adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Straight lines

Mark scheme

3 questions

Exam questions and marking instructions remain credited to their board.

Contents

1 June 2025 - Paper 1 - Q12

2 June 2024 - Paper 2 - Q13

3 June 2019 - Paper 2 - Q10

Question	Scheme	Marks	AOs
12 (a)	e.g. Substitutes $x=6$ into **both** $(y=)\dfrac{15x}{(2x+3)(x-3)}$ **and** $(y=)\ 2x-10$ and finds the y values for both	M1	1.1b
	e.g. Achieves 2 for both and makes a valid conclusion *	A1*	2.4
		(2)	
(b)	Sets $\dfrac{15x}{(2x+3)(x-3)}=2x-10$ and attempts to cross multiply	M1	1.1b
	$4x^3-26x^2-3x+90=0$ *	A1*	2.1
		(2)	
(c)	Deduces that $(x-6)$ is a factor and attempts to divide	M1	2.1
	$4x^3-26x^2-3x+90=(x-6)\left(4x^2-2x-15\right)$	A1	1.1b
	Solves their $4x^2-2x-15$ using suitable method	M1	1.1b
	Deduces $x=\dfrac{1-\sqrt{61}}{4}$ (see note)	A1	2.2a
		(4)	
			(8 marks)

Notes:

(a) Must be seen in (a) to score

M1: Examples to verify include:

- substitutes $x=6$ into **both** $(y=)\dfrac{15x}{(2x+3)(x-3)}$ **and** $(y=)\ 2x-10$ and finds the y values for both
- substitutes $x=6$ into one of the two equations to find $y=2$ and then substitutes this into the other equation and solves to find x
- sets $\dfrac{15x}{(2x+3)(x-3)}=2x-10 \Rightarrow$ cubic equation (=0 implied) and either substitutes $x=6$ into the expression, attempts $\mathrm{f}(6)$ or else attempts to divide the cubic = 0 by $(x-6)$.
  Condone without calculations for this mark only. i.e. $\mathrm{f}(6)=0$
  Condone slips for this mark in any of the outlined approaches. There may be variations of these as well.

A1*: Correct calculations must be seen with a minimal conclusion that they intersect

e.g. $\dfrac{15\times6}{(2\times6+3)(6-3)}=2$ and $12-10=2$ so they intersect (allow eg meet/cross/intercept)

e.g. $\dfrac{15\times6}{(2\times6+3)(6-3)}=2 \Rightarrow 2=2x-10 \Rightarrow x=6$ so they intersect

Acceptable alternatives are:

$\mathrm{f}(x)=4x^3-26x^2-3x+90$, $\mathrm{f}(6)=4\times6^3-26\times6^2-3\times6+90=0 \Rightarrow$ so they intersect

$\mathrm{f}(x)=4x^3-26x^2-3x+90 \Rightarrow (x-6)(4x^2-2x-15)$ so $x=6$ is a root so they intersect

OR $(x-6)$ is a factor hence they intersect

Only accept verified, QED etc if there is a preamble mentioning intersection about how it will be shown.

M1A0 with or without a conclusion. Requires all 3 roots.

(b) Work seen in (a) must be used in (b) to score

M1: Sets $\frac{15x}{(2x+3)(x-3)} = 2x-10$ and attempts to cross multiply to form a cubic equation.

Score for $15x = (2x-10)(2x+3)(x-3)$ or may be implied by an attempt to multiply two of the three brackets out e.g. $15x = (2x-10)\left(2x^2-3x-9\right)$ or

$15x = (x-3)\left(4x^2-14x-30\right)$ or $15x = (2x+3)(2x^2-16x+30)$ (may be unsimplified)

Condone slips provided the intention is clear.
Condone invisible brackets to be implied by further work which is not the given answer.

Do not allow this mark to be scored for proceeding straight from $\frac{15x}{(2x+3)(x-3)} = 2x-10$ to a simplified expression of $15x = 4x^3-26x^2+12x+90$

A1*: Achieves the given answer with sufficient intermediate steps seen and **no errors including invisible brackets** in the main body of their solution (ignore "side workings") Expect to see on the right hand side the product of a linear and a quadratic expression and an expression with all of the brackets multiplied out before simplifying to the given answer.

e.g.

$$\frac{15x}{(2x+3)(x-3)} = 2x-10 \Rightarrow 15x = \left(4x^2-14x-30\right)(x-3) \Rightarrow 15x = 4x^3-26x^2+12x+90$$

$\Rightarrow 4x^3-26x^2-3x+90=0$ * which is A1*

(c) Work seen in (a) or (b) must be used in (c) to score

M1: For the key step in dividing the cubic by $(x-6)$. There are various methods or ways to present their solution so typically methods

- by inspection look for first and last terms e.g. $(x-6)(4x^2 \pm ...x \pm 15)$
- by division look for a three term quadratic quotient of $4x^2-2x \pm ...$

A1: $4x^3-26x^2-3x+90 = (x-6)\left(4x^2-2x-15\right)$ This may be implied by sight of $\left(4x^2-2x-15\right)$ in their working.

M1: **Whilst not dependent on the previous method mark, it does require an attempt at dividing by $(x-6)$ however poor to achieve a three term quadratic factor**

Solves their $4x^2-2x-15=0$ using either the quadratic formula or completing the square. Usual rules apply for solving a quadratic. **It cannot be scored for stating the roots directly via use of a calculator which is M0A0**

A1: $(x=)\ \frac{1-\sqrt{61}}{4}$ or exact equivalent e.g. $\frac{1}{4}-\sqrt{\frac{61}{16}}$

The root $(x=)\ \frac{1+\sqrt{61}}{4}$ must be rejected if found. Isw if they put their exact value in decimal form.

13(a)	$\log_{10} b = 0.0054 \Rightarrow b = 10^{0.0054}$ **or** $\log_{10} a = 0.81 \Rightarrow a = 10^{0.81}$	M1	3.1a
	$b = 1.01$ **or** $a = 6.46$	A1	1.1b
	$\log_{10} b = 0.0054 \Rightarrow b = 10^{0.0054}$ **and** $\log_{10} a = 0.81 \Rightarrow a = 10^{0.81}$	M1	2.1
	$b = 1.013$ **and** $a = 6.457$	A1	1.1b
		(4)	
(b)(i)	e.g. The world population **in billions** in 2004	B1ft	3.2a
(ii)	b = 1.013 represents the scale factor of the yearly increase in the world population	B1ft	3.2a
		(2)	
(c)	$P = 6.457...(1.013...)^{26}$ or e.g. $\log P = 0.81 + 26 \times 0.0054 \Rightarrow P = ...$	M1	3.4
	awrt 9 billion	A1	2.2b
		(2)	
(d)	Not reliable since the data used for the model covered the years 2004 – 2007 and it would not be sensible to assume that the model still holds in 2030	B1	3.2b
		(1)	
			(9 marks)

Notes

(a) Must be using base 10 in (a). Ignore any units associated with a and b in part (a).

M1: Correct strategy to get a numerical expression or value for a **or** b e.g. $a = 10^{0.81}$ **or** $b = 10^{0.0054}$. This may be implied by a = awrt 6.46 or b = awrt 1.01 if no incorrect work is seen.

A1: Correct value for a or b. Allow 3 sf for **this mark** so allow a = awrt 6.46 or b = awrt 1.01.
May be seen embedded in their formula.

M1: Correct strategy to get a numerical expression or value for a **and** b e.g. $a = 10^{0.81}$ **and** $b = 10^{0.0054}$. This may be implied by a = awrt 6.46 and b = awrt 1.01 if no incorrect work is seen.

A1: Correct values. This requires a = awrt 6.457 and b = awrt 1.013 for this mark.
May be seen embedded in their formula.
Isw once correct answers are seen.

Special case: Constants the wrong way round:

$a = 1.013$ **and** $b = 6.457$ with or without working scores M1A1M1A0 unless the equation is formed correctly in which case the final A mark can be recovered.

Note that having found the value of a, it is possible to find b by substituting e.g. t = 1 as follows:

$$a = 10^{0.81} = 6.457 \quad t = 1 \Rightarrow P = ab \Rightarrow b = \frac{P}{a}$$

$$t = 1 \Rightarrow \log_{10} P = 0.0054 + 0.81 = 0.8154 \Rightarrow P = 10^{0.8154} \Rightarrow b = \frac{P}{a} = \frac{10^{0.8154}}{6.457} = 1.013$$

Note that a misread of 0.0054 as 0.054 is quite common and may score 1110 as it does not simplify the question.

(b)(i) Follow through their a.

B1ft: Correct interpretation for a but must reference "billions".

Allow equivalent alternatives e.g.

- The original/initial population **in billions**
- The population in 2004 was "6.46" **billion**

(b)(ii) Follow through their b.

B1ft: Correct interpretation for b but must reference "each year" or e.g. "yearly" oe

Allow equivalent alternatives e.g.

- The proportional increase/change in each year.
- The population will rise by "1.3%" each year. Must follow their value for b.
- The rate/factor at which the population is rising/increasing/changing per annum.
- "1.013" is the multiplier representing the year on year increase.

Do **not** accept

- The amount it is rising
- How much it is rising
- The rate the population increases
- The percentage increase each year
- The rate of increase in billions annually

If they are not labelled (b)(i) and (b)(ii) mark in the order given but accept any way round as long as clearly labelled " *a* is..... " and "*b* is ....."

(c)

M1: Substitutes $t = 25$ or 26 or 27 into their model to find a value for P

Must be using **their** a and b correctly in $P = ab^t$

May be implied by sight of "9" or 9 billion if no incorrect working is seen.

A1: Correct value including units (allow awrt 9 billion) **from a correct model but condone incorrect/premature rounding or truncating in an otherwise correct model that leads to the correct value of awrt 9 billion.**

Allow e.g. awrt 9 000 000 000 or e.g. awrt 9×10^9

Just awrt 9 without the "billions" is A0

(d)

B1: The response must refer to the fact that the answer is unreliable together with a reference to the fact that the data used for the model is a long way from 2030

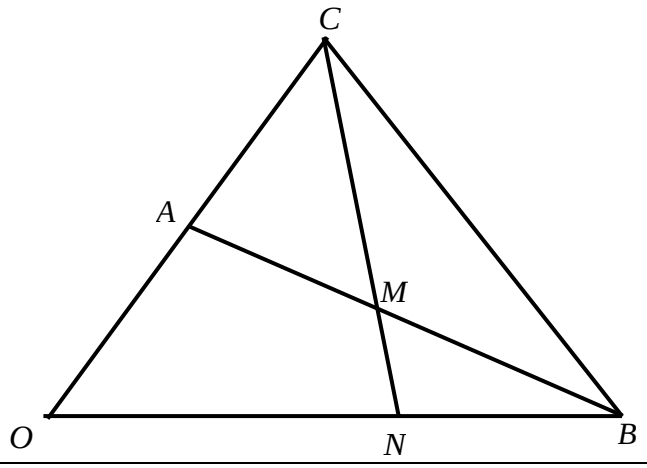
Examples:

- Not good as 2030 is a long way from 2004 – 2007
- Unreliable as based on old data
- Questionable as it has been extrapolated over a long time
- Not reliable due to how far out we have extrapolated
- By the time 2030 arrives it will be unreliable

But not e.g.

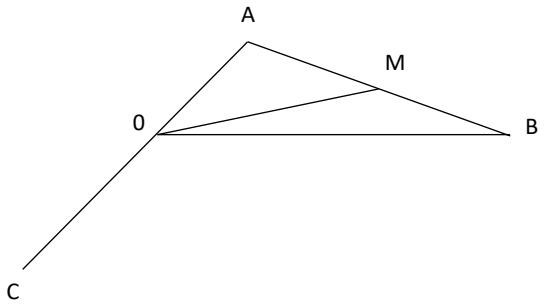
- Unreliable, extrapolation
- Not good as outside the range
- Not good as the population rises 101.3% each year
- Disease may happen
- Reliable as based on old data

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
10			
	$\overrightarrow{OA}=\mathbf{a}, \overrightarrow{OB}=\mathbf{b}$		
(a)	$\left\{\overrightarrow{CM}=\overrightarrow{CA}+\overrightarrow{AM}=\overrightarrow{CA}+\frac{1}{2}\overrightarrow{AB} \Rightarrow\right\}\ \overrightarrow{CM}=-\mathbf{a}+\frac{1}{2}(\mathbf{b}-\mathbf{a})$	M1	3.1a
	$\left\{\overrightarrow{CM}=\overrightarrow{CB}+\overrightarrow{BM}=\overrightarrow{CB}+\frac{1}{2}\overrightarrow{BA} \Rightarrow\right\}\ \overrightarrow{CM}=(-2\mathbf{a}+\mathbf{b})+\frac{1}{2}(\mathbf{a}-\mathbf{b})$		
	$\Rightarrow \overrightarrow{CM}=-\frac{3}{2}\mathbf{a}+\frac{1}{2}\mathbf{b}$ (***needs to be simplified and seen in (a) only***)	A1	1.1b
		(2)	
(b)	$\overrightarrow{ON}=\overrightarrow{OC}+\overrightarrow{CN} \Rightarrow \overrightarrow{ON}=\overrightarrow{OC}+\lambda\overrightarrow{CM}$	M1	1.1b
	$\overrightarrow{ON}=2\mathbf{a}+\lambda\left(-\frac{3}{2}\mathbf{a}+\frac{1}{2}\mathbf{b}\right) \Rightarrow \overrightarrow{ON}=\left(2-\frac{3}{2}\lambda\right)\mathbf{a}+\frac{1}{2}\lambda\mathbf{b}$ *	A1*	2.1
		(2)	
(c) Way 1	$\left(2-\frac{3}{2}\lambda\right)=0 \Rightarrow \lambda=...$	M1	2.2a
	$\lambda=\frac{4}{3} \Rightarrow \overrightarrow{ON}=\frac{2}{3}\mathbf{b}\ \left\{\Rightarrow \overrightarrow{NB}=\frac{1}{3}\mathbf{b}\ \right\} \Rightarrow ON:NB=2:1$ *	A1*	2.1
		(2)	
(c) Way 2	$\overrightarrow{ON}=\mu\mathbf{b} \Rightarrow \left(2-\frac{3}{2}\lambda\right)\mathbf{a}+\frac{1}{2}\lambda\mathbf{b}=\mu\mathbf{b}$		
	$\mathbf{a}: \left(2-\frac{3}{2}\lambda\right)=0 \Rightarrow \lambda=...\ \left\{\mathbf{b}: \frac{1}{2}\lambda=\mu\ \ \&\ \ \lambda=\frac{4}{3} \Rightarrow \mu=\frac{2}{3}\right\}$	M1	2.2a
	$\lambda=\frac{4}{3}$ or $\mu=\frac{2}{3} \Rightarrow \overrightarrow{ON}=\frac{2}{3}\mathbf{b}\ \left\{\Rightarrow \overrightarrow{NB}=\frac{1}{3}\mathbf{b}\ \right\} \Rightarrow ON:NB=2:1$ *	A1*	2.1
		(2)	
			(6 marks)

Question	Scheme	Marks	AOs
10 (c) Way 3	$\overrightarrow{OB} = \overrightarrow{ON} + \overrightarrow{NB} \Rightarrow \mathbf{b} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b} + K\mathbf{b}$		
	$\mathbf{a}: \left(2 - \frac{3}{2}\lambda\right) = 0 \Rightarrow \lambda = ... \quad \left\{ \mathbf{b}: \ 1 = \frac{1}{2}\lambda + K \ \ \& \ \ \lambda = \frac{4}{3} \Rightarrow K = \frac{1}{3} \right\}$	M1	2.2a
	$\lambda = \frac{4}{3}$ or $K = \frac{1}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}$ or $\overrightarrow{NB} = \frac{1}{3}\mathbf{b} \Rightarrow ON : NB = 2:1$ *	A1	2.1
		(2)	
10 (c) Way 4	$\overrightarrow{ON} = \mu\mathbf{b} \ \ \& \ \ \overrightarrow{CN} = k\overrightarrow{CM} \Rightarrow \overrightarrow{CO} + \overrightarrow{ON} = k\overrightarrow{CM}$		
	$-2\mathbf{a} + \mu\mathbf{b} = k\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)$		
	$\mathbf{a}: -2 = -\frac{3}{2}k \Rightarrow k = \frac{4}{3}, \quad \mathbf{b}: \ \mu = \frac{1}{2}k \Rightarrow \mu = \frac{1}{2}\left(\frac{4}{3}\right) = ...$	M1	2.2a
	$\mu = \frac{2}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b} \left\{ \Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b} \right\} \Rightarrow ON : NB = 2:1$ *	A1	2.1
		(2)	

Notes for Question 10	
(a)	
M1:	Valid attempt to find $\overrightarrow{CM}$ using a combination of known vectors **a** and **b**
A1:	A simplified correct answer for $\overrightarrow{CM}$
Note:	Give M1 for $\overrightarrow{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$ or $\overrightarrow{CM} = (-2\mathbf{a} + \mathbf{b}) + \frac{1}{2}(\mathbf{a} - \mathbf{b})$ or for $\left\{\overrightarrow{CM} = \overrightarrow{OM} - \overrightarrow{OC} \Rightarrow\right\} \ \overrightarrow{CM} = \frac{1}{2}(\mathbf{a} + \mathbf{b}) - 2\mathbf{a}$ **only o.e.**
(b)	
M1:	Uses $\overrightarrow{ON} = \overrightarrow{OC} + \lambda\overrightarrow{CM}$
A1*:	Correct proof
Note:	**Special Case**
	Give SC M1 A0 for the solution $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \lambda\overrightarrow{CM}$
	$\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \lambda\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) \ \left\{= \left(\frac{1}{2} - \frac{3}{2}\lambda\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}\lambda\right)\mathbf{b}\right\}$
Note:	**Alternative 1:** Give M1 A1 for the following alternative solution: $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \mu\overrightarrow{CM}$ $\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \mu\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) = \left(\frac{1}{2} - \frac{3}{2}\mu\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}\mu\right)\mathbf{b}$ $\mu = \lambda - 1 \Rightarrow \quad \overrightarrow{ON} = \left(\frac{1}{2} - \frac{3}{2}(\lambda - 1)\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}(\lambda - 1)\right)\mathbf{b} \Rightarrow \overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$
(c)	**Way 1, Way 2 and Way 3**
M1:	Deduces that $\left(2 - \frac{3}{2}\lambda\right) = 0$ and attempts to find the value of λ
A1*:	Correct proof
(c)	**Way 4**
M1:	Complete attempt to find the value of μ
A1*:	Correct proof

Notes for Question 10 Continued	
Note:	Part (b) and part (c) can be marked together.
(a) Special Case	**<u>Special Case where the point *C* is believed to be below the origin *O*</u>** 
	Give Special Case M1 A0 in part (a) for $\left\{\overrightarrow{CM}=\overrightarrow{CA}+\overrightarrow{AM}\Rightarrow\right\}\ \overrightarrow{CM}=3\mathbf{a}+\frac{1}{2}(\mathbf{b}-\mathbf{a})$
	$\left\{\text{which leads to } \overrightarrow{CM}=\frac{5}{2}\mathbf{a}+\frac{1}{2}\mathbf{b}\right\}$

Question	Scheme	Marks	AOs