adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Trigonometry

Mark scheme

30 questions

Exam questions and marking instructions remain credited to their board.

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Question	Scheme	Marks	AOs
14(a)	Attempts to use $\sin(x+30°)=\sin x\cos 30°\pm\cos x\sin 30°$ $\cos(x+30°)=\cos x\cos 30°\mp\sin x\sin 30°$ $\Rightarrow \pm\sin x\cos 30°\pm\cos x\sin 30°\pm\sqrt{3}(\pm\cos x\cos 30°\pm\sin x\sin 30°)$	M1	2.1
	Correct expression $\sin x\cos 30°+\cos x\sin 30°+\sqrt{3}(\cos x\cos 30°-\sin x\sin 30°)$	A1	1.1b
	States or implies that $\sin 30°=\frac{1}{2}$ and $\cos 30°=\frac{\sqrt{3}}{2}$ $=\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x+\sqrt{3}\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x\right)=2\cos x$ *	A1*	2.1
		(3)	
(b)	$2\cos\theta=3\sin 2\theta\Rightarrow 2\cos\theta=6\sin\theta\cos\theta$	M1	2.1
	$\sin\theta=\frac{1}{3}$	A1	1.1b
	$\theta=\arcsin\frac{1}{3}\Rightarrow\theta=...$	dM1	1.1b
	$(\theta=)$ awrt 19.5°, awrt 160.5°, 90°	A1	2.2a
		(4)	
			(7 marks)

Notes:

(a) **Condone a complete proof entirely in θ (or another variable) instead of *x*.**
Do not be concerned with the omission of degrees.

M1: Attempts to use both compound angle expansions to set up an expression in sin *x* and cos *x*
i.e. $\pm\sin x\cos 30°\pm\cos x\sin 30°\pm\sqrt{3}(\pm\cos x\cos 30°\pm\sin x\sin 30°)$
The terms must be correct but condone sign errors and a slip on the multiplication of the $\sqrt{3}$ if they attempt to multiply out the brackets. (The $\sqrt{3}$ may be omitted entirely)
This mark may be implied by further work
e.g. $\pm\frac{\sqrt{3}}{2}\sin x\pm\frac{1}{2}\cos x\pm\sqrt{3}\left(\pm\frac{\sqrt{3}}{2}\cos x\pm\frac{1}{2}\sin x\right)$

A1: Correct expression $\sin x\cos 30°+\cos x\sin 30°+\sqrt{3}(\cos x\cos 30°-\sin x\sin 30°)$ o.e.
May be implied by
$\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x+\sqrt{3}\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x\right)$ (or implied if multiplied out)

A1*: Proceeds to the given answer with **no errors seen including invisible brackets** (condone a missing trailing bracket). We must see the exact numerical values used for $\sin 30°$ and $\cos 30°$ before proceeding to the given answer.
Minimum required
$\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x+\sqrt{3}\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x\right)=2\cos x$ which scores M1A1A1*
$\sin x\cos 30°+\cos x\sin 30°+\sqrt{3}(\cos x\cos 30°-\sin x\sin 30°)=2\cos x$ scores M1A1A0*
There should not be any notational or bracketing errors and no mixed or missing variables.

M1: Writes the left hand side of the equation as

$$\frac{1}{2}\sin\left(x+30°\right)+\frac{\sqrt{3}}{2}\cos\left(x+30°\right)=\sin 30°\sin\left(x+30°\right)+\cos 30°\cos\left(x+30°\right)$$

A1: Correct expression for $\sin 30°\sin\left(x+30°\right)+\cos 30°\cos\left(x+30°\right)=\cos x$

A1*: Proceeds to the given answer $\sin\left(x+30°\right)+\sqrt{3}\cos\left(x+30°\right)=2\cos\left(x+30°-30°\right)=2\cos x$ with **no errors seen including invisible brackets** (condone a missing trailing bracket). There should not be any notational or bracketing errors and no mixed or missing variables.

Alternative part (a) – using the *R*-alpha method

M1: States e.g. $R\cos(x+30\pm\alpha)=\sqrt{3}\cos(x+30)\mp\sin(x+30)$

and attempts to find either R or α correctly. (may be implied)

A1: Achieves $2\cos(x+30-30)$

A1*: Achieves $2\cos x$ with no errors seen and both stages of working shown e.g.

- States $\left(R\cos(x+30\pm\alpha)=\right)\sqrt{3}\cos(x+30)\cos\alpha\mp\sin(x+30)\sin\alpha$ oe
- Shows $R=\sqrt{1+3}=2$ and $\tan\alpha=\left(\frac{1}{\sqrt{3}}\right)$ o.e. e.g. using cosine or sine

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(b) Condone the use of *x* for θ and mixed variables

M1: Sets $2\cos\theta=3\sin 2\theta$ and proceeds to $A\cos\theta=B\sin\theta\cos\theta$. (allow $A=B$)
May be implied by $\sin\theta=k$ $(k\neq 0,1)$

A1: $\sin\theta=\frac{1}{3}$

dM1: Finds at least one of their values of θ for their $\sin\theta=k$ $(k\neq 0,1)$ It is dependent on the previous method mark. You may need to check their value(s) (in degrees or radians) but may be implied by e.g. awrt $19°$ (or awrt $20°$) or awrt $161°$ (or awrt $160°$)
(awrt 0.34 or awrt 2.8 in radians)

A1: Deduces that $\left(\theta=\right)90°$ as well as giving $\left(\theta=\right)$ awrt $19.5°$, awrt $160.5°$ with no other values in the given range (ignore any found outside of the range)
The degree symbol is not required. (Note the angles are 19.4712206…and 160.528779…)
Answers in radians score A0.

Question	Scheme	Marks	AOs
15 (a)	$\frac{1}{2}r^2\theta+\frac{1}{10}r^2=240\Rightarrow r\theta=\frac{240-\frac{1}{10}r^2}{\frac{1}{2}r}$ or $\theta=\frac{240-\frac{1}{10}r^2}{\frac{1}{2}r^2}$	M1 A1	3.4 1.1b
	Substitutes into the expression for P $r\theta=\frac{240-\frac{1}{10}r^2}{\frac{1}{2}r}$ into $\left(P=\right)\ r\theta+2r+\frac{1}{5}r$	dM1	3.4
	$P=\frac{240-\frac{1}{10}r^2}{\frac{1}{2}r}+2r+\frac{1}{5}r=\frac{480}{r}-\frac{1}{5}r+2r+\frac{1}{5}r=2r+\frac{480}{r}$ *	A1*	2.1
		(4)	
(b)	$\left(\frac{\mathrm{d}P}{\mathrm{d}r}=\right)\ 2-\frac{480}{r^2}$	M1	1.1b
	Sets $\frac{\mathrm{d}P}{\mathrm{d}r}=0\Rightarrow r^2=240$ $r=$ awrt 15.5	dM1 A1	2.1 1.1b
		(3)	
(c)	$\left(\frac{\mathrm{d}^2P}{\mathrm{d}r^2}=\right)\ \frac{960}{r^3}$	M1	1.1b
	$\left(\frac{\mathrm{d}^2P}{\mathrm{d}r^2}=\right)$ awrt $0.26>0$ proving a minimum value of P	A1	1.1b
		(2)	
			(9 marks)

Notes:

(a) **Note that just finding a correct equation for the area and/or a correct equation for the perimeter (before any substitution) is insufficient to score any marks.**

M1: Uses area formulae to form an equation of the form $\alpha r^2\theta+\beta r^2=240$ o.e. $(\alpha,\beta\neq 0)$ and rearranges to make $r\theta$, θ or $r\theta+\frac{1}{5}r$ the subject. Look for:

$r\theta=\frac{M\pm Nr^2}{r}\left(=\frac{M}{r}\pm Nr\right)$ o.e. or $\theta=\frac{M\pm Nr^2}{r^2}\left(=\frac{M}{r^2}\pm N\right)$ o.e. where $M,N\neq 0$

or $r\theta+\frac{1}{5}r=\frac{L}{r}$ $L\neq 0$ o.e. May work in degrees.

A1: A correct rearrangement for θ or $r\theta$ or $r\theta+\frac{1}{5}r$ which may be unsimplified (may be in degrees)

$r\theta=\frac{240-\frac{1}{10}r^2}{\frac{1}{2}r}$ o.e. e.g. $r\theta=\frac{2400-r^2}{5r}$ or $r\theta=\frac{480-0.2r^2}{r}$

or $r\theta+\frac{1}{5}r=\frac{480}{r}$ o.e.

or $\theta = \dfrac{240 - \frac{1}{10}r}{\frac{1}{2}r^2}$ o.e. e.g. $\theta = \dfrac{2400 - r^2}{5r^2}$ or $\theta = \dfrac{480}{r^2} - \dfrac{1}{5}$ or $\theta = 480r^{-2} - 0.2$

dM1: Substitutes their $r\theta = \dfrac{M \pm Nr^2}{r}$ o.e. or $\theta = \dfrac{M \pm Nr^2}{r^2}$ o.e. or $r\theta + \dfrac{1}{5}r = \dfrac{L}{r}$ into an expression of the form $(P =)\ r\theta + Qr$, $Q \neq 0$ (typically $P = r\theta + \dfrac{11}{5}r$) which may be unsimplified or in degrees. It is dependent on the previous method mark. It is acceptable for their valid expression for θ, $r\theta$ or $r\theta + \dfrac{1}{5}r$ to be substituted into the perimeter expression directly (without first seeing them in the perimeter expression).

A1*: $P = 2r + \dfrac{480}{r}$ following a correct method (condone slips to be recovered) and all previous marks scored. Condone invisible brackets to be recovered.
$P =$, Perimeter = must be seen at least once in their solution in the correct place.

(b) Mark (b) and (c) together. There is no requirement to see the notation $\dfrac{\mathrm{d}P}{\mathrm{d}r}$ in part (b). It may even be called $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. Allow use of e.g. P' or e.g. y'

M1: $\left(\dfrac{\mathrm{d}P}{\mathrm{d}r} =\right)\ p \pm \dfrac{q}{r^2}$ where p and q are non-zero constants

dM1: Sets or implies that their $\dfrac{\mathrm{d}P}{\mathrm{d}r} = 0$ and proceeds to $mr^{\pm 2} = n, \quad m \times n > 0$. It is dependent on the previous method mark. Do not be concerned by the mechanics of the rearrangement. This mark may be implied by a correct answer to their $p - \dfrac{q}{r^2} = 0$. You may need to check this on your calculator.

A1: $r =$ awrt 15.5 or $\sqrt{240}$ $\left(= 4\sqrt{15}\right)$ Do not accept $\pm$ (ignore any units if given)

(c) Condone other letters used instead of *P* and *r* for $\dfrac{\mathrm{d}^2P}{\mathrm{d}r^2}$ e.g. $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ for M1 only.

Just using $\dfrac{\mathrm{d}P}{\mathrm{d}r}$ and considering a sign change is M0A0

M1: Differentiates and finds $\left(\dfrac{\mathrm{d}^2P}{\mathrm{d}r^2} =\right) \pm \dfrac{f}{r^3}$ (do not be concerned about the sign)

A1: **Note if they score A0 in (b) then this mark cannot be scored.**
Requires

- a correct a correct expression for $\dfrac{\mathrm{d}^2P}{\mathrm{d}r^2}$
- a correct value for $\left(\dfrac{\mathrm{d}^2P}{\mathrm{d}r^2} =\right) \dfrac{960}{r^3} =$ awrt 0.26 using awrt 15.5 (but allow 0.23(43..) if using 16)
- a correct comparison with 0 **and** a conclusion e.g. minimum

The expression for the second derivative does not need to be labelled but if it is then it must be $\dfrac{\mathrm{d}^2P}{\mathrm{d}r^2}$ o.e. or accept e.g. P'' BUT $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ used in their conclusion is A0

Question	Scheme	Marks	AOs
4(a)	e.g. $6.4^2 = 8^2 + 13^2 - 2(8)(13)\cos ABC$	M1	1.1b
	$(\text{angle } ABC =)\ \ \cos^{-1}\left(\dfrac{8^2+13^2-6.4^2}{2\times 8\times 13}\right) = 0.394 \text{ (radians)}$*	A1*	1.1b
		(2)	
(b)	e.g. $(\text{Area}(ABC) =)\ \ \dfrac{1}{2}(8)^2(0.394)\ \ (= \text{awrt } 12.6)$ **or** $(\text{Area}(ABD) =)\ \ \dfrac{1}{2}(8)(13)\sin 0.394\ \ (= \text{awrt } 20.0)$	M1	1.1b
	$(\text{Area}(R) =)\ \ \dfrac{1}{2}(8)(13)\sin 0.394 - \dfrac{1}{2}(8)^2(0.394) = \ldots$	dM1	3.1a
	$(\text{Area}(R) =)$ awrt 7.34 to 7.37	A1	1.1b
		(3)	
			(5 marks)

Notes

It is acceptable in this question to work in degrees and convert if necessary.
NB Angle *ABC* in degrees is 22.591…°

(a)

M1: Attempts to use the cosine rule with values correctly placed. May be implied by, e.g.,

$$\cos\theta = \frac{8^2+13^2-6.4^2}{2\times 8\times 13} \text{ or } \cos^{-1}\left(\frac{6.4^2-8^2-13^2}{-2\times 8\times 13}\right)$$

Condone slips in substitution if a correct cosine formula is seen. The placement of the values should be correct. So, condone e.g. a missing 2 once a correct cosine formula is seen.

Alt 1: attempts the cosine rule and finds angle *ADB* $(= \text{awrt } 0.5 \text{ or } 29°)$ **or** angle *BAD* $(= \text{awrt } 2.2 \text{ or } 2.3 \text{ or } 129°)$ **and** uses the sine rule correctly with angle *ABC* involved.

Alt 2: attempts the cosine rule and finds angle *ADB* $(= \text{awrt } 0.5 \text{ or } 29°)$ **and** angle *BAD* $(= \text{awrt } 2.2 \text{ or } 2.3 \text{ or } 129°)$ **and** sums angles *ADB*, *BAD*, and *ABC* to π (or 180°)

Alt 3: using Pythagoras **and** simultaneous equations to find the length of *AN* or *BN* (see diagram on the next page) **and** uses e.g. $\cos ABC = \dfrac{BN}{8}$ or $\sin ABC = \dfrac{AN}{8}$

A1*: cso Correct proof.

If starting with $6.4^2 = 8^2 + 13^2 - 2(8)(13)\cos ABC$ then there must be an intermediate line such as

- $\cos ABC = \dfrac{8^2+13^2-6.4^2}{2\times 8\times 13}$ or $\text{angle } ABC = \cos^{-1}\left(\dfrac{8^2+13^2-6.4^2}{2\times 8\times 13}\right)$
- $\cos ABC = \text{awrt } 0.92$ or $\dfrac{4801}{5200}$
- angle *ABC* = awrt 0.3943

The minimum required is e.g. $\cos^{-1}\left(\dfrac{8^2+13^2-6.4^2}{2\times 8\times 13}\right)$ or $\cos\theta = \dfrac{8^2+13^2-6.4^2}{2\times 8\times 13} \rightarrow \theta = 0.394$

both of which score M1A1*.

The final value must be 0.394 and not e.g. 0.3943. There should be no obvious incorrect statements in the proof e.g. $\cos(0.92) = 0.394$ and no obvious incorrect work.

Allow the use of, e.g., θ, *x*, or even e.g. *A* throughout. Mention of radians is not required.

Those working in degrees and achieve 22.6 can convert to 0.394 without calculation for M1A1

Attempts via verification e.g. $AD^2 = 8^2 + 13^2 - 2(8)(13)\cos 0.394 \Rightarrow AD =$ awrt 6.4 score maximum M1A0*.

(b) **Note: you may need to check the diagram for working.**

M1: Attempts the area of the sector **or** the area of the triangle *ABD* via a **correct** method. May be implied by a full attempt at the area of *R*.

The sector area may be implied by $\frac{1576}{125}$ or by $\frac{0.394}{2\pi}\times\pi\times8^2$ or $\frac{22.6}{360}\times\pi\times8^2$

The angle *ABC* is given in the question as 0.394 and should be used (or a more accurate value). Do not allow use of a different value for angle *ABC* (other than the angle in degrees).

There are many acceptable alternative approaches to find the area of triangle *ABD*, e.g. using their angle *ADB* **or** their angle *BAD* **or** using right-angled triangles

e.g. $\frac{1}{2}(13)(\text{"3.07"})$ **or** $\frac{1}{2}(\text{"7.39"})(\text{"3.07"})+\frac{1}{2}(\text{"5.614"})(\text{"3.07"})$ and could be implied by e.g. 11.3 + 8.6

Alternatively, attempts the area of the triangle *ACD* $\left(\text{e.g. }\frac{1}{2}(5)(6.4)\sin\text{"0.501"} = \text{awrt } 7.68\right)$

or the area of the segment (in sector *ABC*) $\left(\frac{1}{2}(8^2)(0.394)-\frac{1}{2}(8^2)\sin 0.394 = \text{awrt } 0.324\right)$

via a **correct** method. The area of the triangle *ABC* alone is insufficient for this mark.

Note that $\sin 0.394$ might be seen as $\sqrt{1-\left(\frac{4801}{5200}\right)^2}$

For values, see the diagram below.

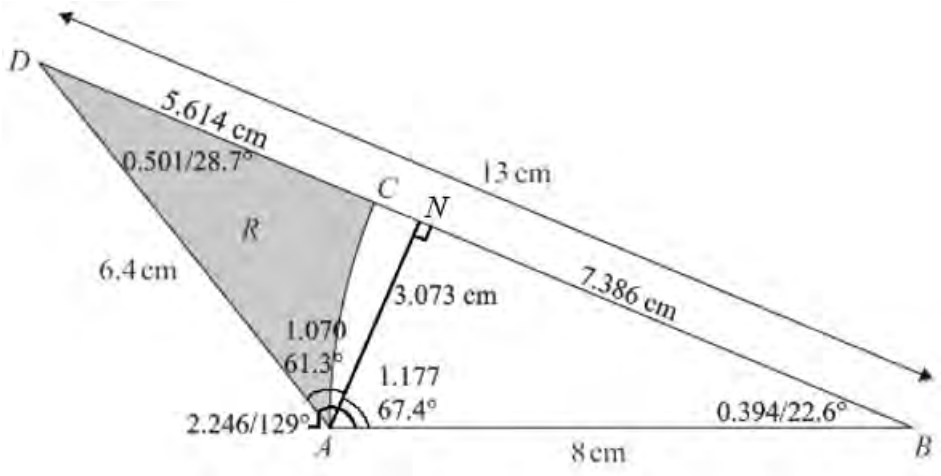
dM1: Complete and correct method for the area of *R*. Requires correct attempts at both the area of the sector **and** the area of the triangle *ABD* **and** subtraction of the two values (or expressions). Allow sector – triangle *ABD* if this is then recovered by making the area positive.

Alternatively, correct attempts at both the area of the triangle *ACD* **and** the area of the segment (in sector *ABC*) **and** subtraction of the two values (or expressions).

Condone slips in substitution provided a correct formula is seen.
May be implied by a correct answer.

A1: awrt 7.34 to 7.37 no units required but penalise incorrect units.
ISW after an acceptable answer is seen.

Helpful Diagram:



Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
13(i)	$(n =)\ 3$	B1	2.2a
		(1)	
(ii)	30 solutions **or** e.g. each interval of length π has 6 solutions	B1	2.2a
	e.g., Each interval of length π has 6 solutions, and there are 5 intervals of length π, so $5\times6=30$ solutions.	dB1	2.4
		(2)	
			(3 marks)

Notes

(i)

B1: cao $(n =)\ 3$

(ii)

B1: Either:

- Deduces 30 solutions
- **or** deduces the correct **total** number of solutions for $\sin(nx) = k$ **and** $\sin(nx) = -k$ (or $\sin^2(nx) = k^2$) in any relevant interval (0 to π, 2π, 4π or 5π)
- **or** deduces the correct number of solutions in 0 to 5π for either $\sin(nx) = k$ or $\sin(nx) = -k$
- **or** deduces the correct **total** number of solutions for $\sin(x) = k$ **and** $\sin(x) = -k$ in any relevant interval (0 to π, 2π, 4π or 5π)

dB1: Requires 30 (solutions) and correct justification comprising all the elements in **one** of the bullet points below. See the tables on the next page for the correct values.

- Deduces the correct **total** number of solutions for $\sin(nx) = k$ **and** $\sin(nx) = -k$ (or $\sin^2(nx) = k^2$) in any relevant interval (0 to π, 2π or 4π) **and** scales the interval to 0 to 5π
- **or** deduces the correct number of solutions in 0 to 5π for $\sin(nx) = k$ **and** for $\sin(nx) = -k$ and then adds
- **or** finds the **total** correct number of solutions for $\sin(x) = k$ **and** $\sin(x) = -k$ in any relevant interval (0 to π, 2π, 4π or 5π) **and** then scales the interval to 0 to 5π **and** multiplies by 3

Just stating e.g. $2\times6\times2.5$ is insufficient for the dB1 mark without further justification. There are many acceptable variations, and some examples are below and on the next page.

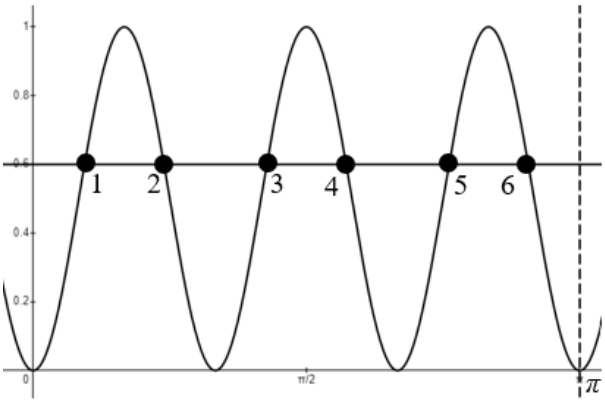
Some examples:

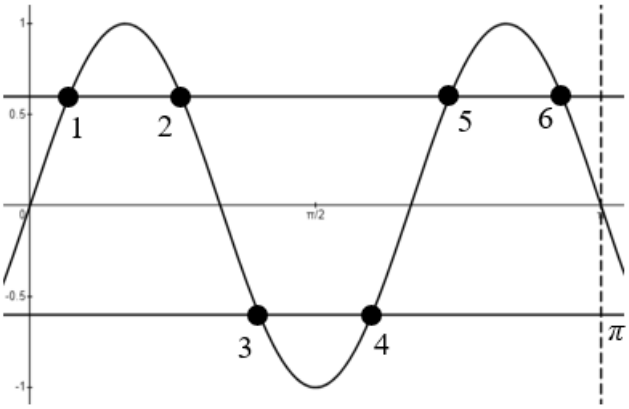
- 6 solutions in each interval of length π (B1) **and** 5 intervals of length π so 30 (dB1)
- 12 solutions in each interval of length 2π (B1) **and** 2.5 intervals of length 2π so 30 (dB1)
- 2 solutions for $\sin x = k$ up to 2π, so for $\sin^2 x$ there are 4 per 2π (B1)
  and since $0 \leqslant x < 5\pi$ then $0 \leqslant 3x < 15\pi$ so $\frac{15}{2}\times4=30$ (dB1)
- Double the 6 solutions since $\sin(nx) = \pm k$ (or $\sin^2(nx) = k^2$) so 12 (B1) **and** so 24 in 4π, hence 30 in 5π (dB1)
- 16 solutions for $\sin(nx) = k$ (B1) **and** 14 (solutions) for $\sin(nx) = -k \rightarrow 30$ (dB1)
- $\sin^2(3x)=\frac{1-\cos(6x)}{2}$ and as $\cos x = k$ has 5 solutions in 0 to 5π (B1) **and** so $\cos 6x$ has $5\times6 = 30$ (dB1)

Graphical approaches may be used but these must be convincing, clearly showing the correct number of solutions in one of the relevant intervals. Be lenient with the shape of the curve.

e.g. 30 solutions in $0 \leq x < 5\pi$ [with 6 solutions on, for example, either sketch below]

$y = \sin^2(3x)\ 0 \leq x < \pi$ $y = \sin(3x)\ 0 \leq x < \pi$





Note that a suitable graph of sin$x$ showing, e.g. 10 solutions up to 5π, followed by 30 (calculation of ×3 clearly implied) would also be eligible to score the dB1.

The following are examples of an incorrect justification but scores B1dB0 for reaching 30:

- sin(nx) = k has 3×6 solutions in the interval 0 to 5π, sin(nx) = –k has 3×4 solutions (in the interval 0 to 5π) so 18 + 12 = 30 solutions in total.
- sin(nx) = k has 15 solutions in the interval 0 to 5π, so for sin^2(nx) has 15×2 = 30 solutions in total.

For reference, the tables below show the total number of solutions for each branch of $\sin^2(nx) = k^2$ with n = 3 and the bold values score the first B1 **provided they are in the correct interval**.

	$0 \leq x < \pi$	$0 \leq x < 2\pi$	$0 \leq x < 4\pi$	$0 \leq x < 5\pi$
$\sin(nx) = k$	4	6	12	**16**
$\sin(nx) = -k$	2	6	12	**14**
Total solutions	**6**	**12**	**24**	**30**

and n = 1 for those that deal with nx (3x) last.

	$0 \leq x < \pi$	$0 \leq x < 2\pi$	$0 \leq x < 4\pi$	$0 \leq x < 5\pi$
$\sin(x) = k$	2	2	4	6
$\sin(x) = -k$	0	2	4	4
Total solutions	**2**	**4**	**8**	**10**

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
15(a)	$\left(f'(x)=\right)\ \dfrac{\lambda x\left(1+x^2\right)^2-\mu x\left(1-x^2\right)\left(1+x^2\right)}{\left(1+x^2\right)^4}$	M1	1.1b
	$=\dfrac{-2x\left(1+x^2\right)^2-4x\left(1-x^2\right)\left(1+x^2\right)}{\left(1+x^2\right)^4}$	A1	1.1b
	e.g. $\dfrac{-2x\left(1+x^2\right)-4x\left(1-x^2\right)}{\left(1+x^2\right)^3}=0$ $\Rightarrow\left(1+x^2\right)+2\left(1-x^2\right)=0$ $\Rightarrow 3-x^2=0$ or e.g. $=\dfrac{-2x\left(1+x^2\right)\left[\left(1+x^2\right)+2\left(1-x^2\right)\right]}{\left(1+x^2\right)^4}$ $=\dfrac{-2x\left(1+x^2\right)\left(3-x^2\right)}{\left(1+x^2\right)^4}$	dM1	3.1a
	e.g. $3-x^2=0$ or $2x\left(1+x^2\right)\left(x^2-3\right)=0$ $\Rightarrow x=...\Rightarrow y=\ldots$	ddM1	2.1
	$\left(-\sqrt{3},-\dfrac{1}{8}\right)$	A1	2.3
		(5)	

Notes

(a) There may be other valid ways to differentiate or to solve the resulting equation.

M1: Attempts the quotient rule to achieve $\dfrac{\pm\lambda x\left(1+x^2\right)^2\pm\mu x\left(1-x^2\right)\left(1+x^2\right)}{\left(1+x^2\right)^4}$

but condone $\dfrac{\pm\lambda x\left(1+x^2\right)^2\pm\mu x\left(1-x^2\right)\left(1+x^2\right)}{\left(1+x^2\right)^2}$ provided an incorrect quotient rule is not seen.

Alternatively, attempts the product rule on $f(x)=\left(1-x^2\right)\left(1+x^2\right)^{-2}$ to achieve

$\pm\lambda x\left(1+x^2\right)^{-2}\pm\mu x\left(1-x^2\right)\left(1+x^2\right)^{-3}$

There may be attempts at splitting the numerator to $f(x)=\left(1+x^2\right)^{-2}-x^2\left(1+x^2\right)^{-2}$ which should differentiate to $\pm\lambda x\left(1+x^2\right)^{-3}\pm\mu x\left(1+x^2\right)^{-2}\pm\gamma x^3\left(1+x^2\right)^{-3}$

In **all** cases, do not penalise signs of λ, μ and/or γ unless an incorrect quotient rule or an incorrect product rule is stated.

Any occurrences of $\left(1+x^2\right)^2$ may be replaced with $1+2x^2+x^4$

There is no need for a LHS e.g. $f'(x)=$ to be present so ignore an incorrect LHS.

Invisible brackets may be recovered/implied by later work.

A1: Correct differentiation. May be unsimplified. Ignore absence of (or incorrect) LHS.

The correct derivative using the quotient rule is $=\dfrac{-2x\left(1+x^2\right)^2-4x\left(1-x^2\right)\left(1+x^2\right)}{\left(1+x^2\right)^4}$ o.e.

using the product rule is $-2x\left(1+x^2\right)^{-2}-4x\left(1-x^2\right)\left(1+x^2\right)^{-3}$ o.e.

and splitting the numerator is $-4x\left(1+x^2\right)^{-3}-2x\left(1+x^2\right)^{-2}+4x^3\left(1+x^2\right)^{-3}$ o.e.

You may need to check carefully for equivalent derivatives and ISW after a correct expression is seen. Note that some candidates may set = 0 at the start and may omit the denominator as a result – send to review if you are unsure in such cases.

dM1: Attempts to reduce the expression to a suitable form so that the roots can be found, either by cancelling a factor of $\left(1+x^2\right)$ and simplifying the other brackets

e.g. $\dfrac{-2x\left(1+x^2\right)-4x\left(1-x^2\right)}{\left(1+x^2\right)^3} \rightarrow$ e.g. $\dfrac{-6x+2x^3}{\left(1+x^2\right)^3}$ or $\dfrac{2x\left(x^2-3\right)}{\left(1+x^2\right)^3}$ or $2x\left(x^2-3\right)\ (=0)$

or by attempting to take x, $\left(1+x^2\right)$ or $\pm 2x\left(1+x^2\right)$ out as a factor and simplify the other brackets. If they only take out a factor of x then they must have $x\left(\pm Ax^4 \pm Bx^2 \pm C\right)$

e.g. $\dfrac{-2x\left(1+x^2\right)\left[\left(1+x^2\right)+2\left(1-x^2\right)\right]}{\left(1+x^2\right)^4} \rightarrow$ e.g. $\dfrac{-2x\left(1+x^2\right)\left(3-x^2\right)}{\left(1+x^2\right)^4}$ or $\dfrac{x\left(2x^4-4x^2-6\right)}{\left(1+x^2\right)^4}$

Depends on the first M mark.

Note: they might attempt to expand the brackets first, or set = 0 and multiply through by $\left(1+x^2\right)^k$ and/or divide through by x or $2x$ at any stage.

Do not be concerned if any factors of $\left(1+x^2\right)$, x or 2 go "missing" during their work.

The = 0 does not explicitly need to be seen.

Look for $\pm\dfrac{2x\left(1+x^2\right)\left(Ax^2+B\right)}{\left(1+x^2\right)^k}$ or $\pm\dfrac{\left(1+x^2\right)\left(Ax^3+Bx\right)}{\left(1+x^2\right)^k}$ or $\pm\dfrac{x\left(Ax^4+Bx^2+C\right)}{\left(1+x^2\right)^k}$ where

- k may be 0, 1, 2, 3 or 4
- A and B (and C if present) are non-zero
- any of $2x$ or $\left(1+x^2\right)$ in the numerator and/or $\left(1+x^2\right)^k$ in the denominator may be absent

ddM1: Attempts to solve their numerator set = 0 using a valid non-calculator method **and** uses a solution for x to find a corresponding value for y.

The substitution may be implied by their value of y but, if not, the substitution must be seen.

- For either Ax^2+B or Ax^3+Bx, we require $A \neq B$ and $A \times B < 0$ From either expression they can write down their $x=\pm\sqrt{\dfrac{-B}{A}}$ without working.
- For Ax^4+Bx^2+C they must show a valid non-calculator method for solving the quartic, treating it as a quadratic in x^2 and using the usual rules for solving a quadratic algebraically, e.g. $a=x^2 \rightarrow 2a^2-4a-6\,(=0) \rightarrow (a+1)(2a-6)\,(=0) \rightarrow x=\pm\sqrt{3}$. They must reach a value for x (and not just x^2) as well as finding a value for y.

Dependent on both previous method marks. They must use a value for x that is not –1, 0 or 1.

A1: Deduces the correct **exact** coordinates for $P\left(-\sqrt{3}, -\dfrac{1}{8}\right)$ o.e. $\left(-\sqrt{3}, -0.125\right)$

Requires all the previous marks to have been scored.

Condone $x=-\sqrt{3}$, $y=-0.125$ or e.g. $-\sqrt{3}, -\dfrac{1}{8}$

If there is more than one pair of coordinates given, then the correct coordinates must be clearly selected or any others clearly rejected.

Ignore any mistakes that occur if they multiply out the denominator $\left(1+x^2\right)^4$ after differentiation.

Question	Scheme	Marks	AOs
15(b)	$x=-1\Rightarrow\alpha=-\dfrac{\pi}{4}$ and $x=1\Rightarrow\beta=\dfrac{\pi}{4}$	B1	2.2a
	$\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=\sec^2\theta$	B1	1.1b
	$\displaystyle\int\frac{1-x^2}{\left(1+x^2\right)^2}\mathrm{d}x=\int\frac{1-\tan^2\theta}{\left(1+\tan^2\theta\right)^2}\text{"}\sec^2\theta\text{"}\,\mathrm{d}\theta$ or $=\displaystyle\int\frac{1-\tan^2\theta}{\sec^4\theta}\text{"}\sec^2\theta\text{"}\,\mathrm{d}\theta$	M1	1.1b
	$=\int\left(1-\tan^2\theta\right)\cos^2\theta\,\mathrm{d}\theta$ $=\displaystyle\int\left(1-\frac{\sin^2\theta}{\cos^2\theta}\right)\cos^2\theta\,\mathrm{d}\theta$ $=\int\left(\cos^2\theta-\sin^2\theta\right)\mathrm{d}\theta$ $=\displaystyle\int\frac{2-1-\tan^2\theta}{\sec^2\theta}\mathrm{d}\theta$ $=\displaystyle\int\left(\frac{2}{\sec^2\theta}-\frac{1+\tan^2\theta}{\sec^2\theta}\right)\mathrm{d}\theta$ $=\int\left(2\cos^2\theta-1\right)\mathrm{d}\theta$	dM1	3.1a
	$=\int\cos 2\theta\,\mathrm{d}\theta$ *	A1*	2.1
		(5)	

Notes

(b) **Mark parts (b) and (c) together.**

B1: Deduces the correct limits for the integral in θ which may be seen separately as side working or within their integral work. This mark cannot be scored working in degrees.

Allow $\dfrac{3\pi}{4}$ instead of $-\dfrac{\pi}{4}$ but not decimal approximations.

B1: $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=\sec^2\theta$ or equivalent e.g. $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=1+x^2$ or $\dfrac{\cos^2\theta--\sin^2\theta}{\cos^2\theta}$ (coming from $x=\dfrac{\sin\theta}{\cos\theta}$) but must be correct, so not e.g. $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=\sec^2 x$ unless recovered.

M1: Makes a complete attempt at using the substitution $x=\tan\theta$

Requires:

- An attempt at using $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}$ to replace dx with $\mathrm{d}\theta$ either way round, so if $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=\mathrm{g}(\theta)$ allow either $\mathrm{d}x=\mathrm{g}(\theta)\{\mathrm{d}\theta\}$ **or** $\mathrm{d}x=\dfrac{1}{\mathrm{g}(\theta)}\{\mathrm{d}\theta\}$. $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}$ must be a function of θ and not a constant.
- All terms in x replaced with $\tan\theta$ (or $1+x^2$ with $\sec^2\theta$)

Condone the absence of $\mathrm{d}\theta$ but dx must no longer be present.

Condone if they fail to square the denominator or e.g. a slip in missing a θ

Use of notation such as $\mathrm{d}(\tan\theta)$ is correct but does not score the M1 until replaced with $\sec^2\theta\{\mathrm{d}\theta\}$ or their derivative of $\tan\theta$ (which must not be a constant multiple of $\tan\theta$).

dM1: Uses trigonometric identities e.g. $\pm1\pm\tan^2\theta=\pm\sec^2\theta$, $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$, $\sec\theta=\dfrac{1}{\cos\theta}$, $\pm\cos^2\theta\pm\sin^2\theta=\pm1$ to simplify the integral to an expression of the form $\pm a\sin^2\theta\pm b\cos^2\theta\pm c$ where one of a, b or c may be 0.

The algebra should essentially be correct but condone e.g. sign slips or errors collecting terms.

Alternatively, e.g. $\int \frac{1-\tan^2\theta}{1+\tan^2\theta}\mathrm{d}\theta = \int \frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta}\mathrm{d}\theta = \int \frac{\cos^2\theta-\sin^2\theta}{1}\mathrm{d}\theta$

They must have replaced dx with $\mathrm{d}\theta$ the correct way round for this mark, so if $\frac{\mathrm{d}x}{\mathrm{d}\theta} = \mathrm{g}(\theta)$ then they must have $\mathrm{d}x = \mathrm{g}(\theta)\{\mathrm{d}\theta\}$ and **not** $\mathrm{d}x = \frac{1}{\mathrm{g}(\theta)}\{\mathrm{d}\theta\}$

It is acceptable to replace e.g. $\tan^2\theta\cos^2\theta$ with $\sin^2\theta$
Dependent on the previous method mark.

A1*: cso Arrives at $\int \cos 2\theta\,\mathrm{d}\theta$ with sufficient working shown and no incorrect work ignoring limits. The final line must be fully correct, including the integral sign and $\mathrm{d}\theta$ (ignoring limits). Note that this may be seen in part (c) and may score the mark.
All trigonometric identities must be fully correct.
Condone one or two slips in a missing θ but not frequent omissions.
Condone missing integral signs in their intermediate work, but it must be present on the final line.
dx must be replaced with $\mathrm{d}\theta$ at some stage before the final line but does not need to be present in every intermediate line of working.
Condone notational errors throughout e.g. $\sin\theta^2$ provided they are recovered.
This mark is independent from any work to do with limits, i.e., B0B1M1dM1A1* is possible.

Question	Scheme	Marks	AOs
15(c)	$\int \cos 2\theta \, \mathrm{d}\theta = \frac{1}{2}\sin 2\theta$	M1	1.1b
	$\left[\frac{1}{2}\sin 2\theta\right]_{"-\frac{\pi}{4}"}^{"\frac{\pi}{4}"} = \frac{1}{2}\sin\left(2\times\frac{\pi}{4}\right) - \frac{1}{2}\sin\left(2\times-\frac{\pi}{4}\right)$	dM1	1.1b
	$=1$	A1	2.1
		(3)	
			(13 marks)

Notes

(c) **Mark parts (b) and (c) together.**

M1: $\int \cos 2\theta \, \mathrm{d}\theta \rightarrow \pm\frac{1}{2}\sin 2\theta \{+c\}$

dM1: Substitutes their changed limits (not 1 and –1) into $\pm\frac{1}{2}\sin 2\theta$ and subtracts either way round.

May be implied e.g. $\frac{1}{2}+\frac{1}{2}$ provided they have the correct limits.

Dependent on the previous method mark.

A1: Area of 1 found following correct work. This mark requires clear substitution of the correct limits into $\frac{1}{2}\sin 2\theta$ the correct way round. If they have the limits the wrong way round and achieve an answer of –1 they cannot just make the answer positive for this mark.

They may use limits from 0 to $\frac{\pi}{4}$ and multiply their result by 2.

$\frac{3\pi}{4}$ may be used in place of $-\frac{\pi}{4}$ as the lower limit which is acceptable.

Condone any spurious integral symbol accompanying $\frac{1}{2}\sin 2\theta$

The substitution may be implied by e.g. $\frac{1}{2}--\frac{1}{2}$ or $\frac{1}{2}\sin\left(\frac{\pi}{2}\right)-\frac{1}{2}\sin\left(-\frac{\pi}{2}\right)$ but must follow sight of $\frac{1}{2}\sin 2\theta$

Condone use of limits –45 and 45 (degrees) the correct way round for full marks in part (c).
Note that use of a calculator on the original integral will give the correct answer. An answer of 1 scores no marks without evidence of scoring both method marks above including the substitution.

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Question	Scheme	Marks	AOs
12(a)	$K = 500$	B1	1.1b
	$\tan\alpha = \dfrac{480}{140} \Rightarrow \alpha = ...$	M1	1.1b
	$\alpha =$ awrt $73.74°$ or $500\cos(\theta + 73.74)°$	A1	1.1b
		(3)	
(b)(i)	$R = 1000 + 500\cos(30t + 73.74)°$ or $R = 1000 + 140\cos(30t)° - 480\sin(30t)°$	B1ft	3.3
(b)(ii)	$\{R_{\min} =\} 500$	B1ft	3.4
		(2)	
(c)	$t = 3.5 \Rightarrow R = "1000" + "500"\cos(30(3.5) + "73.74")° = ...$	M1	3.4
	$R =$ awrt 500.1… so the model is reliable	A1	3.5a
		(2)	
(d)	$\sin(30t + 70)° = -1 \Rightarrow 30t + 70 = 270 \Rightarrow 30t = ...$ (or $t = ...$)	M1	3.4
	$30t = 200$ $\left(\text{or } t = \dfrac{20}{3}\right)$	A1	1.1b
	$R = "1000" + "500"\cos\left(30\left("\dfrac{20}{3}"\right) + "73.74"\right)°$ or $R = "1000" + 140\cos("200")° - 480\sin("200")°$	dM1	3.4
	$R = 1032$ (or 1033)	A1	1.1b
		(4)	
			(11 marks)

Notes

Note: Candidates working in radians are able to score all the M and B marks in this question. Condone the absence of the degrees symbol throughout the whole question.

(a)

B1: Correct value for K. Condone $R = 500$

M1: Award for $\tan\alpha = \pm\dfrac{480}{140} \Rightarrow \alpha = ...$, $\tan\alpha = \pm\dfrac{140}{480} \Rightarrow \alpha = ...$, $\sin\alpha = \pm\dfrac{480}{"500"} \Rightarrow \alpha = ...$ or $\cos\alpha = \pm\dfrac{140}{"500"} \Rightarrow \alpha = ...$

Note $\alpha =$ awrt 1.3 (rad) implies this mark.

A1: $\alpha =$ awrt $73.74\{°\}$ or correct expression $500\cos(\theta + 73.74)\{°\}$

(b)(i) Note: mark parts (b)(i) and (b)(ii) together.

B1ft: Correct equation of the model in either form including the $R =$ following through on their numerical K ($0 < K$,, 750) and their numerical α.

Allow for e.g. $R = 1500 - "500" + "500"\cos(30t + "73.74")\{°\}$ or for $R = 1500 - "500" + 140\cos(30t)° - 480\sin(30t)°$ but not e.g. $R = 1500 - K + K\cos(30t + \alpha)°$

$R = 1000 + 140\cos 30t - 480\sin 30t$ (without the brackets) is correct.

Allow this mark if they have truncated or rounded an otherwise correct α (to 3s.f.)

(b)(ii)

B1ft: 500 or follow through on (their A – their K) or (1500 – 2 × their K) provided it is non-negative and less than 1500. It must be clear this is their answer to (b)(ii) so expect to see e.g. (b) or $R_{\min} =$ or an indication it is the minimum.

(c) Note: if θ is used in place of $30t$ then they must revert back to $30t$ correctly to access the marks.

M1: Substitutes $t = 3.5$ into their model for the number of rabbits (you may need to check if no method is shown)

or substitutes $t = 3.5$ into their $\cos(30t + \alpha)°$

Condone substitution of a value of t in the range $3 \text{ „ } t \text{ „ } 4.5$ for this mark.

A1: R = awrt 500.1… **or** 500 (not awrt) following substitution of $t = 3.5$, suggesting that the model is valid/reliable/appropriate/good.

or $\cos(30(3.5) + 73.74)\{°\} \approx -1$ suggesting that the model is valid/reliable/appropriate/good.

Allow this mark if they have truncated or rounded an otherwise correct α (to 3s.f.)

Alt:

M1: Minimum occurs when $A + K\cos(30t + \alpha)° = R_{\min} \Rightarrow \cos(30t + \alpha)° = \lambda$ with $|\lambda| \text{ „ } 1$ leading to $t = ...$

May just see $\cos(30t + 73.74)° = -1 \Rightarrow t = ...$ (or their $\cos(30t + \alpha)° = -1 \Rightarrow t = ...$)

$30t + \alpha = 180 \Rightarrow t = ...$ implies this mark. Condone $30t + \alpha = \pi \Rightarrow t = ...$ for this mark.

A1: $t = 3.54...$ (i.e. the middle of April) so the model is valid/reliable/appropriate/good.

Do not condone incorrect statements, e.g., $t = 3.54...$ i.e. the middle of March so close to middle of April. If using $A + K\cos(30t + \alpha)° = R_{\min}$ then $R_{\min}$ must be = their A – their K

Allow this mark if they have truncated or rounded an otherwise correct α (to 3s.f.)

Alt 2 using differentiation (Condoned)

M1: Condone finding the minimum using $...\sin(30t + 73.74)° = 0 \Rightarrow t = ...$ (or their $\sin(30t + \alpha)° = 0 \Rightarrow t = ...$)

$30t + \alpha = 180 \Rightarrow t = ...$ implies this mark. Condone $30t + \alpha = \pi \Rightarrow t = ...$ for this mark.

A1: $t = 3.54...$ (i.e. the middle of April) suggesting that the model is valid/reliable/appropriate.

Do not condone incorrect statements, e.g., $t = 3.54...$ i.e. the middle of March so close to middle of April.

The complete derivative for $\frac{\mathrm{d}R}{\mathrm{d}t}$ does not need to be seen.

Allow this mark if they have truncated or rounded an otherwise correct α (to 3s.f.)

(d) Note: if θ is used in place of $30t$ then they must revert back to $30t$ correctly to access the marks.

M1: Realises that $\sin(30t + 70)° = -1$, reaches $30t + 70 = 270$ or –90 and attempts to find t (or $30t$)

Condone attempts using differentiation. The minimum occurs when $\cos(30t + 70)° = 0 \Rightarrow$ $30t + 70 = 270 \Rightarrow 30t = ...$ (or t = …). They must use 270 or –90 and **not** 90 to achieve the minimum. Condone $30t + \alpha = \frac{3\pi}{2} \Rightarrow t = ...$ for this mark but not $30t + \alpha = \frac{\pi}{2} \Rightarrow t = ...$

A1: Correct value for $30t$ (or t) Accept rounded or truncated values to at least 3s.f. e.g. 6.66 or 6.67

dM1: Substitutes their value of $t > 0$ (or $30t > 0$) coming from $30t + 70 =$ **270** into their model for R

A1: Correct number of rabbits. Allow 1032 or 1033 but must be whole numbers and not just 1030.

Allow this mark if they have truncated or rounded an otherwise correct α (to 3s.f.)

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
13(a)	$x = a\sin^2\theta \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 2a\sin\theta\cos\theta$	B1	1.1b
	$\int x^{\frac{1}{2}}\sqrt{a-x}\,\mathrm{d}x = \int \sqrt{a}\sin\theta\sqrt{a-a\sin^2\theta}\times 2a\sin\theta\cos\theta\{\mathrm{d}\theta\}$	M1	2.1
	$= \int \sqrt{a}\sin\theta\sqrt{a}\cos\theta\times 2a\sin\theta\cos\theta\,\mathrm{d}\theta = 2a^2\int \sin^2\theta\cos^2\theta\,\mathrm{d}\theta$ $= 2a^2\int\left(\frac{1}{2}\sin 2\theta\right)^2\mathrm{d}\theta$	dM1	3.1a
	Replaces or considers limits $\{x=0\Rightarrow\}\,\theta=0,\ \{x=a\Rightarrow\}\,\theta=\dfrac{\pi}{2}$ $= \frac{1}{2}a^2\int_0^{\frac{\pi}{2}}\sin^2 2\theta\,\mathrm{d}\theta$ *	A1*	1.1b
		(4)	
(b)	$\ldots\int \sin^2 2\theta\,\mathrm{d}\theta \rightarrow \ldots\int \frac{1-\cos 4\theta}{2}\mathrm{d}\theta$	M1	1.1b
	$\rightarrow \ldots\left(\frac{\theta}{2}-\frac{1}{8}\sin 4\theta\right)$	dM1	2.1
	$\mu\int \sin^2 2\theta\,\mathrm{d}\theta \rightarrow \frac{\mu}{2}\left(\theta-\frac{1}{4}\sin 4\theta\right)$	A1	1.1b
	$\left\{\int_0^a x^{\frac{1}{2}}\sqrt{a-x}\,\mathrm{d}x\right\} = \frac{a^2}{4}\left(\frac{\pi}{2}-0-0\right) = \frac{1}{8}\pi a^2$	A1	1.1b
		(4)	
			(8 marks)

Notes

(a)

B1: $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 2a\sin\theta\cos\theta$ or $\mathrm{d}x = 2a\sin\theta\cos\theta\,\mathrm{d}\theta$ o.e. seen or implied by their substitution.

Note that writing $x = a\sin^2\theta = \frac{a}{2}(1-\cos 2\theta) \rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}\theta} = a\sin 2\theta$ is correct. Condone use of $\dfrac{\mathrm{d}x}{\mathrm{d}a}$

M1: Attempts to substitute, fully replacing $x^{\frac{1}{2}}$ and $\sqrt{a-x}$ with θ's and dx with their dx = …

Look for $x^{\frac{1}{2}}\sqrt{a-x}\,\mathrm{d}x \rightarrow \mathrm{f}(\theta)\mathrm{g}(\theta)\mathrm{h}(\theta)$ where

- $\mathrm{f}(\theta)$ is an attempt at $\sqrt{a\sin^2\theta}$ e.g. allow $a\sin\theta$ but just $a\sin^2\theta$ is not condoned
- $\mathrm{g}(\theta)$ is an attempt at $\sqrt{a-a\sin^2\theta}$ but not $\sqrt{a}-\sqrt{a\sin^2\theta}$ unless $\sqrt{a-a\sin^2\theta}$ is attempted first
- $\mathrm{h}(\theta)$ = their dx or their $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}$ or $\dfrac{1}{\text{their }\frac{\mathrm{d}x}{\mathrm{d}\theta}}$ (in terms of θ only but condone da seen)

Condone slips provided the intention is clear, e.g. $x^{\frac{1}{2}} \rightarrow \sqrt{a}\sin^2\theta$ but x must be eliminated. There is no need to replace the limits and condone the absence of dθ and/or the integral sign for this mark.

dM1: Attempts to use $\sin 2\theta = 2\sin\theta\cos\theta$ to convert an integral of the form $\int \sin^2\theta\cos^2\theta\,\mathrm{d}\theta$ or

e.g. the form $\int \sin\theta\cos\theta\sin 2\theta\,\mathrm{d}\theta$ to $\int ...\sin^2 2\theta\,\mathrm{d}\theta$

There is no need to replace the limits and condone the absence of $\mathrm{d}\theta$ and/or the integral sign for this mark.

A1*: Replaces or considers limits $\{x=0\Rightarrow\}\theta=0,\ \{x=a\Rightarrow\}\theta=\dfrac{\pi}{2}$ at some stage before the given answer and proceeds with no errors to the given answer. The replaced limits may appear with their integral symbol and do not have to be justified and do not have to appear on every line. Condone infrequent slips in notation, e.g. $\sin\theta^2$ in a line as long as it is not consistently poor. You must see the integral sign with the correct limits and the $\mathrm{d}\theta$ together in the given answer.

(b)

M1: Adopts an appropriate strategy by using the double angle identity to obtain an integrable form $...\int \sin^2 2\theta\,\mathrm{d}\theta \to ...\int \dfrac{\pm1\pm\cos 4\theta}{2}\mathrm{d}\theta$ which may be seen as

$\lambda\int \sin^2 2\theta\,\mathrm{d}\theta \to \dfrac{\lambda}{2}\int \pm1\pm\cos 4\theta\,\mathrm{d}\theta$ with the $\dfrac{1}{2}$ absorbed into their coefficient of the integral.

dM1: Integrates into the form $\pm p\theta \pm q\sin 4\theta$

A1: Correct integration of $\mu\int \sin^2 2\theta\,\mathrm{d}\theta$ to $\dfrac{\mu}{2}\left(\theta-\dfrac{1}{4}\sin 4\theta\right)$. Here μ may be 1.

Condone lack of limits here.

A1: Applies limits to the correct integral and proceeds to $\dfrac{1}{8}\pi a^2$ following correct work.

There is no need to see 0 substituted in and condone any omission of integral signs and/or $\mathrm{d}\theta$

Note that $\dfrac{a^2}{4}\left[\theta+\dfrac{1}{4}\sin 4\theta\right]_0^{\frac{\pi}{2}}=\dfrac{1}{8}\pi a^2$ is incorrect and scores M1dM1A0A0

Use of $\sin^2 2\theta=\dfrac{\pm1\pm\cos k\theta}{2}$ with $k\neq 4$ scores M0dM0A0A0 but may lead to $\dfrac{1}{8}\pi a^2$

Condone use of x in place of θ e.g. $\dfrac{a^2}{4}\left[x-\dfrac{1}{4}\sin 4x\right]_0^{\frac{\pi}{2}}=\dfrac{1}{8}\pi a^2$

See overleaf for some alternative approaches.

Alternative 13(a) working backwards:

$\frac{1}{2}a^2\int_0^{\frac{\pi}{2}}\sin^2 2\theta\,\mathrm{d}\theta$ $\qquad$ $\frac{\mathrm{d}x}{\mathrm{d}\theta}=2a\sin\theta\cos\theta$ score B1 (as in main scheme)

$=\frac{1}{2}a^2\int_0^{\frac{\pi}{2}}(2\sin\theta\cos\theta)(2\sin\theta\cos\theta)\,\mathrm{d}\theta$

$=a\int_0^{\frac{\pi}{2}}\sin\theta\cos\theta\,\mathrm{d}x$

Score M1 here for using the double angle identity and replacing $\ldots\sin\theta\cos\theta\,\mathrm{d}\theta$ with $\mathrm{d}x$

$=a\int_0^{a}\sqrt{\frac{x}{a}}\sqrt{1-\frac{x}{a}}\,\mathrm{d}x$

Score dM1 here for a full attempt to replace all trig leading to everything in terms of x only

Must come from the form $\int\ldots\sin\theta\cos\theta\,\mathrm{d}x$

A1 fully correct with limits replaced / considered before the final line and the final line fully correct with limits, integral sign and dx as per the main scheme.

Alternative 13(b) via IBP Way 1:

$\int_0^{\frac{\pi}{2}}\sin^2 2\theta\,\mathrm{d}\theta$ $\qquad$ $\left\{\begin{array}{ll}u=\sin^2 2\theta & v'=1\\ u'=4\sin 2\theta\cos 2\theta & v=\theta\end{array}\right\}$

$=\left[\theta\sin^2 2\theta\right]_0^{\frac{\pi}{2}}-4\int_0^{\frac{\pi}{2}}\theta\sin 2\theta\cos 2\theta\,\mathrm{d}\theta$

$=\left[\theta\sin^2 2\theta\right]_0^{\frac{\pi}{2}}-2\int_0^{\frac{\pi}{2}}\theta\sin 4\theta\,\mathrm{d}\theta$ $\qquad$ $\left\{\begin{array}{ll}u=\theta & v'=\sin 4\theta\\ u'=1 & v=-\frac{\cos 4\theta}{4}\end{array}\right\}$

$=\left[\theta\sin^2 2\theta\right]_0^{\frac{\pi}{2}}-2\left(\left[-\frac{\theta\cos 4\theta}{4}\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}-\frac{\cos 4\theta}{4}\,\mathrm{d}\theta\right)$ $\qquad$ Score M1 here.

$=\left[\theta\sin^2 2\theta\right]_0^{\frac{\pi}{2}}-2\left[-\frac{\theta\cos 4\theta}{4}+\frac{\sin 4\theta}{16}\right]_0^{\frac{\pi}{2}}$ Score dM1 here, A1 if correct (ignoring limits).

$=\left[\theta\sin^2 2\theta+\frac{\theta\cos 4\theta}{2}-\frac{\sin 4\theta}{8}\right]_0^{\frac{\pi}{2}}$

$=\left(0+\frac{\pi}{4}-0\right)-(0+0-0)=\frac{\pi}{4}\ \rightarrow\ \left\{\int_0^{a}x^{\frac{1}{2}}\sqrt{a-x}\,\mathrm{d}x=\right\}=\frac{1}{8}\pi a^2$ score A1* (with no errors).

Alternative 13(b) via IBP Way 2:

$$\int_0^{\frac{\pi}{2}} \sin^2 2\theta \, \mathrm{d}\theta \qquad \left\{ \begin{array}{ll} u = \sin 2\theta & v' = \sin 2\theta \\ u' = 2\cos 2\theta & v = -\dfrac{\cos 2\theta}{2} \end{array} \right\}$$

$$= \left[-\frac{\sin 2\theta \cos 2\theta}{2} \right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} -\cos^2 2\theta \, \mathrm{d}\theta$$

$$\Rightarrow \int_0^{\frac{\pi}{2}} \sin^2 2\theta \, \mathrm{d}\theta + \int_0^{\frac{\pi}{2}} \sin^2 2\theta \, \mathrm{d}\theta = \left[-\frac{\sin 2\theta \cos 2\theta}{2} \right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos^2 2\theta \, \mathrm{d}\theta + \int_0^{\frac{\pi}{2}} \sin^2 2\theta \, \mathrm{d}\theta$$

$$\Rightarrow 2\int_0^{\frac{\pi}{2}} \sin^2 2\theta \, \mathrm{d}\theta = \left[-\frac{\sin 2\theta \cos 2\theta}{2} \right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} 1 \, \mathrm{d}\theta$$ Score M1 here.

$$\Rightarrow 2\int_0^{\frac{\pi}{2}} \sin^2 2\theta \, \mathrm{d}\theta = \left[-\frac{\sin 2\theta \cos 2\theta}{2} + \theta \right]_0^{\frac{\pi}{2}}$$ Score dM1 here, A1 if correct including the 2 (ignoring limits).

$$\Rightarrow \int_0^{\frac{\pi}{2}} \sin^2 2\theta \, \mathrm{d}\theta = \frac{1}{2}\left(0 + \frac{\pi}{2} \right) - \frac{1}{2}(0+0) = \frac{\pi}{4}$$

$$\rightarrow \left\{ \int_0^a x^{\frac{1}{2}} \sqrt{a-x} \, \mathrm{d}x = \right\} = \frac{1}{8}\pi a^2$$ score A1* (with no errors).

Question	Scheme	Marks	AOs

8(a) Way 1	$\left(\frac{1}{\text{cosec}\theta-1}+\frac{1}{\text{cosec}\theta+1}\equiv\right)\frac{\text{cosec}\theta+1+\text{cosec}\theta-1}{(\text{cosec}\theta-1)(\text{cosec}\theta+1)}$	B1	1.1b
	$\equiv\frac{2\text{cosec}\theta}{\text{cosec}^2\theta-1}\equiv\frac{2\text{cosec}\theta}{\cot^2\theta}$ or e.g. $\equiv\frac{2\sin\theta}{1-\sin^2\theta}=\frac{2\sin\theta}{\cos^2\theta}$	M1	1.1b
	$\frac{2\text{cosec}\theta}{\cot^2\theta}\equiv\frac{2}{\sin\theta}\times\frac{\sin^2\theta}{\cos^2\theta}\equiv 2\tan\theta\sec\theta*$ or $\frac{2\sin\theta}{\cos^2\theta}\equiv 2\tan\theta\sec\theta*$	A1*	2.1
		(3)	
(a) Way 2	**"Meets in the middle"**		
	$\left(\text{LHS}=\frac{1}{\text{cosec}\theta-1}+\frac{1}{\text{cosec}\theta+1}\equiv\right)\frac{\text{cosec}\theta+1+\text{cosec}\theta-1}{(\text{cosec}\theta-1)(\text{cosec}\theta+1)}$	B1	1.1b
	$\equiv\frac{2\text{cosec}\theta}{\text{cosec}^2\theta-1}\equiv\frac{2\text{cosec}\theta}{\cot^2\theta}$	M1	1.1b
	$\text{RHS}=2\tan\theta\sec\theta\equiv\frac{2\sin\theta}{\cos^2\theta}\equiv\frac{2\sin^2\theta}{\sin\theta\cos^2\theta}$ $\equiv\frac{2}{\sin\theta}\times\frac{\sin^2\theta}{\cos^2\theta}\equiv\frac{2\,\text{cosec}\theta}{\cot^2\theta}=\text{LHS}$ or e.g. QED or e.g. Proven	A1*	2.1
		(3)	

Part (a) Notes

(a) **Condone a complete proof entirely in *x* (or another variable) instead of θ**
Condone "=" for "≡"

Note that we are marking this as **B1M1A1** not **M1M1A1**

B1: Adds the fractions to obtain a **correct** single fraction (not fractions over fractions) in any form. Condone missing brackets when they combine their fractions as long as they are recovered to give a correct fraction.

This can be done in a variety of ways but when combined, the fraction must be **correct** e.g.

$$\frac{\text{cosec}\theta+1+\text{cosec}\theta-1}{(\text{cosec}\theta-1)(\text{cosec}\theta+1)} \text{ or } \frac{2\text{cosec}\theta}{(\text{cosec}\theta-1)(\text{cosec}\theta+1)} \text{ or } \frac{2\text{cosec}\theta}{\text{cosec}^2\theta-1}$$

$$\text{or e.g. } \left(\frac{1}{\frac{1}{\sin\theta}-1}+\frac{1}{\frac{1}{\sin\theta}+1}=\frac{\sin\theta}{1-\sin\theta}+\frac{\sin\theta}{1+\sin\theta}\right)=\frac{2\sin\theta}{1-\sin^2\theta} \text{ etc.}$$

M1: **Uses** a **correct** Pythagorean identity anywhere in their attempt e.g.
$\text{cosec}^2\theta-1=\cot^2\theta$, $\sin^2\theta+\cos^2\theta=1$ etc. or equivalent

A1*: Correct work with all necessary steps shown leading to the given answer. See scheme for the necessary steps. They need to proceed via sine and cosine to the given answer. There should be **no notational or bracketing errors and no mixed or missing variables**. E.g. we would consider $\cos^2\theta$ written as $\cos\theta^2$ a notational error.
Condone reaching $2\sec\theta\tan\theta*$

Way 2 (Meet in the middle)

B1: See Way 1

M1: See Way 1

A1*: Correct work on the RHS with all necessary steps shown leading to showing the equivalence with the LHS. See scheme for the necessary steps. There should be no notational or bracketing errors and no mixed or missing variables.
For this approach there must be a (minimal) conclusion e.g. "= LHS", "QED", "Hence proven" etc.

It is possible to start with the rhs e.g.:

$$2\tan\theta\sec\theta = 2\frac{\sin\theta}{\cos\theta}\times\frac{1}{\cos\theta}$$
$$= \frac{2\sin\theta}{\cos^2\theta} = \frac{2\text{cosec}\,\theta}{\cot^2\theta}$$
$$= \frac{2\text{cosec}\,\theta}{\text{cosec}^2\theta - 1} = \frac{2\text{cosec}\,\theta}{(\text{cosec}\,\theta - 1)(\text{cosec}\,\theta + 1)}$$
$$= \frac{\text{cosec}\,\theta + 1 + \text{cosec}\,\theta - 1}{(\text{cosec}\,\theta - 1)(\text{cosec}\,\theta + 1)}$$
$$= \frac{1}{\text{cosec}\,\theta - 1} + \frac{1}{\text{cosec}\,\theta + 1}$$

B1: Correctly reaches $2\tan\theta\sec\theta = \dfrac{2\text{cosec}\,\theta}{(\text{cosec}\,\theta - 1)(\text{cosec}\,\theta + 1)}$

M1: **Uses** a **correct** Pythagorean identity anywhere in their attempt e.g.
$\text{cosec}^2\theta - 1 = \cot^2\theta$, $\sin^2\theta + \cos^2\theta = 1$ etc. or equivalent

A1*: Correct work with all necessary steps shown leading to the lhs. See scheme for the necessary steps. There should be no notational or bracketing errors and no mixed or missing variables.

8(b)	$2\tan 2x\sec 2x = \cot 2x\sec 2x$	B1	2.2a
	$2\tan 2x\sec 2x - \cot 2x\sec 2x = 0$ $\Rightarrow \sec 2x\left(2\tan 2x - \cot 2x\right) = 0$ $2\tan 2x - \cot 2x = 0 \Rightarrow 2\tan 2x = \cot 2x \Rightarrow \tan^2 2x = \frac{1}{2}$ **or** $2\tan 2x - \cot 2x = 0 \Rightarrow 2\frac{\sin 2x}{\cos 2x} = \frac{\cos 2x}{\sin 2x} \Rightarrow 2\sin^2 2x = \cos^2 2x$ $\Rightarrow 2\sin^2 2x = 1 - \sin^2 2x \Rightarrow \sin^2 2x = \frac{1}{3}$ **or** $2\left(1 - \cos^2 2x\right) = \cos^2 2x \Rightarrow \cos^2 2x = \frac{2}{3}$ **or** $2\tan 2x = \cot 2x \Rightarrow \frac{4\tan x}{1-\tan^2 x} = \frac{1-\tan^2 x}{2\tan x}$ $\Rightarrow \tan^4 x - 10\tan^2 x + 1 = 0$ **or** $2\tan 2x - \cot 2x = 0 \Rightarrow 2\frac{\sin 2x}{\cos 2x} = \frac{\cos 2x}{\sin 2x} \Rightarrow 2\sin^2 2x = \cos^2 2x$ $2\sin^2 2x = \cos^2 2x \Rightarrow 1 - \cos 4x = \frac{1}{2}\left(\cos 4x + 1\right) \Rightarrow \cos 4x = \frac{1}{3}$	M1	2.1
	$2x = \tan^{-1}\frac{1}{\sqrt{2}} = K \Rightarrow x = \frac{K}{2}$ **or** $2x = \sin^{-1}\frac{1}{\sqrt{3}} = K \Rightarrow x = \frac{K}{2}$ **or** $2x = \cos^{-1}\sqrt{\frac{2}{3}} = K \Rightarrow x = \frac{K}{2}$ **or** $\tan^2 x = 5 \pm 2\sqrt{6} \Rightarrow x = \tan^{-1}\left(\sqrt{5 \pm 2\sqrt{6}}\right)$ **or** $\cos 4x = \frac{1}{3} \Rightarrow 4x = \cos^{-1}\left(\frac{1}{3}\right) = K \Rightarrow x = \frac{1}{4}K$	M1	1.1b
	$x = 17.6°,\ 72.4°$	A1	1.1b
		(4)	
			(7 marks)
(b) Notes			

(b) Note that attempts solve an equation of the form:
$2\tan x\sec x=\cot 2x\sec 2x$ or e.g. $2\tan\theta\sec\theta=\cot 2\theta\sec 2\theta$ or e.g. $2\tan\theta\sec\theta=\cot 2x\sec 2x$
Will generally score no marks in part (b)

Condone the use of θ instead of x here.

B1: Deduces the correct equation using the result from part (a)

M1: Factors out or cancels the $\sec 2x$ to obtain $...\tan 2x\pm...\cot 2x=0$ oe e.g. $...\tan 2x=\pm...\cot 2x$ leading to an equation of the form:

$\tan^2 2x=\alpha$ oe e.g. $\cot^2 2x=\dfrac{1}{\alpha}$ where $\alpha>0$

Or uses $\tan 2x=\dfrac{\sin 2x}{\cos 2x}$ and $\cot 2x=\dfrac{\cos 2x}{\sin 2x}$ and then $\sin^2 2x=\pm 1\pm\cos^2 2x$ or $\cos^2 2x=\pm 1\pm\sin^2 2x$ to obtain an equation of the form $\sin^2 2x=\beta$ or $\cos^2 2x=\gamma$ oe e.g. $\text{cosec}^2 2x=\dfrac{1}{\beta}$ or $\sec^2 2x=\dfrac{1}{\gamma}$ where $0<\beta<1$ or $0<\gamma<1$

Or uses $\tan 2x=\dfrac{2\tan x}{1-\tan^2 x}$ and $\cot 2x=\dfrac{1-\tan^2 x}{2\tan x}$ to obtain a 3TQ in $\tan^2 x$ (or possibly in $\sec^2 x$)

Or uses $\tan 2x=\dfrac{\sin 2x}{\cos 2x}$ and $\cot 2x=\dfrac{\cos 2x}{\sin 2x}$ and then $2\sin^2 2x=\pm 1\pm\cos 4x$ and $2\cos^2 2x=\pm 1\pm\cos 4x$ to obtain an equation of the form $\cos 4x=k,\quad 0<k<1$

M1: Correct order of operations from $\tan^2 2x=\alpha$ or $\sin^2 2x=\beta$ or $\cos^2 2x=\gamma$ or $\cos 4x=k$ or equivalents e.g. $\text{cosec}^2 2x=\dfrac{1}{\beta}$ where $\alpha>0$ or $0<\beta<1$ or $0<\gamma<1$ or $0<k<1$ leading to at least one value for x e.g. square roots, finds inverse tan/sin/cos/cosec and divides by 2 or inverse cos and divides by 4

or from $\tan^2 x=k,\ k>0$ which follows their equation (may need to check) and then finds $x=\tan^{-1}\sqrt{k}$

You may need to check their value(s) (in degrees or radians) to see if the correct order of operations has been used. May be implied by e.g. 17.6° or 17.7° provided no incorrect work is seen.

A1: Correct values. Allow awrt 17.6° and awrt 72.4°. The degrees symbol is not required. Ignore any values outside the range (correct or incorrect) but if there are extra angles in range score A0. Answers in radians score A0.

Note that some candidates may convert to $\sin x$ or $\cos x$ and then solve:

E.g.

$$\tan^2 2x=\frac{1}{2}\Rightarrow\frac{\sin^2 2x}{\cos^2 2x}=\frac{1}{2}\Rightarrow\frac{4\sin^2 x\cos^2 x}{\left(2\cos^2 x-1\right)^2}\rightarrow 12\cos^4 x-12\cos^2 x+1=0$$

$$\text{or}\ \frac{4\sin^2 x\cos^2 x}{\left(1-2\sin^2 x\right)^2}\rightarrow 12\sin^4 x-12\sin^2 x+1=0$$

$$\Rightarrow\cos^2 x/\sin^2 x=\frac{3\pm\sqrt{6}}{6}\Rightarrow\cos x/\sin x=\pm\sqrt{\frac{3\pm\sqrt{6}}{6}}\Rightarrow x=17.6°,\ 72.4°$$

These can be marked in a similar way.

Alternative not using part (a):

$$\frac{1}{\text{cosec}2x-1}+\frac{1}{\text{cosec}2x+1}=\cot 2x\sec 2x$$

$$\Rightarrow \frac{2\text{cosec}2x}{\text{cosec}^2 2x-1}=\cot 2x\sec 2x$$

$$\Rightarrow \frac{2\text{cosec}2x}{\cot^2 2x}=\frac{\cos 2x}{\sin 2x}\times\frac{1}{\cos 2x}=\text{cosec}2x$$

$$\Rightarrow 2\tan^2 2x=1\Rightarrow \tan^2 2x=\frac{1}{2}$$

Score as:

M1: For correct work leading to one of the forms in the main scheme e.g.

$$\tan^2 2x=\alpha \text{ oe e.g. } \cot^2 2x=\frac{1}{\alpha} \text{ where } \alpha>0$$

or

$$\sin^2 2x=\beta \text{ or } \cos^2 2x=\gamma \text{ oe e.g. } \text{cosec}^2 2x=\frac{1}{\beta} \text{ or } \sec^2 2x=\frac{1}{\gamma}$$

$$\text{where } 0<\beta<1 \text{ or } 0<\gamma<1$$

B1: Any correct equation e.g. $\tan^2 2x=\frac{1}{2}$, $\cot^2 2x=2$, $\sin^2 2x=\frac{1}{3}$ etc.

Then **M1A1** as main scheme

Question	Scheme	Marks	AOs

8(a)	$OC \times 2.3 = 27.6$	M1	1.1b
	e.g. $OC = \dfrac{27.6}{2.3} = 12\,\text{m}$ *	A1*	2.1
		(2)	
(b)	e.g. $(2AOB =)\ \pi - 2.3$	M1	1.1b
	$\dfrac{\pi - 2.3}{2} \Rightarrow 0.421$ rad *	A1*	2.1
		(2)	
(c)	Area $OCDE = \dfrac{1}{2} \times 12^2 \times 2.3$	M1	1.1b
	$= 165.6\,(\text{m}^2)$ (accept awrt 166)	A1	1.1b
	$(OB =)\ \dfrac{35 - 27.6}{2} + 12 = 15.7\,\text{m}$	B1	2.1
	Area of OAB (or OFG) $= \dfrac{1}{2} \times \text{"15.7"} \times 7.5 \times \sin 0.421 \quad (= 24.0...\text{m}^2)$	M1	1.1b
	Total area = $\text{"165.6"} + 2 \times \text{"24.1"}$	dM1	3.1a
	= awrt $214\,(\text{m}^2)$	A1	1.1b
		(6)	
			(10 marks)

Notes

(a)

M1: Uses $l = r\theta$ with $l = 27.6$ and $\theta = 2.3$ correctly substituted in (may be labelled differently in their equation). Values just need to be embedded in an equation or accept an expression for *OC* e.g. $\dfrac{27.6}{2.3}$.

May work in degrees which is acceptable.
Condone an alternative letter being used to denote *OC* such as *r*
Alternatively, they use $l = r\theta$ with $r = 12$ and $\theta = 2.3$ and verify that $l = 27.6\,\text{m}$

A1*: Achieves an expression for *OC* before proceeding to $OC = 12\,(\text{m})$ with no errors seen (condone lack of units)

They must show at least $\dfrac{27.6}{2.3} \Rightarrow OC = 12\,(\text{m})$ which can score M1A1*

$r = \dfrac{27.6}{2.3} = 12$ is M1A1* (condone alternative letters for *OC*)

BUT e.g. $\dfrac{27.6}{2.3} = 12(\text{m})$ on its own is M1A0*

e.g. $OC \times 2.3 = 27.6 \Rightarrow OC = 12(\text{m})$ is M1A0*

In the alternative method they verify $l = 27.6$ and conclude that $OC = 12\,\text{m}$
We must see the calculation $12 \times 2.3 = 27.6$ and conclude that $OC = 12\,(\text{m})$
e.g. $\text{arc} = 12 \times 2.3 = 27.6$ so $OC = 12(\text{m})$ is M1A1* whereas $12 \times 2.3 = 27.6$ is M1A0*
Also allow e.g. if $OC = 12(\text{m})$ then $12 \times 2.3 = 27.6$✓ is M1A1*

If they work in degrees and use rounded values this scores A0* (If they work with e.g. $\dfrac{414}{\pi}$ to keep the angle exact then A1* can still be scored)

(b)

M1: Attempts to subtract 2.3 from π (which may be implied by an expression for *AOB* which is not the given answer)

e.g. $\frac{1}{2}(\pi - 2.3)$ or $\frac{\pi}{2} - 1.15$ score M1

May work in degrees e.g. $180 - \text{awrt} 132$ is M1

Condone invisible brackets e.g. $\pi - 2.3 \div 2$ can still score M1.

A1*: Achieves 0.421 (rad) with no errors seen (ignore any side working which is not part of their main solution). Look for a correct expression which is awrt 0.421 before proceeding to the answer. Alternatively, they may write

e.g $2AOB = \pi - 2.3 \left(= 0.8415..\right) \Rightarrow AOB = 0.421$

Condone if they do not round their answer at the end to 0.421.

Condone lack of units. Condone poor labelling of other angles and it does not require *AOB* = to score this mark, but do not accept e.g. *ABO* =

If they work in degrees then withhold this mark if they do not show the conversion back to radians.

e.g. $\frac{\pi - 2.3}{2} = 0.421 (\text{rad})$ is M1A1*

e.g. $\frac{180 - \text{awrt} 131.8}{2} \div \frac{180}{\pi} = 0.421$ (rad) is M1A1* (conversion from degrees to radians seen)

e.g. $\pi - 2.3 \div 2 = 0.421 (\text{rad})$ is M1A0* (invisible/lack of brackets)

e.g. $\pi - 2.3 = \frac{0.842...}{2} = 0.421$ M1A0* (incorrect joined statement)

(c)

M1: Attempts to use $A = \frac{1}{2} r^2 \theta$ with $r = 12$ and $\theta = 2.3$ The values embedded in the formula is sufficient for this mark. May be implied by a correct answer or further work. Look out for alternative more complex ways to find the area of the sector. e.g. area of semicircle – area of two sectors with $r = 12$ and $\theta = 0.421$

A1: awrt 166 (may be implied by later work)

B1: A correct expression or value for the length *OB* or *OF* which may be a part of a calculation (may see 15.7 in the equation to find the area of *AOB*). May be implied by a correct value for the area of a congruent triangle (or both)

M1: Attempts to find the area of at least one of the two congruent triangles using their *OB* found from $\frac{35 - 27.6}{2} + 12 \left(= 15.7\right)$, $OA = 7.5$ and $\theta = 0.421$ in $\frac{1}{2} \times OA \times OB \times \sin C$ (may work in degrees)

Be aware that omitting sine in the formula may give a value close to the area of the triangle which would be M0. **Condone use of** $\theta = 0.4$ **or** $\theta = 0.42$ if they have rounded angle *AOB*.

The values embedded in the expression is sufficient to score the mark or may be implied by the value.

Look out for more complex methods to find the area of one or both of the two congruent triangles

e.g. they may split the congruent triangle into two right angled triangles and add the separate areas.

Other alternatives

e.g. finding the area of the trapezium *ABFG*:

$BF = 2\times15.7\cos0.421$

Area of $AOB = \frac{1}{2}\left(\left(\frac{15+2\times15.7\cos0.421}{2}\right)\times15.7\sin0.421 - \frac{1}{2}\times15.7^2\times\sin2.3\right)$ o.e.

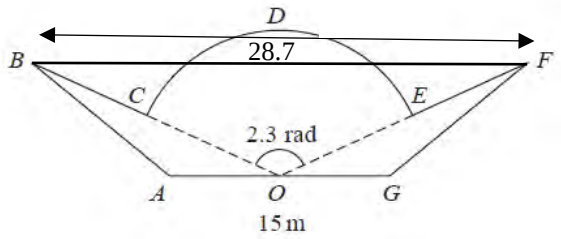
e.g. finding the length *AB* and either angle *OAB* or angle *ABO*:

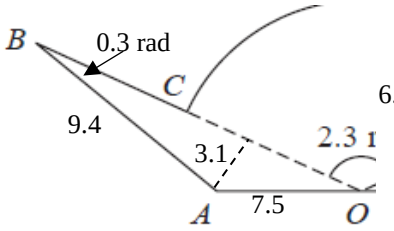
$AB^2 = 15.7^2 + 7.5^2 - 2\times15.7\times7.5\times\cos0.421 \Rightarrow AB = 9.37...$

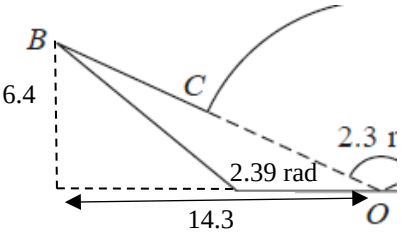
$\frac{\sin ABO}{7.5} = \frac{\sin0.421}{9.37...} \Rightarrow ABO = 0.333...$ or $\frac{\sin OAB}{15.7} = \frac{\sin0.421}{9.37...} \Rightarrow OAB = 2.4$

Area of ABO $\frac{1}{2}\times15.7\times9.37...\times\sin0.333...$ or $\frac{1}{2}\times7.5\times9.37...\times\sin2.4$

Approximate values are shown below for some of the lengths you may see in calculations:







dM1: Solves the problem by combining appropriate areas together which result in the total area of the concert stage (usually the sum of the areas of the **two** congruent triangles and the area of the sector).
It is dependent on the previous method marks and the B mark.

A1: awrt 214 (m^2) (condone lack of units). Must follow from a correct method. Isw if they round incorrectly.

Question	Scheme	Marks	AOs

13(a)	$a = 60$	B1	3.1b
	$2 = "60" - b(-20)^2 \Rightarrow b = ...$	M1	3.4
	$H = 60 - 0.145(t-20)^2$	A1	3.3
		(3)	
(b)	Height = 2 m	B1	3.4
		(1)	
(c)	$\alpha = 180$ or $\beta = 31$	M1	3.4
	$H = 29\cos(9t + 180)° + 31$	A1	3.3
		(2)	
(d)	e.g. "The model allows for more than one circuit"	B1	3.5a
		(1)	
			(7 marks)

Notes

(a)

B1: $a = 60$ (may be seen in their final equation of the model or implied by 60 substituted for *a* in the model)

M1: Attempts to find *b* by substituting in $t = 0$, $H = 2$ and their *a* and proceeding to a value for *b*.
May be seen as two simultaneous equations formed:
$2 = a - b(-20)^2$ and $60 = a - b(20-20)^2$ proceeding to a value for *b*

A1: $H = 60 - 0.145(t-20)^2$ or equivalent such as $H = -\frac{29}{200}t^2 + 5.8t + 2$ or $H = 60 - \frac{29}{200}(t-20)^2$ isw once a correct equation for the model is seen. Must be in terms of *H* and *t*. If they just state $a = 60,\ b = 0.145$ then A0
A correct answer with no working seen scores full marks.

(b)

B1: 2 cao (condone lack of units) This can be scored even if their model in (a) is incorrect (they may have used symmetry to determine this value)

(c)

M1: $(\alpha =)\ 180$ or $(\beta =)\ 31$ Condone $(\alpha =)\ \pi$

A1: $H = 29\cos(9t + 180)° + 31$ or equivalent e.g. $H = -29\cos(9t) + 31$ isw once a correct equation for the model is seen. Must be in terms of *H* and *t*. If they just state $\alpha = 180,\ \beta = 31$ then A0.
A correct equation with no working seen scores both marks. Does not require the degree symbol.

(d)

B1: Score for a reason which makes reference to any of

- the alternative model allows repetition (allow phrases e.g. "multiple cycles", "repeated circuits", "cyclical", "periodic", "loops around", "the original model can only go up and down once")
- the alternative model after 2 minutes the carriage will be back at the start (e.g."at 2 mins, $H = 2$")
- the original/quadratic model after 40 seconds (or any time after this) will be negative (e.g. "the height will be negative which cannot happen")
- the original model after 2 minutes would not be back at the start

Do not allow vague responses on their own e.g. "the original model is a parabola"
If calculations are used then they must be correct using a correct model (allow rounded or truncated)
Look for a valid reason and ignore reference to anything else as long as it does not contradict

t	0	5	10	15	20	25	30	35	40	45	50	55	60	80	100	120
h	2	27	46	56	60	56	46	27	2	-31	-71	-118	-172	-462	-868	-1390

Question	Scheme	Marks	AOs

2(a)(i) **(ii)**	e.g. $(u_2 =)35+7\cos\left(\frac{\pi}{2}\right)-5(-1)^1=40$ *	**B1***	2.1
	$u_3=40+7\cos\left(\frac{2\pi}{2}\right)-5(-1)^2\ (=28)$ **or** $u_4=\text{"28"}+7\cos\left(\frac{3\pi}{2}\right)-5(-1)^3\ (=33)$	**M1**	1.1b
	$u_3=28$ and $u_4=33$	**A1**	1.1b
		(3)	
(b)(i) **(ii)**	$(u_5 =)35$	**B1**	2.2a
	e.g. $\sum_{r=1}^{25}u_r=6(35+40+\text{"28"}+\text{"33"})+35$	**M1**	3.1a
	$=851$	**A1**	1.1b
		(3)	
			(6 marks)

Notes

(a)

(i)

B1*: Correct application of the formula with $n = 1$ and proceeds correctly to achieve an answer of 40 with no errors. Note that e.g., $(u_2 =)35+7\cos\left(\frac{35\pi}{2}\right)-5(-1)^{35}=35+0+5=40$ scores B0

As a minimum need to see e.g. $(u_2 =)35+7\cos\left(\frac{\pi}{2}\right)-5(-1)^1=40,\quad 35+0+5=40,\quad 35+5=40,\quad 35-5(-1)^1=40$

(ii)

M1: A correct attempt to use the formula to find a value for u_3 or u_4

Look for $n = 2$ substituted correctly into the given formula with $u_2=40$. May be implied by $u_3=28$

Or their calculated value of u_3 used with $n = 3$ substituted correctly into the given formula to find u_4

Condone use of calculator in degree mode which gives $u_3=41.989...$ which may imply this mark if no working is shown. If there is **no** working and u_3 is incorrect and u_4 is correct score M0A0

A1: Both correct $u_3=28$ and $u_4=33$ If 28, 33 are listed then allow M1A1.

For both correct values only score M1A1

(b)(i)

B1: $(u_5 =)35$

(ii)

M1: Attempts a **correct** method to find $\sum_{r=1}^{25}u_r$

There are various ways e.g. attempts to add 35 to $6\times$the sum of their four values.

Some other examples are:

$\sum_{r=1}^{25}u_r=7\times35+6\times40+6\times\text{"28"}+6\times\text{"33"}$, $\sum_{r=1}^{25}u_r=7(35+40+\text{"28"}+\text{"33"})-(40+\text{"28"}+\text{"33"})$,

$\sum_{r=1}^{25}u_r=\frac{25}{4}(35+40+\text{"28"}+\text{"33"})+1$, $2(35+40+\text{"28"}+\text{"33"})=272$, $272\times3=816$, $816+35$

There may be other methods seen but the calculation must be correct for their values.

If there is no working, with incorrect u_3 and/or u_4 you will need to check if their answer implies a correct method using $6(35+40+\text{"28"}+\text{"33"})+35$

Attempts to use an AP/GP formula score M0

A1: 851 (Correct answer with no working scores both marks)

Question	Scheme	Marks	AOs

4(a)	$(A=)\ 55$	**B1**	3.4
		(1)	
(b)	$\left\{\frac{\mathrm{d}H}{\mathrm{d}t}=\right\}-AB\mathrm{e}^{-Bt}$ or $\left\{\frac{\mathrm{d}H}{\mathrm{d}t}=\right\}-"55"B\mathrm{e}^{-Bt}$	**M1**	3.1b
	$-B\times"55"=-7.5\Rightarrow B=...\ \left(\frac{3}{22}=\text{awrt } 0.136\right)$	**M1**	1.1b
	$H=55\mathrm{e}^{-0.136t}+30$	**A1cso**	3.3
		(3)	
			(4 marks)

Notes

(a)

B1: 55 only. Just look for this value e.g. "A =" is not required. Ignore any "units" if given e.g. 55 °C

(b)

M1: Differentiates to obtain an expression of the form $\pm AB\mathrm{e}^{-Bt}$ which may have their A already substituted in so allow for $\pm AB\mathrm{e}^{-Bt}$ or $\pm"55"B\mathrm{e}^{-Bt}$

M1: Substitutes $t = 0$ and their A into their $\frac{\mathrm{d}H}{\mathrm{d}t}$, sets $=\pm 7.5$ and proceeds to find a value for B which may be implied by $\frac{3}{22}$ or awrt 0.136

Their $\frac{\mathrm{d}H}{\mathrm{d}t}$ must not be H. i.e. it must be a "changed" function.

A1cso: Correct **equation** which follows **fully correct work** $H=55\mathrm{e}^{-0.136t}+30$ but condone $H=55\mathrm{e}^{-\frac{3}{22}t}+30$

The final equation must be correct but you can ignore spurious notation within their solution such as integral signs and "+ c" which do not affect their solution.

Marking guidance is as follows for particular cases in (b)

Case 1: $\left\{\frac{\mathrm{d}H}{\mathrm{d}t}=\right\}-"55"B\mathrm{e}^{-Bt},\ -"55"B\mathrm{e}^{-Bt}=7.5\Rightarrow\ B=-0.136\Rightarrow H=55\mathrm{e}^{-0.136t}+30$ scores **M1M1A0**

Error: it should be – 7.5

Case 2: $\left\{\frac{\mathrm{d}H}{\mathrm{d}t}=\right\}"55"B\mathrm{e}^{-Bt},\ "55"B\mathrm{e}^{-Bt}=-7.5\Rightarrow\ B=-0.136\Rightarrow H=55\mathrm{e}^{-0.136t}+30$ scores **M1M1A0**

Error: incorrect derivative

Case 3: $\left\{\frac{\mathrm{d}H}{\mathrm{d}t}=\right\}"55"B\mathrm{e}^{-Bt},\ "55"B\mathrm{e}^{-Bt}=7.5\Rightarrow\ B=0.136\Rightarrow H=55\mathrm{e}^{-0.136t}+30$ scores **M1M1A0**

Error: incorrect derivative

Case 4: $\left\{\frac{\mathrm{d}H}{\mathrm{d}t}=\right\}-"55"B\mathrm{e}^{-Bt},\ "55"B=7.5\Rightarrow\ B=0.136\Rightarrow H=55\mathrm{e}^{-0.136t}+30$ scores **M1M1A1**

No errors

Question	Scheme	Marks	AOs

8(a)	$R=\sqrt{2^2+8^2}=\sqrt{68}=2\sqrt{17}$	**B1**	1.1b
	$2\cos\theta+8\sin\theta=R\cos\theta\cos\alpha+R\sin\theta\sin\alpha$ $2=R\cos\alpha \quad 8=R\sin\alpha$ $\tan\alpha=\frac{8}{2}\Rightarrow\alpha=...$	**M1**	1.1b
	$\alpha=$ awrt 1.326	**A1**	2.2a
		(3)	
(b)(i)	$4.5\times"2\sqrt{17}"$	**M1**	1.1b
	$9\sqrt{17}$	**A1**	2.2a
(ii)	awrt 1.33	**B1ft**	2.2a
		(3)	
			(6 marks)

Notes

(a)

B1: $R=2\sqrt{17}$ or $\sqrt{68}$.

$\pm2\sqrt{17}$ or $\pm\sqrt{68}$ score B0

(Condone if this comes from e.g., $8=R\cos\alpha \quad 2=R\sin\alpha$)

Decimal answers score B0 unless the exact value is seen then apply isw.

M1: Proceeds to a value for α from $\tan\alpha=\pm\frac{8}{2}$, $\cos\alpha=\pm\frac{2}{"\sqrt{68}"}$, $\sin\alpha=\pm\frac{8}{"\sqrt{68}"}$

May be implied by awrt 1.33 radians or 76 degrees

A1: awrt 1.326 for α. Apply isw if this is then subsequently rounded to e.g. 1.33

(b)(i)

M1: For a value of $\pm4.5\times$their R or allow $\pm4.5R$ (with the letter R)

But not embedded in an expression e.g. $9\sqrt{17}\cos(\theta-\alpha)$ unless extracted later.

Note that the sum may be found as $9\cos x+36\sin x$ with the maximum then found using calculus

e.g. $S=9\cos x+36\sin x\Rightarrow\frac{\mathrm{d}S}{\mathrm{d}x}=-9\sin x+36\cos x=0\Rightarrow\tan x=4\Rightarrow\sin x=\frac{4}{\sqrt{17}},\ \cos x=\frac{1}{\sqrt{17}}$

$\Rightarrow9\cos x+36\sin x=9\sqrt{17}$. This will score M1 once they reach $\pm4.5\times$their R

May be implied by $9\sqrt{17}$ or awrt 37.1 (which may come from a graphical method)

May also see e.g. $\mathrm{Max}(9\cos x+36\sin x)=\sqrt{9^2+36^2}=...$

A1: $9\sqrt{17}$ or exact equivalent e.g. $\sqrt{1377}$, $4.5\sqrt{68}$, $4.5(2\sqrt{17})$ and apply isw once a correct answer is seen

(ii)

B1ft: awrt 1.33 (or follow through on their α even if in degrees (76), no matter how accurate)

Question	Scheme	Marks	AOs

15	$(\sin x - \cos x)^2 \leqslant 1 \Rightarrow \sin^2 x - 2\sin x\cos x + \cos^2 x \ \ (< 1)$ o.e.	**M1**	1.1b
	Examples: $1 - 2\sin x\cos x < 1,\ 1 - \sin 2x < 1,\ -2\sin x\cos x < 0,\ -\sin 2x < 0$	**A1**	2.2a
	As x is obtuse then $-2\sin x\cos x$ is positive because $\sin x > 0$ and $\cos x < 0$ so we have a contradiction. Therefore $\sin x - \cos x \ldots 1$ *	**A1***	2.4
			(3 marks)

Notes

Condone poor notation e.g. $\sin x^2$ **or e.g.** $-2\sin\theta\cos x < 1$ **for the first two marks only.**

M1: Expands $(\sin x - \cos x)^2$ to obtain $\sin^2 x \pm k\sin x\cos x + \cos^2 x$ where $k = 1$ or 2 o.e. May be implied.

A1: Uses a correct identity $\sin^2 x + \cos^2 x = 1$ or e.g. $-\sin^2 x - \cos^2 x = -1$ to obtain a correct inequality in any form that does not include the $\sin^2 x$ and $\cos^2 x$ terms. Condone e.g. $-2\sin\cos x < 0$

A1*: Fully correct work which includes

- a convincing argument that explains why their inequality is not true
- a statement that indicates there is a contradiction
- a conclusion that $\sin x - \cos x \ldots 1$ (there is no need to repeat "when x is obtuse")
- no contradictory statements
- no mixed/missed variables, e.g., $-2\sin\theta\cos x < 1$ or $1 - \sin 2 < 1$

Examples:

From $-2\sin x\cos x < 0$:

In the second quadrant $-2\sin x\cos x$ is $-\times+\times-=+$

"(this is a) contradiction" or equivalent (therefore) $\sin x - \cos x \ldots 1$

or

As x is obtuse, $\sin x > 0$, $\cos x < 0$ so $-2\sin x\cos x > 0$

"(this is a) contradiction" or equivalent (therefore) $\sin x - \cos x \ldots 1$

From $-\sin 2x < 0$:

As x is obtuse, $2x$ is reflex o.e. (i.e. $\pi < 2x < 2\pi$) so $-\sin 2x > 0$

"(this is) wrong" or equivalent (therefore) $\sin x - \cos x \ldots 1$

From $1 - \sin 2x < 1$:

As x is obtuse, $2x$ is reflex o.e. (i.e. $180 < 2x < 360$) so $\sin 2x < 0$ so $1 - \sin 2x > 1$

"(this is a) contradiction" or equivalent (therefore) $\sin x - \cos x \ldots 1$

From $\sin 2x > 0$:

As x is obtuse, $2x$ is reflex o.e. (i.e. $180 < 2x < 360$) so $\sin 2x < 0$

"(this is) incorrect" or equivalent (therefore) $\sin x - \cos x \ldots 1$

Note that you may condone the absence of a statement referring to the fact that $(\sin x - \cos x)^2 < 1$ is only valid since $\sin x - \cos x > 0$ when x is obtuse.

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9(a)	Deduces that $A = \pm 50$ **or** $b = \frac{1}{4}$	B1	3.4
	Deduces that $A = \pm 50$ **and** $b = \frac{1}{4}$	B1	3.4
	Uses $t = 0, H = 1 \Rightarrow \alpha = ...$ E.g. $1 = "50"\sin(\alpha)° \Rightarrow \alpha = ...$	M1	3.4
	$H = \left\| \pm 50\sin\left(\frac{1}{4}t + 1.15\right)° \right\|$	A1	3.3
		(4)	
(b)	E.g. the minimum height above the ground of the passenger on the original model was 0 m or Adding "*d*" means the passenger does not touch the ground.	B1	3.5b
		(1)	
			(5 marks)

Notes:

(a) Note that B0B1 is not possible

B1: Uses the equation of the given model to deduce that $A = \pm 50$ **or** $b = \frac{1}{4}$ o.e.

May be seen embedded within their equation.

B1: Uses the equation of the given model to deduce that $A = \pm 50$ **and** $b = \frac{1}{4}$ o.e.

May be seen embedded within their equation.

M1: Uses $t = 0$ and $H = 1$ in the equation of the model to find a value for α.

Follow through on their value for *A*. Allow for $\pm 1 = "50"\sin(\alpha)° \Rightarrow \alpha = ...$ where α is in degrees or radians.

Note that in radians $\sin^{-1}\left(\frac{1}{50}\right) \approx \frac{1}{50}$ (0.0200…) which may appear incorrect but is in fact ok.

Also in degrees a value of e.g. 1.14 (truncated) would indicate the method.

A1: Writes down the correct full equation of the model: $H = \left| "\pm" 50\sin\left(\frac{1}{4}t + 1.15\right)^{\circ} \right|$ o.e.

Condone omission of degrees symbol and allow awrt 1.15 for α.

Allow if a correct equation is seen anywhere in their solution.

(b)

B1: Gives a suitable explanation with no contradictory statements.

Condone "so that pod/capsule/seat/passenger/ferris wheel/it etc. will not hit/touch the ground"

Responses that focus on the starting point of the model are likely to score B0

6(a)	$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = -3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k} + \mathbf{i} + \mathbf{j} + 4\mathbf{k} = ...$	M1	1.1b
	$= -2\mathbf{i} - 3\mathbf{j} - \mathbf{k}$	A1	1.1b
		(2)	
(b)	At least 2 of $\left(AC^2\right) = "2^2 + 3^2 + 1^2",\left(AB^2\right) = 3^2 + 4^2 + 5^2,\ \left(BC^2\right) = 1^2 + 1^2 + 4^2$	M1	1.1b
	$2^2 + 3^2 + 1^2 = 3^2 + 4^2 + 5^2 + 1^2 + 1^2 + 4^2 - 2\sqrt{3^2 + 4^2 + 5^2}\sqrt{1^2 + 1^2 + 4^2}\cos ABC$	M1	3.1a
	$14 = 50 + 18 - 2\sqrt{50}\sqrt{18}\cos ABC$ $\Rightarrow \cos ABC = \dfrac{50 + 18 - 14}{2\sqrt{50}\sqrt{18}} = \dfrac{9}{10}*$	A1*	2.1
		(3)	
	(b) Alternative		
	$AB^2 = 3^2 + 4^2 + 5^2,\ BC^2 = 1^2 + 1^2 + 4^2$	M1	1.1b
	$\overrightarrow{BA}.\overrightarrow{BC} = \left(3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}\right)\bullet\left(\mathbf{i} + \mathbf{j} + 4\mathbf{k}\right) = 27 = \sqrt{3^2 + 4^2 + 5^2}\sqrt{1^2 + 1^2 + 4^2}\cos ABC$	M1	3.1a
	$27 = \sqrt{50}\sqrt{18}\cos ABC \Rightarrow \cos ABC = \dfrac{27}{\sqrt{50}\sqrt{18}} = \dfrac{9}{10}*$	A1*	2.1
			(5 marks)

Notes

(a)

M1: Attempts $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

There must be attempt to add not subtract.

If no method shown it may be implied by **two** correct components

A1: Correct vector. Allow $-2\mathbf{i} - 3\mathbf{j} - \mathbf{k}$ and $\begin{pmatrix} -2 \\ -3 \\ -1 \end{pmatrix}$ but not $\begin{pmatrix} -2\mathbf{i} \\ -3\mathbf{j} \\ -1\mathbf{k} \end{pmatrix}$

(b)

M1: Attempts to "square and add" for at least 2 of the 3 sides. Follow through on their $\overrightarrow{AC}$

Look for an attempt at either $a^2 + b^2 + c^2$ or $\sqrt{a^2 + b^2 + c^2}$

M1: A correct attempt to apply a correct cosine rule to the given problem; Condone **slips** on the lengths of the sides but the sides must be in the correct position to find angle ABC

A1*: Correct completion with sufficient intermediate work to establish the printed result.

Condone different labelling, e.g. $ABC \leftrightarrow \theta$ as long as it is clear what is meant

It is OK to move from a correct cosine rule $14 = 50 + 18 - 2\sqrt{50}\sqrt{18}\cos ABC$

via $\cos ABC = \dfrac{54}{2\sqrt{50}\sqrt{18}}$ o.e. such as $\cos ABC = \dfrac{\left(5\sqrt{2}\right)^2 + \left(3\sqrt{2}\right)^2 - \left(\sqrt{14}\right)^2}{2 \times 5\sqrt{2} \times 3\sqrt{2}}$ to $\cos ABC = \dfrac{9}{10}$

Alternative:

M1: Correct application of Pythagoras for sides AB and BC or their squares

M1: Recognises the requirement for and applies the scalar product

A1*: Correct completion with sufficient intermediate work to establish the printed result

6(a)	Angle $AOB = \dfrac{\pi-\theta}{2}$	B1	2.2a
		(1)	
(b)	Area $= 2\times\dfrac{1}{2}r^2\left(\dfrac{\pi-\theta}{2}\right)+\dfrac{1}{2}(2r)^2\theta$	M1	2.1
	$=\dfrac{1}{2}r^2\pi-\dfrac{1}{2}r^2\theta+2r^2\theta=\dfrac{3}{2}r^2\theta+\dfrac{1}{2}r^2\pi=\dfrac{1}{2}r^2(3\theta+\pi)$*	A1*	1.1b
		(2)	
(c)	Perimeter $= 4r+2r\left(\dfrac{\pi-\theta}{2}\right)+2r\theta$	M1	3.1a
	$=4r+r\pi+r\theta$ or e.g. $r(4+\pi+\theta)$	A1	1.1b
		(2)	
			(5 marks)

Notes

(a)

B1: Deduces the correct expression for angle AOB

Note that $\dfrac{180-\theta}{2}$ scores B0

(b)

M1: Fully correct strategy for the area using their angle from (a) appropriately.

Need to see $2\times\dfrac{1}{2}r^2\alpha$ or just $r^2\alpha$ where α is their angle in terms of θ from part (a) + $\dfrac{1}{2}(2r)^2\theta$ with or without the brackets.

A1*: Correct proof. For this mark you can condone the omission of the brackets in $\dfrac{1}{2}(2r)^2\theta$ as long as they are recovered in subsequent work e.g. when this term becomes $2r^2\theta$

The first term must be seen expanded as e.g. $\dfrac{1}{2}r^2\pi-\dfrac{1}{2}r^2\theta$ or equivalent

(c)

M1: Fully correct strategy for the perimeter using their angle from (a) appropriately

Need to see $4r+2r\alpha+2r\theta$ where α is their angle from part (a) in terms of θ

A1: Correct simplified expression

Note that some candidates may change the angle to degrees at the start and all marks are available e.g.

(a) $\dfrac{180-\dfrac{180\theta}{\pi}}{2}$

(b) $2\left(\dfrac{180-\dfrac{180\theta}{\pi}}{2}\right)\times\dfrac{1}{360}\times\pi r^2+\dfrac{\theta}{360}\times\dfrac{180}{\pi}\times\pi(2r)^2=\dfrac{1}{2}\pi r^2-\dfrac{1}{2}r^2\theta+2r^2\theta=\dfrac{1}{2}r^2(3\theta+\pi)$

(c) $4r+2\left(\dfrac{180-\dfrac{180\theta}{\pi}}{2}\right)\times\dfrac{1}{360}\times2\pi r+\dfrac{180\theta}{\pi}\times\dfrac{1}{360}\times2\pi(2r)=4r+\pi r+r\theta$

9	$a=\left(\frac{3}{4}\right)^2 \quad \text{or} \quad a=\frac{9}{16}$ or $r=-\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n \cos(180n)^\circ = \frac{\frac{9}{16}}{1-\left(-\frac{3}{4}\right)} = \ldots$	M1	3.1a
	$=\frac{9}{28}*$	A1*	1.1b
		(3)	
	Alternative 1:		
	$\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n \cos(180n)^\circ = \frac{-\frac{3}{4}}{1-\left(-\frac{3}{4}\right)} = \ldots \text{ or } r=-\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n \cos(180n)^\circ = -\frac{3}{7}-\left(-\frac{3}{4}\right)$	M1	3.1a
	$=\frac{9}{28}*$	A1*	1.1b
	Alternative 2:		
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n \cos(180n)^\circ = \left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^3+\left(\frac{3}{4}\right)^4-\ldots$	B1	2.2a
	$=\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^4+\ldots-\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^5-\ldots$ $\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^4+\ldots=\left(\frac{3}{4}\right)^2\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right) \text{ or } -\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^5-\ldots=-\left(\frac{3}{4}\right)^3\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$ $\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n \cos(180n)^\circ = \left(\frac{3}{4}\right)^2\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)-\left(\frac{3}{4}\right)^3\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$	M1	3.1a
	$=\frac{9}{28}*$	A1*	1.1b
	Alternative 3:		
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n \cos(180n)^\circ = S = \left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^3+\left(\frac{3}{4}\right)^4-\ldots$	B1	2.2a
	$=\left(\frac{3}{4}\right)^2\left(1-\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)^2-\ldots\right)=\left(\frac{3}{4}\right)^2\left(\frac{1}{4}+S\right)\Rightarrow\frac{7}{16}S=\frac{9}{64}\Rightarrow S=\ldots$	M1	3.1a
	$=\frac{9}{28}*$	A1*	1.1b
			(3 marks)

Notes

B1: Deduces the correct value of the **first** term or the common ratio. The correct first term can be

M1: Recognises that the series is infinite geometric and applies the sum to infinity GP formula with $a = \frac{9}{16}$ and $r = \pm\frac{3}{4}$

A1*: Correct proof

Alternative 1:

B1: Deduces the correct value for the sum to infinity (starting at $n = 1$) or the common ratio

M1: Calculates the required value by subtracting the first term from their sum to infinity

A1*: Correct proof

Alternative 2:

B1: Deduces the correct value of the **first** term or the common ratio.

M1: Splits the series into "odds" and "evens", attempts the sum of both parts and calculates the required value by adding both sums

A1*: Correct proof

Alternative 3:

B1: Deduces the correct value of the **first** term

M1: A complete method by taking out the first term, expresses the rhs in terms of the original sum and rearranges for "S"

A1*: Correct proof

15(a)	$R=\sqrt{5}$	B1	1.1b
	$\tan\alpha=\frac{1}{2}$ or $\sin\alpha=\frac{1}{\sqrt{5}}$ or $\cos\alpha=\frac{2}{\sqrt{5}}\Rightarrow\alpha=...$	M1	1.1b
	$\alpha=0.464$	A1	1.1b
		(3)	
(b)(i)	$3+2\sqrt{5}$	B1ft	3.4
(ii)	$\cos(0.5t+0.464)=1\Rightarrow 0.5t+0.464=2\pi$ $\Rightarrow t=...$	M1	3.4
	t = 11.6	A1	1.1b
		(3)	
(c)	$3+2\sqrt{5}\cos(0.5t+0.464)=0$ $\cos(0.5t+0.464)=-\frac{3}{2\sqrt{5}}$	M1	3.4
	$\cos(0.5t+0.464)=-\frac{3}{2\sqrt{5}}\Rightarrow 0.5t+0.464=\cos^{-1}\left(-\frac{3}{2\sqrt{5}}\right)$ $\Rightarrow t=2\left(\cos^{-1}\left(-\frac{3}{2\sqrt{5}}\right)-0.464\right)$	dM1	1.1b
	So the time required is e.g.: $2(3.977...-0.464)-2(2.306...-0.464)$	dM1	3.1b
	= 3.34	A1	1.1b
		(4)	
(d)	e.g. the "3" would need to vary	B1	3.5c
		(1)	
			(11 marks)

Notes

(a)

B1: $R=\sqrt{5}$ only.

M1: Proceeds to a value for α from $\tan\alpha=\pm\frac{1}{2}$ or $\sin\alpha=\pm\frac{1}{"R"}$ or $\cos\alpha=\pm\frac{2}{"R"}$

It is implied by either awrt 0.464 (radians) or awrt 26.6 (degrees)

A1: $\alpha=$ awrt 0.464

(b)(i)

B1ft: For $(3+2\sqrt{5})$m or awrt 7.47 m and remember to isw. Condone lack of units.

Follow through on their R value so allow 3 + 2×Their R. (Allow in decimals with at least 3sf accuracy)

(b)(ii)

M1: Uses $0.5t\pm"0.464"=2\pi$ to obtain a value for t

Follow through on their 0.464 but this angle must be in radians.

It is possible in degrees but only using $0.5t\pm"26.6"=360$

A1: Awrt 11.6

$$H = 3 + 4\cos(0.5t) - 2\sin(0.5t) \Rightarrow \frac{\mathrm{d}H}{\mathrm{d}t} = -2\sin(0.5t) - \cos(0.5t) = 0$$

$$\Rightarrow \tan(0.5t) = -\frac{1}{2} \Rightarrow 0.5t = 2.677..., 5.819... \Rightarrow t = 5.36,\ 11.6$$

$$t = 11.6 \Rightarrow H = 7.47$$

Score as follows:

M1: For a complete method:

Attempts $\frac{\mathrm{d}H}{\mathrm{d}t}$ and attempts to solve $\frac{\mathrm{d}H}{\mathrm{d}t} = 0$ for t

A1: For t = awrt 11.6

B1ft: For awrt 7.47 or 3 + 2×Their R

(c)

M1: Uses the model and sets $3 + 2"\sqrt{5}"\cos(...) = 0$ and proceeds to $\cos(...) = k$ where $|k| < 1$.

Allow e.g. $3 + 2"\sqrt{5}"\cos(...) < 0$

dM1: Solves $\cos(0.5t \pm "0.464") = k$ where $|k| < 1$ to obtain at least one value for t

This requires e.g. $2\left(\pi + \cos^{-1}(k) \pm \tan^{-1}\left(\frac{1}{2}\right)\right)$ or e.g. $2\left(\pi - \cos^{-1}(k) \pm \tan^{-1}\left(\frac{1}{2}\right)\right)$

Depends on the previous method mark.

dM1: A fully correct strategy to find the required duration. E.g. finds 2 consecutive values of t when H = 0 and subtracts. Alternatively finds t when H is minimum and uses the times found correctly to find the required duration.

Depends on the previous method mark.

Examples:

Second time at water level – first time at water level:

$$2\left(\pi + \cos^{-1}\left(\frac{3}{2\sqrt{5}}\right) - \tan^{-1}\left(\frac{1}{2}\right)\right) - 2\left(\pi - \cos^{-1}\left(\frac{3}{2\sqrt{5}}\right) - \tan^{-1}\left(\frac{1}{2}\right)\right) = 7.02685... - 3.68492...$$

2×(first time at minimum point – first time at water level):

$$2\left(2\left(\pi - \tan^{-1}\left(\frac{1}{2}\right)\right) - 2\left(\pi - \cos^{-1}\left(\frac{3}{2\sqrt{5}}\right) - \tan^{-1}\left(\frac{1}{2}\right)\right)\right) = 2(5.35589... - 3.68492...)$$

Note that both of these examples equate to $4\cos^{-1}\left(\frac{3}{2\sqrt{5}}\right)$ which is not immediately obvious but may be seen as an overall method.

There may be other methods – if you are not sure if they deserve credit send to review.

A1: Correct value. Must be 3.34 (**not** awrt).

Special Cases in (c):

Note that if candidates have an incorrect α and have e.g. $3 + 2\sqrt{5}\cos(0.5t - 0.464)$, this has no impact on the final answer. So for candidates using $3 + 2\sqrt{5}\cos(0.5t \pm \alpha)$ in (c) allow all the marks including the A mark as a correct method should always lead to 3.34

Some values to look for:

$$0.5t \pm "0.464" = \pm 2.306,\ \pm 3.977,\ \pm 8.598,\ \pm 10.26$$

(d)

B1: Correct refinement e.g. As in scheme. If they suggest a specific function to replace the "3" then it must be sensible e.g. a trigonometric function rather than e.g. a quadratic/linear one.

Question	Scheme	Marks	AOs
6 (a)	$R=\sqrt{5}$	B1	1.1b
	$\tan\alpha=2\Rightarrow\alpha=...$	M1	1.1b
	$\alpha=1.107$	A1	1.1b
		(3)	
	$\theta=5+\sqrt{5}\sin\left(\frac{\pi t}{12}+1.107-3\right)$		
(b)	$\left(5+\sqrt{5}\right)$ °C or awrt 7.24℃	B1ft	2.2a
		(1)	
(c)	$\frac{\pi t}{12}+1.107-3=\frac{\pi}{2}\Rightarrow t=$	M1	3.1b
	$t=$ awrt 13.2	A1	1.1b
	Either 13:14 or 1:14 pm or 13 hours 14 minutes after midnight.	A1	3.2a
		(3)	
			(7 marks)

Notes:

(a)

B1: $R=\sqrt{5}$ only.

M1: Proceeds to a value of α from $\tan\alpha=\pm2,\tan\alpha=\pm\frac{1}{2},\sin\alpha=\pm\frac{2}{"R"}$ OR $\cos\alpha=\pm\frac{1}{"R"}$

It is implied by either awrt 1.11 (radians) or 63.4 (degrees)

A1: $\alpha=$ awrt 1.107

(b)

B1ft: Deduces that the maximum temperature is $\left(5+\sqrt{5}\right)$ °C or awrt 7.24℃ Remember to isw

Condone a lack of units. Follow through on their value of *R* so allow $\left(5+"R"\right)$ °C

(c)

M1: An complete strategy to find *t* from $\frac{\pi t}{12}\pm1.107-3=\frac{\pi}{2}$.

Follow through on their 1.107 but the angle must be in radians.

It is possible via degrees but only using $15t\pm63.4-171.9=90$

A1: awrt $t=13.2$

A1: The question asks for the time of day so accept either 13:14, 1:14 pm, 13 hours 14 minutes after midnight, 13h 14, or 1 hour 14 minutes after midday. If in doubt use review

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It is possible to attempt parts (b) and (c) via differentiation but it is unlikely to yield correct results.

$\frac{d\theta}{dt}=\frac{\pi}{12}\cos\left(\frac{\pi t}{12}-3\right)-\frac{2\pi}{12}\sin\left(\frac{\pi t}{12}-3\right)=0\Rightarrow\tan\left(\frac{\pi t}{12}-3\right)=\frac{1}{2}\Rightarrow t=13.23=13:14$ scores M1 A1 A1

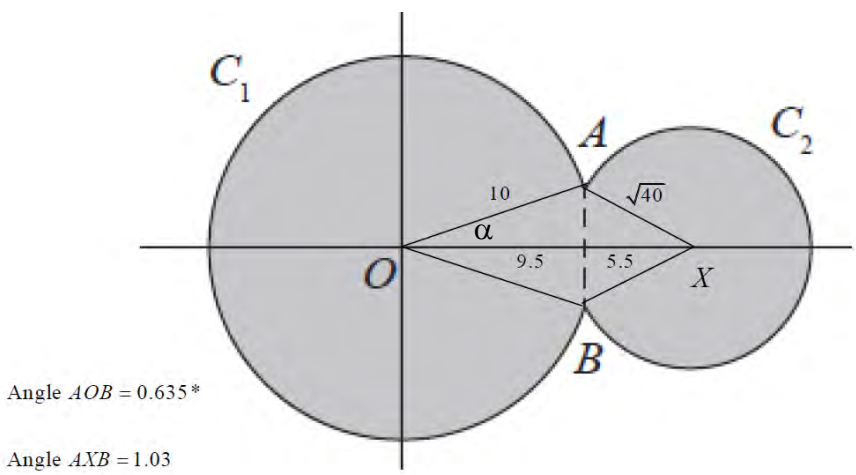
$\frac{d\theta}{dt}=\cos\left(\frac{\pi t}{12}-3\right)-2\sin\left(\frac{\pi t}{12}-3\right)=0\Rightarrow\tan\left(\frac{\pi t}{12}-3\right)=\frac{1}{2}\Rightarrow t=13.23=13:14$ they can score M1 A0 A1 (SC)

A value of $t=1.23$ implies the minimum value has been found and therefore incorrect method M0.

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Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
11 (a)	Solves $x^2+y^2=100$ and $(x-15)^2+y^2=40$ simultaneously to find x or y E.g. $(x-15)^2+100-x^2=40 \Rightarrow x=...$	M1	3.1a
	Either $\Rightarrow -30x+325=40 \Rightarrow x=9.5$ Or $y=\frac{\sqrt{39}}{2}$ = awrt ± 3.12	A1	1.1b
	Attempts to find the angle AOB in circle C_1 Eg Attempts $\cos\alpha=\frac{"9.5"}{10}$ to find α then $\times 2$	M1	3.1a
	Angle AOB = $2\times\arccos\left(\frac{9.5}{10}\right)=0.635$ rads (3sf) *	A1*	2.1
		(4)	
(b)	Attempts $10\times(2\pi-0.635)$ = 56.48	M1	1.1b
	Attempts to find angle AXB or AXO in circle C_2 (see diagram) E.g. $\cos\beta=\frac{15-"9.5"}{\sqrt{40}}\Rightarrow\beta=...$ (Note AXB =1.03 rads)	M1	3.1a
	Attempts $10\times(2\pi-0.635)+\sqrt{40}\times(2\pi-2\beta)$	dM1	2.1
	= 89.7	A1	1.1b
		(4)	
			(8 marks)
Notes:			



(a)

M1: For the key step in an attempt to find either coordinate for where the two circles meet.
Look for an attempt to set up an equation in a single variable leading to a value for x or y.

A1: $x=9.5$ (or $y=\frac{\sqrt{39}}{2}$ = awrt ± 3.12)

M1: Uses the radius of the circle and correct trigonometry in an attempt to find angle AOB in circle C_1

E.g. Attempts $\cos\alpha = \frac{"9.5"}{10}$ to find α then $\times 2$

Alternatives include $\tan\alpha = \frac{\sqrt{100-"9.5"^2}}{"9.5"} = (0.3286...)$ to find α then $\times 2$

And $\cos AOB = \frac{10^2+10^2-\left(\sqrt{39}\right)^2}{2\times 10\times 10} = \frac{161}{200}$

A1*: Correct and careful work in proceeding to the given answer. Condone an answer with greater accuracy. Condone a solution where the intermediate value has been truncated, provided the trig equation is correct.

E.g. $\sin\alpha = \frac{\sqrt{39}}{20} \Rightarrow \alpha = 0.317 \Rightarrow AOB = 2\alpha = 0.635$

Condone a solution written down from awrt $36.4°$ (without the need to shown any calculation.)

E

(b)

M1: Attempts to use the formula $s = r\theta$ with $r = 10$ and $\theta = 2\pi - 0.635$

The formula may be embedded. You may see $\underline{\underline{2\pi 10}} + 2\pi\sqrt{40} \underline{\underline{-10\times 0.635}}...$ which is fine for this M1

M1: Attempts to use a correct method in order to find angle AXB or AXO in circle C_2

Amongst many other methods are $\tan\beta = \frac{"3.12"'''}{15-9.5}$ and $\cos AXB = \frac{40+40-\left(\sqrt{39}\right)^2}{2\times\sqrt{40}\times\sqrt{40}} = \frac{41}{80}$

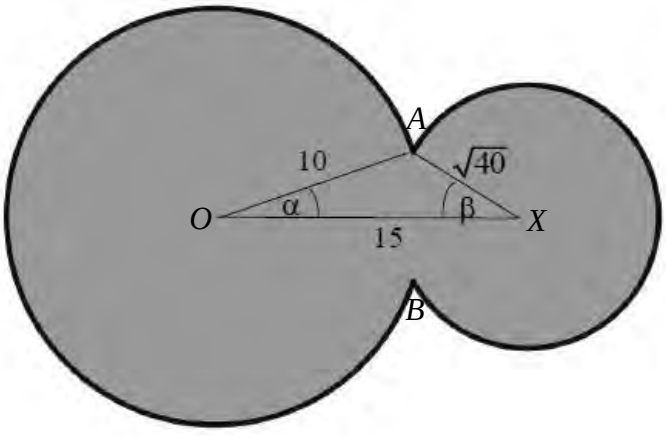
Note that many candidates believe this to be 0.635. This scores M0 dM0 A0

dM1: A full and complete attempt to find the perimeter of the region.

It is dependent upon having scored both M's.

A1: awrt 89.7

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(a)

M1: For the key step in attempting to find all lengths in triangle OAX, condoning slips

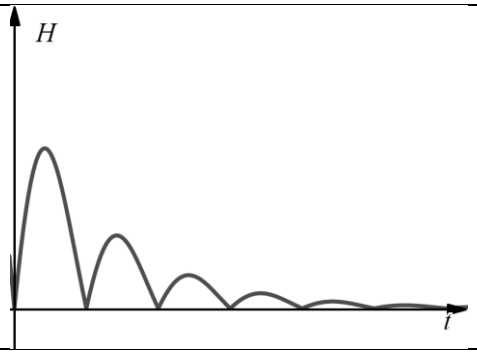
A1: All three lengths correct

M1: Attempts cosine rule to find α then $\times 2$

A1*: Correct and careful work in proceeding to the given answer

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Question	Scheme	Marks	AOs

12 (a)	$f(x) = 10e^{-0.25x} \sin x$		
	$\Rightarrow f'(x) = -2.5e^{-0.25x} \sin x + 10e^{-0.25x} \cos x$ oe	M1 A1	1.1b 1.1b
	$f'(x) = 0 \Rightarrow -2.5\cancel{e^{-0.25x}} \sin x + 10\cancel{e^{-0.25x}} \cos x = 0$	M1	2.1
	$\frac{\sin x}{\cos x} = \frac{10}{2.5} \Rightarrow \tan x = 4$*	A1*	1.1b
		(4)	
(b)	 "Correct" shape for 2 loops Fully correct with decreasing heights	M1 A1	1.1b 1.1b
		(2)	
(c)	Solves $\tan x = 4$ and substitutes answer into $H(t)$	M1	3.1a
	$H(4.47) = \lvert 10e^{-0.25\times 4.47} \sin 4.47 \rvert$	M1	1.1b
	awrt 3.18 (metres)	A1	3.2a
		(3)	
(d)	The **times between** each bounce should not stay the same when the heights of each bounce is getting smaller	B1	3.5b
		(1)	
			(10 marks)

(a)

M1: For attempting to differentiate using the product rule condoning slips, for example the power of e .

So for example score expressions of the form $\pm ...e^{-0.25x} \sin x \pm ...e^{-0.25x} \cos x$ M1

Sight of $vdu - udv$ however is M0

A1: $f'(x) = -2.5e^{-0.25x} \sin x + 10e^{-0.25x} \cos x$ which may be unsimplified

M1: For clear reasoning in setting their $f'(x) = 0$, factorising/ cancelling out the $e^{-0.25x}$ term leading to a trigonometric equation in only $\sin x$ and $\cos x$

Do not allow candidates to substitute $x = \arctan 4$ into $f'(x)$ to score this mark.

A1*: Shows the steps $\frac{\sin x}{\cos x} = \frac{10}{2.5}$ or equivalent leading to $\Rightarrow \tan x = 4$*. $\frac{\sin x}{\cos x}$ must be seen.

(b)

M1: Draws at least two "loops". The height of the second loop should be lower than the first loop.

Condone the sight of rounding where there should be cusps

A1: At least 4 loops with decreasing heights and no rounding at the cusps.

The intention should be that the graph should 'sit' on the x -axis but be tolerant.

It is possible to overwrite Figure 3, but all loops must be clearly seen.

(c)

M1: Understands that to solve the problem they are required to substitute an answer to $\tan t = 4$ into $H(t)$

This can be awarded for an attempt to substitute $t =$ awrt 1.33 or $t =$ awrt 4.47 into $H(t)$

$H(t) = 6.96$ implies the use of $t = 1.33$ Condone for this mark only, an attempt to substitute $t =$ awrt $76°$ or awrt $256°$ into $H(t)$

M1: Substitutes $t =$ awrt 4.47 into $H(t) = \left|10e^{-0.25t} \sin t\right|$. Implied by awrt 3.2

A1: Awrt 3.18 metres. Condone the lack of units. If two values are given the correct one must be seen to have been chosen

It is possible for candidates to sketch this on their graphical calculators and gain this answer. If there is no incorrect working seen and 3.18 is given, then award 111 for such an attempt.

(d)

B1: Makes reference to the fact that the time between each bounce should not stay the same when the heights of each bounce is getting smaller.

Look for " time (or gap) between the bounces will change''

'bounces would not be equal times apart'

'bounces would become more frequent'

But do not accept 'the times between each bounce would be longer or slower'

Do not accept explanations such as there are other factors that would affect this such as "wind resistance", friction etc

Question	Scheme	Marks	AOs
3 (a)	Allow explanations such as • student should have worked in radians • they did not convert degrees to radians • 40 should be in radians • θ should be in radians • angle (or θ) should be $\frac{40\pi}{180}$ or $\frac{2\pi}{9}$ • correct formula is $\pi r^2\left(\frac{\theta}{360}\right)$ {where θ is in degrees} • correct formula is $\pi r^2\left(\frac{40}{360}\right)$	B1	2.3
		(1)	
(b) **Way 1**	{Area of sector = } $\frac{1}{2}(5^2)\left(\frac{2\pi}{9}\right)$	M1	1.1b
	$= \frac{25}{9}\pi$ {cm^2} or awrt 8.73 {cm^2}	A1	1.1b
		(2)	
(b) **Way 2**	{Area of sector = } $\pi(5^2)\left(\frac{40}{360}\right)$	M1	1.1b
	$= \frac{25}{9}\pi$ {cm^2} or awrt 8.73 {cm^2}	A1	1.1b
		(2)	
			(3 marks)

Notes for Question 3

(a)	
B1:	Explains that the formula use is only valid when angle *AOB* is applied in radians. See scheme for examples of suitable explanations.
(b)	**Way 1**
M1:	Correct application of the sector formula using a correct value for θ in radians
Note:	Allow exact equivalents for θ e.g. $\theta = \frac{40\pi}{180}$ or θ in the range [0.68, 0.71]
A1*:	Accept $\frac{25}{9}\pi$ or awrt 8.73 **Note:** Ignore the units
(b)	**Way 2**
M1:	Correct application of the sector formula in degrees
A1:	Accept $\frac{25}{9}\pi$ or awrt 8.73 **Note:** Ignore the units.
Note:	Allow exact equivalents such as $\frac{50}{18}\pi$
Note:	Allow M1 A1 for $500\left(\frac{\pi}{180}\right) = \frac{25}{9}\pi$ {cm^2} or awrt 8.73 {cm^2}

Question	Scheme	Marks	AOs

12	$\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} \equiv 2\cot 2\theta$		
(a) Way 1	$\{\text{LHS}=\}\ \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta}$	M1	3.1a
	$=\dfrac{\cos(3\theta-\theta)}{\sin\theta\cos\theta} \quad \left\{=\dfrac{\cos 2\theta}{\sin\theta\cos\theta}\right\}$	A1	2.1
	$=\dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta}=2\cot 2\theta$ *	dM1	1.1b
		A1 *	2.1
		(4)	
(a) Way 2	$\{\text{LHS}=\}\ \dfrac{\cos 2\theta\cos\theta-\sin 2\theta\sin\theta}{\sin\theta}+\dfrac{\sin 2\theta\cos\theta+\cos 2\theta\sin\theta}{\cos\theta}$		
	$=\dfrac{\cos 2\theta\cos^2\theta-\sin 2\theta\sin\theta\cos\theta+\sin 2\theta\cos\theta\sin\theta+\cos 2\theta\sin^2\theta}{\sin\theta\cos\theta}$	M1	3.1a
	$=\dfrac{\cos 2\theta(\cos^2\theta+\sin^2\theta)}{\sin\theta\cos\theta} \quad \left\{=\dfrac{\cos 2\theta}{\sin\theta\cos\theta}\right\}$	A1	2.1
	$=\dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta}=2\cot 2\theta$ *	dM1	1.1b
		A1 *	2.1
		(4)	
(a) Way 3	$\{\text{RHS}=\}\ \dfrac{2\cos 2\theta}{\sin 2\theta}=\dfrac{2\cos(3\theta-\theta)}{\sin 2\theta}=\dfrac{2(\cos 3\theta\cos\theta+\sin 3\theta\sin\theta)}{\sin 2\theta}$	M1	3.1a
		A1	2.1
	$=\dfrac{2(\cos 3\theta\cos\theta+\sin 3\theta\sin\theta)}{2\sin\theta\cos\theta}$	dM1	1.1b
	$=\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}$ *	A1 *	2.1
		(4)	
(b) Way 1	$\left\{\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=4\Rightarrow\right\} \quad 2\cot 2\theta=4\Rightarrow 2\left(\dfrac{1}{\tan 2\theta}\right)=4$	M1	1.1b
	Rearranges to give $\tan 2\theta=k;\ k\neq 0$ and applies $\arctan k$	dM1	1.1b
	$\left\{90°<\theta<180°,\ \tan 2\theta=\dfrac{1}{2}\Rightarrow\right\}$		
	Only one solution of $\theta=103.3°$ (1 dp) or awrt 103.3°	A1	2.2a
		(3)	
(b) Way 2	$\left\{\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=4\Rightarrow\right\} \quad 2\cot 2\theta=4\Rightarrow\dfrac{2}{\tan 2\theta}=4$	M1	1.1b
	$\dfrac{2}{\left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right)}=4\Rightarrow 2(1-\tan^2\theta)=8\tan\theta$ $\Rightarrow \tan^2\theta+4\tan\theta-1=0\Rightarrow\tan\theta=\dfrac{-4\pm\sqrt{(4)^2-4(1)(-1)}}{2(1)}$ $\{\Rightarrow\tan\theta=-2\pm\sqrt{5}\}\Rightarrow\tan\theta=k;\ k\neq 0\Rightarrow$ applies $\arctan k$	dM1	1.1b
	$\{90°<\theta<180°,\ \tan\theta=-2-\sqrt{5}\ \Rightarrow\}$		
	Only one solution of $\theta=103.3°$ (1 dp) or awrt 103.3°	A1	2.2a
		(3)	
			(7 marks)

Notes for Question 12

(a)	**Way 1 and Way 2**
M1:	Correct valid method forming a common denominator of $\sin\theta\cos\theta$ i.e. correct process of $\dfrac{(...)\cos\theta+(...)\sin\theta}{\cos\theta\sin\theta}$
A1:	Proceeds to show that the numerator of their resulting fraction simplifies to $\cos(3\theta-\theta)$ or $\cos 2\theta$
dM1:	**dependent on the previous M mark** Applies a correct $\sin 2\theta \equiv 2\sin\theta\cos\theta$ to the common denominator $\sin\theta\cos\theta$
A1*	Correct proof
Note:	Writing $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta}{\sin\theta\cos\theta} + \dfrac{\sin 3\theta\sin\theta}{\sin\theta\cos\theta}$ is considered a correct valid method of forming a common denominator of $\sin\theta\cos\theta$ for the 1st M1 mark
Note:	Give 1st M0 e.g. for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=\dfrac{\cos 4\theta+\sin 4\theta}{\sin\theta\cos\theta}$ but allow 1st M1 for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta} = \dfrac{\cos 4\theta+\sin 4\theta}{\sin\theta\cos\theta}$
Note:	Give 1st M0 e.g. for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=\dfrac{\cos^2 3\theta+\sin^2 3\theta}{\sin\theta\cos\theta}$ but allow 1st M1 for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta} = \dfrac{\cos^2 3\theta+\sin^2 3\theta}{\sin\theta\cos\theta}$
Note:	Allow 2nd M1 for stating a correct $\sin 2\theta = 2\sin\theta\cos\theta$ and for attempting to apply it to the common denominator $\sin\theta\cos\theta$
(a)	**Way 3**
M1:	Starts from RHS and proceeds to expand $\cos 2\theta$ in the form $\cos 3\theta\cos\theta \pm \sin 3\theta\sin\theta$
A1:	Shows, as part of their proof, that $\cos 2\theta = \cos 3\theta\cos\theta + \sin 3\theta\sin\theta$
dM1:	**dependent on the previous M mark** Applies $\sin 2\theta \equiv 2\sin\theta\cos\theta$ to their denominator
A1*:	Correct proof
Note:	Allow 1st M1 1st A1 (together) for any of LHS $\rightarrow \dfrac{\cos 2\theta}{\sin\theta\cos\theta}$ or LHS $\rightarrow \dfrac{\cos 2\theta(\cos^2\theta+\sin^2\theta)}{\sin\theta\cos\theta}$ or LHS $\rightarrow \cos 2\theta(\cot\theta+\tan\theta)$ or LHS $\rightarrow \cos 2\theta\left(\dfrac{1+\tan^2\theta}{\tan\theta}\right)$ (i.e. where $\cos 2\theta$ has been factorised out)
Note:	Allow 1st M1 1st A1 for progressing as far as LHS $= ... = \cot x - \tan x$
Note:	The following is a correct alternative solution $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta} = \dfrac{\frac{1}{2}(\cos 4\theta+\cos 2\theta)-\frac{1}{2}(\cos 4\theta-\cos 2\theta)}{\sin\theta\cos\theta}$ $= \dfrac{\cos 2\theta}{\sin\theta\cos\theta} = \dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} = 2\cot 2\theta$ *
Note:	E.g. going from $\dfrac{\cos 2\theta\cos^2\theta-\sin 2\theta\sin\theta\cos\theta+\sin 2\theta\cos\theta\sin\theta+\cos 2\theta\sin^2\theta}{\sin\theta\cos\theta}$ to $\dfrac{\cos 2\theta}{\sin\theta\cos\theta}$ with no intermediate working is 1st A0

Notes for Question 12 Continued	
(b)	**Way 1**
M1:	Evidence of applying $\cot 2\theta = \dfrac{1}{\tan 2\theta}$
dM1:	**dependent on the previous M mark** Rearranges to give $\tan 2\theta = k,\ k \neq 0,$ and applies $\arctan k$
A1:	Uses $90° < \theta < 180°$ to deduce the only solution $\theta =$ awrt $103.3°$
Note:	Give M0M0A0 for writing, for example, $\tan 2\theta = 2$ with no evidence of applying $\cot 2\theta = \dfrac{1}{\tan 2\theta}$
Note:	1st M1 can be implied by seeing $\tan 2\theta = \dfrac{1}{2}$
Note:	Condone 2nd M1 for applying $\dfrac{1}{2}\arctan\left(\dfrac{1}{2}\right)$ $\{= 13.28...\}$
(b)	**Way 2**
M1:	Evidence of applying $\cot 2\theta = \dfrac{1}{\tan 2\theta}$
dM1:	**dependent on the previous M mark** Applies $\tan 2\theta \equiv \dfrac{2\tan\theta}{1-\tan^2\theta}$, forms and uses a correct method for solving a 3TQ to give $\tan\theta = k,\ k \neq 0,$ and applies $\arctan k$
A1:	Uses $90° < \theta < 180°$ to deduce the only solution $\theta =$ awrt $103.3°$
Note:	Give M1 dM1 A1 for no working leading to $\theta =$ awrt $103.3°$ and no other solutions
Note:	Give M1 dM1 A0 for no working leading to $\theta =$ awrt $103.3°$ and other solutions which can be either outside or inside the range $90° < \theta < 180°$

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
3	States or uses $\frac{1}{2}r^2\theta = 11$	B1	1.1b
	States or uses $2r + r\theta = 4r\theta$	B1	1.1b
	Attempts to solve, full method $r = ...$	M1	3.1a
	$r = \sqrt{33}$	A1	1.1b
			[4]

(4 marks)

Notes:

B1: States or uses $\frac{1}{2}r^2\theta = 11$ This may be implied with an embedded found value for θ

B1: States or uses $2r + r\theta = 4r\theta$ or equivalent

M1: Full method to find $r = ...$ This involves combining the equations to eliminate θ or find θ

The initial equations must be of the same "form" (see **) but condone slips when attempting to solve.

It cannot be scored from impossible values for θ Hence only score if $0 < \theta < 2\pi$ FYI $\theta = \frac{2}{3}$ radians

Allow this to be scored from equations such as $...r^2\theta = 11$ and ones that simplify to $...r = ...r\theta$ **

Allow their $2r + r\theta = 4r\theta \Rightarrow \theta = ..$ then substitute this into their $\frac{1}{2}r^2\theta = 11$

Allow their $2r + r\theta = 4r\theta \Rightarrow r\theta = ..$ then substitute this into their $\frac{1}{2}r^2\theta = 11$

Allow their $\frac{1}{2}r^2\theta = 11 \Rightarrow \theta = \frac{..}{r^2}$ then substitute into their $2r + r\theta = 4r\theta \Rightarrow r = ..$

A1: $r = \sqrt{33}$ only but isw after a correct answer.

.....

The whole question can be attempted using θ in degrees.

B1: States or uses $\frac{\theta}{360} \times \pi r^2 = 11$

B1: States or uses $2r + \frac{\theta}{360} \times 2\pi r = 4 \times \frac{\theta}{360} \times 2\pi r$

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
5	$\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\dfrac{(2\sin\theta+2\cos\theta)3\cos\theta-3\sin\theta(2\cos\theta-2\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$	M1 A1	1.1b 1.1b
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator or uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\dfrac{\ldots}{\ldots\ldots C\sin\theta\cos\theta}$	M1	3.1a
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\dfrac{P}{Q+R\sin 2\theta}$	M1	2.1
	$\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\dfrac{3}{2+2\sin 2\theta}=\dfrac{\frac{3}{2}}{1+\sin 2\theta}$	A1	1.1b
			(5 marks)

Notes:

M1: For choosing either the quotient, product rule or implicit differentiation and applying it to the given function. Look for the correct form of $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}$ (condone it being stated as $\dfrac{\mathrm{d}y}{\mathrm{d}x}$) but tolerate slips on the coefficients and also condone $\dfrac{\mathrm{d}(\sin\theta)}{\mathrm{d}\theta}=\pm\cos\theta$ and $\dfrac{\mathrm{d}(\cos\theta)}{\mathrm{d}\theta}=\pm\sin\theta$

For quotient rule look for $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\dfrac{(2\sin\theta+2\cos\theta)\times\pm\ldots\cos\theta-3\sin\theta(\pm\ldots\cos\theta\pm\ldots\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$

For product rule look for

$\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=(2\sin\theta+2\cos\theta)^{-1}\times\pm\ldots\cos\theta\pm3\sin\theta\times(2\sin\theta+2\cos\theta)^{-2}\times(\pm\ldots\cos\theta\pm\ldots\sin\theta)$

Implicit differentiation look for $(\ldots\cos\theta\pm\ldots\sin\theta)y+(2\sin\theta+2\cos\theta)\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\ldots\cos\theta$

A1: A correct expression involving $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}$ condoning it appearing as $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

M1: Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator OR uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\dfrac{\ldots}{\ldots\ldots C\sin\theta\cos\theta}$

M1: Expands and uses $\sin^2\theta+\cos^2\theta=1$ in the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in the denominator to reach an expression of the form $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\dfrac{P}{Q+R\sin 2\theta}$.

A1: Fully correct proof with $A=\dfrac{3}{2}$ stated but allow for example $\dfrac{\frac{3}{2}}{1+\sin 2\theta}$

Allow recovery from missing brackets. Condone notation slips. This is not a given answer

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
8 (a)	$D = 5 + 2\sin(30\times 6.5)° = \text{awrt } 4.48\,\text{m}$ with units	B1	3.4
		(1)	
(b)	$3.8 = 5 + 2\sin(30t)° \Rightarrow \sin(30t)° = -0.6$	M1 A1	1.1b 1.1b
	$t = 10.77$	dM1	3.1a
	10:46 a.m. or 10:47 a.m.	A1	3.2a
		(4)	
			(5 marks)

Notes:

(a)

B1: Scored for using the model ie. substituting $t = 6.5$ into $D = 5 + 2\sin(30t)°$ and stating $D = awrt\ 4.48\text{m}$. The units must be seen somewhere in (a) . So allow when $D = 4.482.. = 4.5\ \text{m}$

Allow the mark for a correct answer without any working.

(b)

M1: For using $D = 3.8$ and proceeding to $\sin(30t)° = k, \quad |k| \le 1$

A1: $\sin(30t)° = -0.6$ This may be implied by any correct answer for t such as $t = 7.2$

If the A1 implied, the calculation must be performed in degrees.

dM1: For finding **the first value** of t for their $\sin(30t)° = k$ after $t = 8.5$.

You may well see other values as well which is not an issue for this dM mark

(Note that $\sin(30t)° = -0.6 \Rightarrow 30t = 216.9°$ as well but this gives $t = 7.2$)

For the correct $\sin(30t)° = -0.6 \Rightarrow 30t = 323.1 \Rightarrow t = \text{awrt } 10.8$

For the incorrect $\sin(30t)° = +0.6 \Rightarrow 30t = 396.9 \Rightarrow t = \text{awrt } 13.2$

So award this mark if you see $30t = \text{inv}\sin\ their - 0.6$ to give the first value of t where $30\ t > 255$

A1: Allow 10:46 a.m. (12 hour clock notation) or 10:46 (24 hour clock notation) oe

Allow 10:47 a.m. (12 hour clock notation) or 10:47 (24 hour clock notation) oe

DO NOT allow 646 minutes or 10 hours 46 minutes.

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
6	(a) $f(x) = -3x^3 + 8x^2 - 9x + 10, \quad x \in \mathbb{R}$		
(a)	(i) $\{f(2) = -24 + 32 - 18 + 10 \Rightarrow\}\ f(2) = 0$	B1	1.1b
	(ii) $\{f(x) =\}\ (x-2)(-3x^2 + 2x - 5)$ or $(2-x)(3x^2 - 2x + 5)$	M1	2.2a
		A1	1.1b
		(3)	
(b)	$-3y^6 + 8y^4 - 9y^2 + 10 = 0 \Rightarrow (y^2 - 2)(-3y^4 + 2y^2 - 5) = 0$		
	Gives a partial explanation by • explaining that $-3y^4 + 2y^2 - 5 = 0$ has no {real} solutions with a reason, e.g. $b^2 - 4ac = (2)^2 - 4(-3)(-5) = -56 < 0$ • or stating that $y^2 = 2$ has 2 {real} solutions or $y = \pm\sqrt{2}$ {only}	M1	2.4
	Complete proof that the given equation has exactly two {real} solutions	A1	2.1
		(2)	
(c)	$3\tan^3\theta - 8\tan^2\theta + 9\tan\theta - 10 = 0;\ 7\pi \leq \theta < 10\pi$		
	{Deduces that} there are 3 solutions	B1	2.2a
		(1)	
			(6 marks)

Notes for Question 6

(a)(i)	
B1:	$f(2) = 0$ or 0 stated by itself in part (a)(i)
(a)(ii)	
M1:	Deduces that $(x-2)$ or $(2-x)$ is a factor and attempts to find the other quadratic factor by • using long division to obtain either $\pm 3x^2 \pm kx + ...,\ k = \text{value} \neq 0$ or $\pm 3x^2 \pm \alpha x + \beta,\ \beta = \text{value} \neq 0,\ \alpha$ can be 0 • factorising to obtain their quadratic factor in the form $(\pm 3x^2 \pm kx \pm c),\ k = \text{value} \neq 0,$ c can be 0, or in the form $(\pm 3x^2 \pm \alpha x \pm \beta),\ \beta = \text{value} \neq 0,\ \alpha$ can be 0
A1:	$(x-2)(-3x^2 + 2x - 5),\ (2-x)(3x^2 - 2x + 5)$ or $-(x-2)(3x^2 - 2x + 5)$ stated together as a product
(b)	
M1:	See scheme
A1:	See scheme. Proof must be correct ***with no errors***, e.g. giving an incorrect discriminant value
Note:	Correct calculation e.g. $(2)^2 - 4(-3)(-5),\ 4 - 60$ or -56 must be given for the first explanation
Note:	Note that M1 can be allowed for • a correct follow through calculation for the discriminant of their "$-3y^4 + 2y^2 - 5$" which would lead to a value < 0 together with an explanation that $-3y^4 + 2y^2 - 5 = 0$ has no {real} solutions • **or** for the omission of < 0
Note:	< 0 must also been stated in a discriminant method for A1
Note:	Do not allow A1 for incorrect working, e.g. $(2)^2 - 4(-3)(-5) = -54 < 0$
Note:	$y^2 = 2 \Rightarrow y = \pm 2$, so 2 solutions is not allowed for A1, but can be condoned for M1
Note:	Using the formula on $-3y^4 + 2y^2 - 5 = 0$ or $-3x^2 + 2x - 5 = 0$ gives y^2 or $x = \dfrac{-2 \pm \sqrt{-56}}{-6}$ or $\dfrac{-1 \pm \sqrt{-14}}{-3}$

Notes for Question 6 Continued	
Note:	Completing the square on $-3x^2+2x-5=0$ gives $x^2-\frac{2}{3}x+\frac{5}{3}=0 \Rightarrow \left(x-\frac{1}{3}\right)^2-\frac{1}{9}+\frac{5}{3}=0 \Rightarrow x=\frac{1}{3}\pm\sqrt{\frac{-14}{9}}$
Note:	Do not recover work for part (b) in part (c)
(c)	
B1:	See scheme
Note:	Give B0 for stating $\theta =$ awrt 23.1, awrt 26.2, awrt 29.4 **without** reference to 3 solutions

Question	Scheme	Marks	AOs

9	$\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$; as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$		
	$\frac{\cos(\theta + h) - \cos\theta}{h}$	B1	2.1
	$= \frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}$	M1	1.1b
		A1	1.1b
	$= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$		
	As $h \to 0$, $-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta \to -1\sin\theta + 0\cos\theta$	dM1	2.1
	so $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$ *	A1*	2.5
		(5)	

(5 marks)

Notes for Question 9

B1:	Gives the correct fraction such as $\frac{\cos(\theta + h) - \cos\theta}{h}$ or $\frac{\cos(\theta + \delta\theta) - \cos\theta}{\delta\theta}$ Allow $\frac{\cos(\theta + h) - \cos\theta}{(\theta + h) - \theta}$ o.e. **Note:** $\cos(\theta + h)$ or $\cos(\theta + \delta\theta)$ may be expanded
M1:	Uses the compound angle formula for $\cos(\theta + h)$ to give $\cos\theta\cos h \pm \sin\theta\sin h$
A1:	Achieves $\frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}$ or equivalent
dM1:	**dependent on both the B and M marks being awarded** Complete attempt to apply the given limits to the gradient of their chord
Note:	They must isolate $\frac{\sin h}{h}$ and $\left(\frac{\cos h - 1}{h}\right)$, and replace $\frac{\sin h}{h}$ with 1 and replace $\left(\frac{\cos h - 1}{h}\right)$ with 0
A1*:	cso. Uses correct mathematical language of limiting arguments to prove $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$
Note:	Acceptable responses for the final A mark include: • $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = \lim_{h \to 0}\left(-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta\right) = -1\sin\theta + 0\cos\theta = -\sin\theta$ • Gradient of chord $= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$. As $h \to 0$, gradient of chord tends to the gradient of the curve, so derivative is $-\sin\theta$ • Gradient of chord $= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$. As $h \to 0$, gradient of ***curve*** is $-\sin\theta$
Note:	Give final A0 for the following example which shows ***no limiting arguments***: when $h = 0$, $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta = -1\sin\theta + 0\cos\theta = -\sin\theta$
Note:	Do not allow the final A1 for stating $\frac{\sin h}{h} = 1$ or $\left(\frac{\cos h - 1}{h}\right) = 0$ and attempting to apply these
Note:	In this question $\delta\theta$ may be used in place of h
Note:	Condone $\mathrm{f}'(\theta)$ where $\mathrm{f}(\theta) = \cos\theta$ or $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ where $y = \cos\theta$ used in place of $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta)$

Notes for Question 9 Continued	
Note:	Condone *x* used in place of θ if this is done consistently
Note:	Give final A0 for • $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos x) = \lim_{h\to 0}\left(-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta\right) = -1\sin\theta + 0\cos\theta = -\sin\theta$ • $\frac{\mathrm{d}}{\mathrm{d}\theta} = \ldots$ • Defining $\mathrm{f}(x) = \cos\theta$ and applying $\mathrm{f}'(x) = \ldots$ • $\frac{\mathrm{d}}{\mathrm{d}x}(\cos\theta)$
Note:	Give final A1 for a correct limiting argument in *x*, followed by $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$ e.g. $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos x) = \lim_{h\to 0}\left(-\frac{\sin h}{h}\sin x + \left(\frac{\cos h - 1}{h}\right)\cos x\right) = -1\sin x + 0\cos x = -\sin x$ $\Rightarrow \frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$
Note:	Applying $h\to 0,\ \sin h\to h,\ \cos h\to 1$ to give e.g. $\lim_{h\to 0}\left(\frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}\right) = \left(\frac{\cos\theta(1) - \sin\theta(h) - \cos\theta}{h}\right) = \frac{-\sin\theta(h)}{h} = -\sin\theta$ is final M0 A0 for incorrect application of limits
Note:	$\lim_{h\to 0}\left(\frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}\right) = \lim_{h\to 0}\left(-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta\right)$ $= \underset{\longleftarrow}{\lim_{h\to 0}}\ (-(1)\sin\theta + 0\cos\theta) = -\sin\theta$. So for $\underset{\longleftarrow}{\text{not removing } \lim_{h\to 0}}$ when the limit was taken is final A0
Note:	**<u>Alternative Method:</u>** Considers $\frac{\cos(\theta+h) - \cos(\theta-h)}{(\theta+h)-(\theta-h)}$ which simplifies to $\frac{-2\sin\theta\sin h}{2h}$

Question	Scheme	Marks	AOs

12	$1-\cos 2\theta \equiv \tan\theta \sin 2\theta,\ \theta \neq \frac{(2n+1)\pi}{2},\ n \in \mathbb{Z}$		
(a) Way 1	$\tan\theta\sin 2\theta = \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta)$	M1	1.1b
	$= \left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)(2\sin\theta\cancel{\cos\theta}) = 2\sin^2\theta = 1-\cos 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
(a) Way 2	$1-\cos 2\theta = 1-(1-2\sin^2\theta) = 2\sin^2\theta$	M1	1.1b
	$= \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta) = \tan\theta\sin 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
	$(\sec^2 x-5)(1-\cos 2x) = 3\tan^2 x\sin 2x,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$		
(b) Way 1	$(\sec^2 x-5)\tan x\sin 2x = 3\tan^2 x\sin 2x$ **or** $(\sec^2 x-5)(1-\cos 2x) = 3\tan x(1-\cos 2x)$		
	Deduces $x=0$	B1	2.2a
	Uses $\sec^2 x = 1+\tan^2 x$ and cancels/factorises out $\tan x$ or $(1-\cos 2x)$ e.g. $(1+\tan^2 x-3\tan x-5)\tan x = 0$ **or** $(1+\tan^2 x-3\tan x-5)(1-\cos 2x) = 0$ **or** $1+\tan^2 x-5 = 3\tan x$	M1	2.1
	$\tan^2 x-3\tan x-4 = 0$	A1	1.1b
	$(\tan x-4)(\tan x+1) = 0 \Rightarrow \tan x = ...$	M1	1.1b
	$x = -\frac{\pi}{4}, 1.326$	A1	1.1b
		A1	1.1b
		(6)	
			(9 marks)

Notes for Question 12

(a)	**Way 1**
M1:	Applies $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to $\tan\theta\sin 2\theta$
M1:	Cancels as scheme (may be implied) and attempts to use $\cos 2\theta = 1-2\sin^2\theta$
A1*:	For a correct proof showing all steps of the argument
(a) Way 2	
M1:	For using $\cos 2\theta = 1-2\sin^2\theta$
Note:	If the form $\cos 2\theta = \cos^2\theta-\sin^2\theta$ or $\cos 2\theta = 2\cos^2\theta-1$ is used, the mark cannot be awarded until $\cos^2\theta$ has been replaced by $1-\sin^2\theta$
M1:	Attempts to write their $2\sin^2\theta$ in terms of $\tan\theta$ and $\sin 2\theta$ using $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and
	$\sin 2\theta = 2\sin\theta\cos\theta$ within the given expression
A1*:	For a correct proof showing all steps of the argument
Note:	If a proof meets in the middle; e.g. they show LHS $= 2\sin^2\theta$ and RHS $= 2\sin^2\theta$; then some indication must be given that the proof is complete. E.g. $1-\cos 2\theta \equiv \tan\theta\sin 2\theta$, QED, box

Notes for Question 12 Continued	
(b)	
B1:	Deduces that the given equation yields a solution $x=0$
M1:	For using the key step of $\sec^2 x=1+\tan^2 x$ and cancels/factorises out $\tan x$ or $(1-\cos 2x)$ or $\sin 2x$ to produce a quadratic factor or quadratic equation in just $\tan x$
Note:	Allow the use of $\pm\sec^2 x=\pm 1\pm\tan^2 x$ for M1
A1:	Correct 3TQ in $\tan x$. E.g. $\tan^2 x-3\tan x-4=0$
Note:	E.g. $\tan^2 x-4=3\tan x$ or $\tan^2 x-3\tan x=4$ are acceptable for A1
M1:	For a correct method of solving their 3TQ in $\tan x$
A1:	Any one of $-\frac{\pi}{4}$, awrt -0.785, awrt 1.326, $-45°$, awrt 75.964°
A1:	Only $x=-\frac{\pi}{4}, 1.326$ **cao** stated in the range $-\frac{\pi}{2}<x<\frac{\pi}{2}$

Note: **Alternative Method (Alt 1)**

$(\sec^2 x-5)\tan x\sin 2x=3\tan^2 x\sin 2x$ **or** $(\sec^2 x-5)(1-\cos 2x)=3\tan x(1-\cos 2x)$			
Deduces $x=0$		B1	2.2a
$\sec^2 x-5=3\tan x \Rightarrow \frac{1}{\cos^2 x}-5=3\left(\frac{\sin x}{\cos x}\right)$ $1-5\cos^2 x=3\sin x\cos x$ $1-5\left(\frac{1+\cos 2x}{2}\right)=\frac{3}{2}\sin 2x$	Complete process (as shown) of using the identities for $\sin 2x$ and $\cos 2x$ to proceed as far as $\pm A\pm B\cos 2x=\pm C\sin 2x$	M1	2.1
$-\frac{3}{2}-\frac{5}{2}\cos 2x=\frac{3}{2}\sin 2x$ $\{3\sin 2x+5\cos 2x=-3\}$	$-\frac{3}{2}-\frac{5}{2}\cos 2x=\frac{3}{2}\sin 2x$ o.e.	A1	1.1b
$\sqrt{34}\sin(2x+1.03)=-3$	Expresses their answer in the form $R\sin(2x+\alpha)=k;\ k\neq 0$ with values for R and α	M1	1.1b
$\sin(2x+1.03)=-\frac{3}{\sqrt{34}}$			
$x=-\frac{\pi}{4}, 1.326$		A1	1.1b
		A1	1.1b

Question	Scheme	Marks	AOs
	$C: y = x\ln x$; l is a normal to C at $P(\mathrm{e}, \mathrm{e})$		