is a **greater/different probability of high cloud cover/more likely to have high cloud cover** if the previous day had high cloud cover • **independence(o.e.)** does not hold	B1	2.4
	…therefore Magali's (binomial) model may not be suitable.	dB1	3.5a
		(2)	
			(9 marks)

Notes	
	Allow fractions, decimals or percentages throughout this question.
(a)	Allow equivalent fraction, e.g. $\frac{33}{46}$
(b)(i)	**M1:** for writing or using $1 - P(X \leq 5)$ or $P(X = 6) + P(X = 7) + P(X = 8)$ **A1:** awrt 0.703 (correct answer scores 2 out of 2)
(b)(ii)	**M1:** for $184 \times P(X = 7)$ o.e. e.g., $184 \times [P(X \leq 7) - P(X \leq 6)]$ **A1:** awrt 51.7
(c)	**B1ft:** comparing '0.717' with '0.703' and '51.7 or '0.281' with 52 or 0.283 **and** concluding that Magali's model **is supported** (must be comparing prob. with prob. and days with days). Allow not supported or mixed conclusions if consistent with their f.t. answers in (a) and (b)
(e)	**B1:** Any bullet point **dB1:** (dep on previous B1) for Magali's model may not be suitable (o.e.) Condone not accurate for not suitable **SC:** part (d) is similar to part (a)/(b)(i) **and** a compatible conclusion (i.e. Magali's model is supported) to score B1B1.

Question	Scheme	Marks	AOs

Qu 1	Scheme	Marks	AO
(a)	c: 0 1 2 3 4 5 6 7 8 P($C = c$): $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$	B1 B1ft (2)	1.2 1.2
(b)	P($C < 4$) = $\frac{4}{9}$ (accept 0.444 or better)	B1 (1)	3.4
(c)	Probability lower than expected suggests model is <u>not</u> good	B1ft (1)	3.5a
(d)	e.g. Cloud cover will vary from month to month and place to place So e.g. use a non-uniform distribution	B1 (1)	3.5c
		(5 marks)	

Notes

(a) 1st B1 for a correct set of values for c. Allow $\left\{\frac{1}{8}, \frac{2}{8}, \ldots \frac{8}{8}\right\}$

2nd B1ft for correct probs from their values for c, consistent with discrete uniform distrib'n
Maybe as a prob. function. Allow P($X = x$) = $\frac{1}{9}$ for $0 \leqslant x \leqslant 8$ provided x = {0, 1, 2, …, 8} is clearly defined somewhere.

(b) B1 for using correct model to get $\frac{4}{9}$ (o.e.)

SC **Sample space {1, …, 8}** If scored B0B1 in (a) for this allow P($C < 4$) = $\frac{3}{8}$ to score B1 in (b)

(c) B1ft for comment that states that the model proposed is or is not a good one based on their model in part (a) and their probability in (b)
|(b) – 0.315| > 0.05 Allow e.g. "it is not suitable"; "it is not accurate" etc
|(b) – 0.315| ⩽ 0.05 Allow a comment that suggests it <u>is</u> suitable
No prob in (b) Allow a comparison that mentions 50% or 0.5 and rejects the model
No prob in (b) and no 50% or 0.5 or **(b) > 1** scores B0
Ignore any comments about location or weather patterns.

(d) B1 for a sensible refinement considering variations in month or location
Just saying "not uniform" is B0
Context & "non-uniform" Allow mention of different locations, months <u>and</u> non-uniform
<u>or</u> use more locations to form a new distribution with probabilities based on frequencies
Context & "binomial" Allow mention of different locations, months <u>and</u> binomial
Just refined model Model must be outlined and discrete and non-uniform
e.g. higher probabilities for more cloud cover <u>or</u> lower probabilities for less cloud cover
Continuous model Any model that is based on a continuous distribution. e.g. normal is B0

Qu 3	Scheme	Marks	AO
(a)	The <u>probability</u> of a dart hitting the target is <u>constant</u> (from child to child and for each throw by each child) (o.e.)	B1	1.2
	The <u>throws</u> of each of the darts are <u>independent</u> (o.e.)	B1 (2)	1.2
(b)	[P($H \geqslant 4$) = 1 – P($H \leqslant 3$) = 1 – 0.9872 = 0.012795.. =] awrt **<u>0.0128</u>**	B1 (1)	1.1b
(c)	P($F = 5$) = $0.9^4 \times 0.1$, = 0.06561 = awrt **<u>0.0656</u>**	M1, A1 (2)	3.4 1.1b
(d)	n | 1 | 2 | … | 10 P($F = n$) | 0.01 | $0.01 + \alpha$ | … | $0.01+9\alpha$	M1	3.1b
	Sum of probs = 1 $\Rightarrow \frac{10}{2}[2\times 0.01+9\alpha]=1$	M1A1	3.1a 1.1b
	[i.e. $5(0.02 + 9\alpha) = 1$ or $0.1 + 45\alpha = 1$] so α = **<u>0.02</u>**	A1 (4)	1.1b
(e)	P($F = 5$| Thomas' model) = **<u>0.09</u>**	B1ft (1)	3.4
(f)	<u>Peta's</u> model assumes the <u>probability</u> of hitting target is <u>constant</u> (o.e.) **and** <u>Thomas</u>' model assumes this <u>probability increases</u> with each attempt(o.e.)	B1 (1)	3.5a
		(11 marks)	

Notes

(a) 1st B1 for stating that the <u>probability</u> (or possibility or chance) is <u>constant</u> (or fixed or same)
2nd B1 for stating that <u>throws</u> are <u>independent</u> ["trials" are independent is B0]

(b) B1 for awrt 0.0128 (found on calculator)

(c) M1 for a probability expression of the form $(1-p)^4 \times p$ where $0 < p < 1$
A1 for awrt 0.0656
SC Allow M1A0 for answer only of 0.066

(d) 1st M1 for setting up the distribution of F with at least 3 correct values of n and P($F = n$) in terms of α. (Can be implied by 2nd M1 or 1st A1)
2nd M1 for use of sum of probs = 1 **and** clear summation or use of arithmetic series formula (allow 1 error or missing term). (Can be implied by 1st A1)
1st A1 for a correct equation for α
2nd A1 for α = 0.02 (must be exact and come from correct working)

(e) B1ft for value resulting from $0.01 + 4\times$"their α" (provided α and the answer are probs)
Beware If their answer is the same as their (c) (or a rounded version of their (c)) score B0

(f) B1 for a suitable comment about the <u>probability</u> of hitting the target
ALT Allow idea that Peta's model suggests the dart may never hit the target but Thomas' says that it will hit at least once (in the first 10 throws).