adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Trigonometry

Question paper

30 questions

Exam questions and marking instructions remain credited to their board.

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14. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Show that

$$\sin(x + 30°) + \sqrt{3}\cos(x + 30°) \equiv 2\cos x$$

(3)

(b) Hence solve, for $0 \leqslant \theta < 180°$

$$\sin(\theta + 30°) + \sqrt{3}\cos(\theta + 30°) = 3\sin 2\theta$$

giving your answers to one decimal place, where appropriate.

(4)

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Question 14 continued

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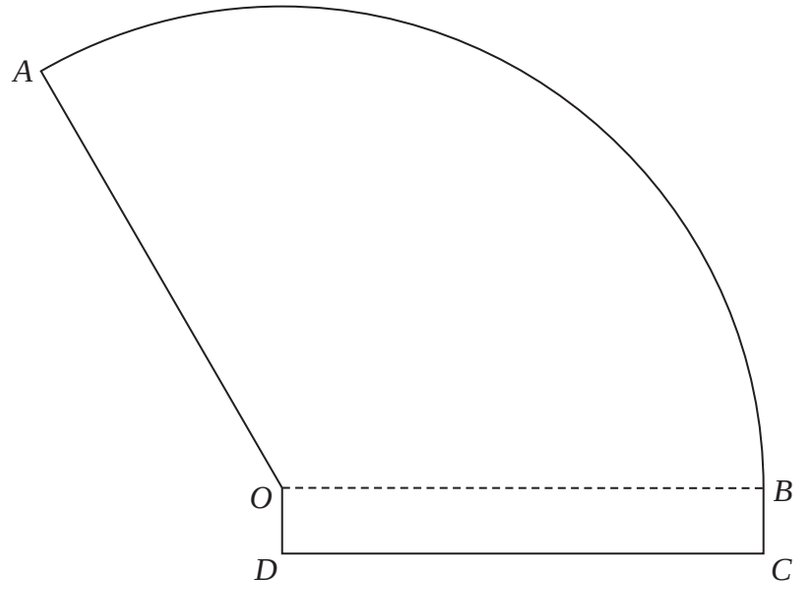


Figure 4

Figure 4 shows the plan view for the design of a stage.

The shape of this design consists of a sector of a circle *AOB* joined to a rectangle *OBCD*.

Given that

- the radius of the sector is r metres and angle AOB is θ radians
- the length and width of the rectangle are r metres and $\frac{1}{10}r$ metres respectively
- the total area of the stage is $240\,\text{m}^2$

(a) show that the perimeter of the stage, P metres, is given by

$$P = 2r + \frac{480}{r}$$

You must make your method clear.

(4)

Using algebraic differentiation,

(b) find the value of r for which P has a stationary value.

(3)

(c) Prove, by further differentiation, that this value of r gives the minimum perimeter of the stage.

(2)

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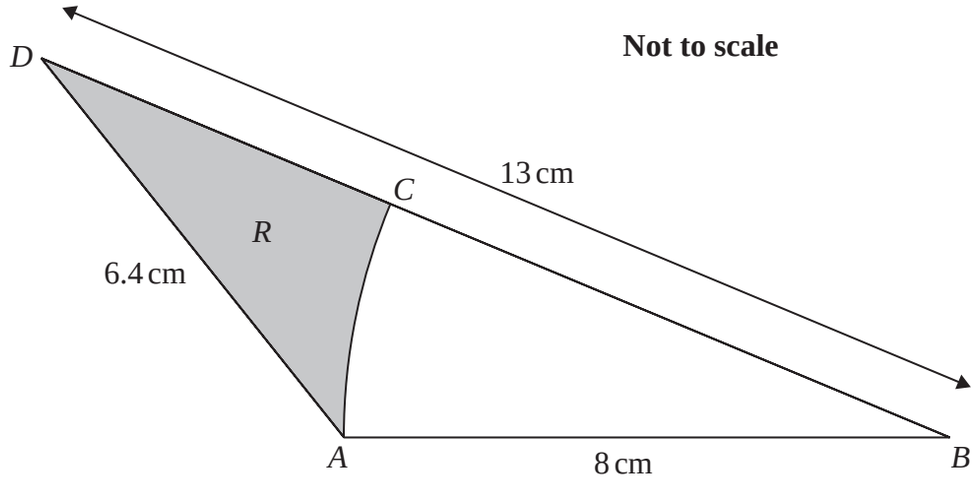


Figure 1

The shape *ABCD*, shown in Figure 1, consists of a triangle *ABD* containing a sector *ABC* of a circle with centre *B*.

Given that

- $AD = 6.4\,\text{cm}$
- $BD = 13\,\text{cm}$
- $BA = BC = 8\,\text{cm}$

(a) show that angle $ABC = 0.394$ radians to 3 significant figures.

(2)

The region *R*, shown shaded in Figure 1, is bounded by the line *CD*, the line *DA* and the arc *AC*.

(b) Find the area of *R*, giving the answer in cm^2 to 3 significant figures.
You must make your method clear.

(3)

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Question 4 continued

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13. Given that for all values of k, where $0 < |k| < 1$, the equation

$$\sin(nx) = k \qquad n \in \mathbb{N}$$

has exactly 6 solutions in the interval $0 \leqslant x < 2\pi$

(i) deduce the value of n

(1)

(ii) deduce the number of solutions of the equation

$$\sin^2(nx) = k^2$$

in the interval $0 \leqslant x < 5\pi$, justifying your answer.

(2)

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Question 13 continued

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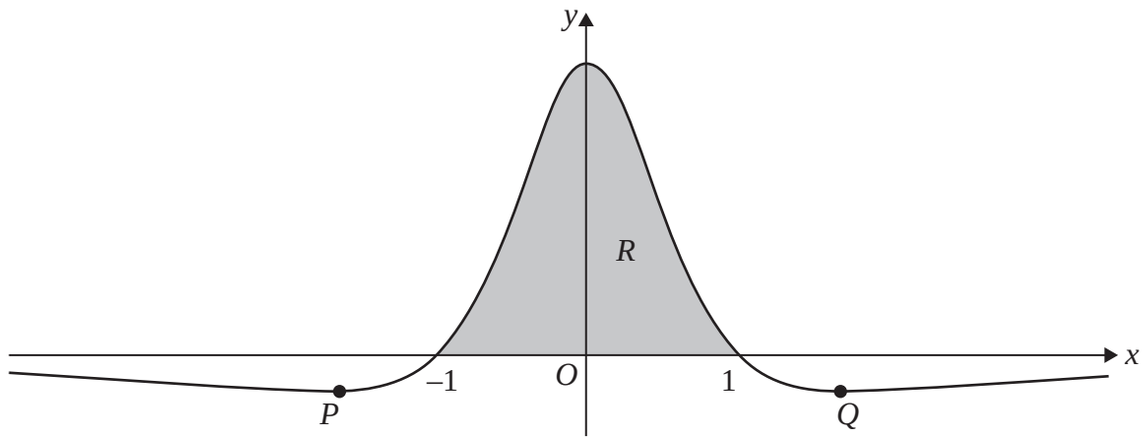


Figure 5

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 5 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{1 - x^2}{\left(1 + x^2\right)^2}$$

The curve

- intersects the x-axis at –1 and 1
- has minimum turning points at P and Q

as shown in Figure 5.

(a) Use calculus to find the exact coordinates of P.

(5)

(b) Using the substitution $x = \tan\theta$ show that

$$\int_{-1}^{1} \mathrm{f}(x)\,\mathrm{d}x = \int_{\alpha}^{\beta} \cos 2\theta\,\mathrm{d}\theta$$

where α and β are constants to be found.

(5)

The finite region R, shown shaded in Figure 5, is bounded by the x-axis and the curve.

(c) Use algebraic integration to find the area of R.

(3)

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12. (a) Express $140\cos\theta - 480\sin\theta$ in the form $K\cos(\theta + \alpha)$

where $K > 0$ and $0 < \alpha < 90°$

State the value of K and give the value of α, in degrees, to 2 decimal places.

(3)

A scientist studies the number of rabbits and the number of foxes in a wood for one year.

The number of rabbits, R, is modelled by the equation

$$R = A + 140\cos(30t)° - 480\sin(30t)°$$

where t months is the time after the start of the year and A is a constant.

Given that, during the year, the maximum number of rabbits in the wood is 1500

(b) (i) find a complete equation for this model.

(ii) Hence write down the minimum number of rabbits in the wood during the year according to the model.

(2)

The actual number of rabbits in the wood is at its minimum value in the middle of April.

(c) Use this information to comment on the model for the number of rabbits.

(2)

The number of foxes, F, in the wood during the same year is modelled by the equation

$$F = 100 + 70\sin(30t + 70)°$$

The number of foxes is at its minimum value after T months.

(d) Find, according to the models, the number of **rabbits** in the wood at time T months.

(4)

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Question 12 continued

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13. (a) Given that a is a positive constant, use the substitution $x = a\sin^2\theta$ to show that

$$\int_0^a x^{\frac{1}{2}}\sqrt{a-x}\ \mathrm{d}x = \frac{1}{2}a^2\int_0^{\frac{\pi}{2}}\sin^2 2\theta\ \mathrm{d}\theta$$

(4)

(b) Hence use algebraic integration to show that

$$\int_0^a x^{\frac{1}{2}}\sqrt{a-x}\ \mathrm{d}x = k\pi a^2$$

where k is a constant to be found.

(4)

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Question 13 continued

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8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\frac{1}{\operatorname{cosec}\theta - 1} + \frac{1}{\operatorname{cosec}\theta + 1} \equiv 2\tan\theta\sec\theta \qquad \theta \neq (90n)^\circ,\ n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $0 < x < 90°$, the equation

$$\frac{1}{\operatorname{cosec} 2x - 1} + \frac{1}{\operatorname{cosec} 2x + 1} = \cot 2x \sec 2x$$

Give each answer, in degrees, to one decimal place.

(4)

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Question 8 continued

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Question 8 continued

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8.

FRONT OF STAGE

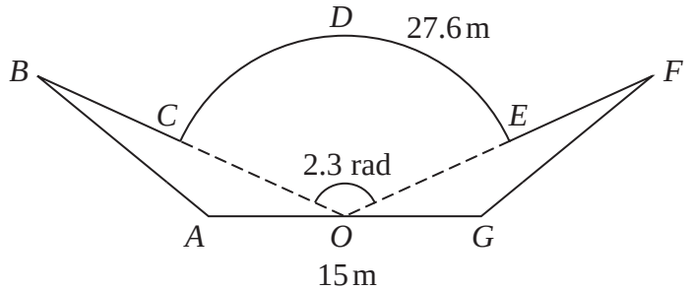


Diagram **NOT** accurately drawn

BACK OF STAGE

Figure 1

Figure 1 shows the plan view of a stage.

The plan view shows two congruent triangles *ABO* and *GFO* joined to a sector *OCDEO* of a circle, centre *O*, where

- angle *COE* = 2.3 radians
- arc length *CDE* = 27.6 m
- *AOG* is a straight line of length 15 m

(a) Show that *OC* = 12 m.

(2)

(b) Show that the size of angle *AOB* is 0.421 radians correct to 3 decimal places.

(2)

Given that the total length of the front of the stage, *BCDEF*, is 35 m,

(c) find the total area of the stage, giving your answer to the nearest square metre.

(6)

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Question 8 continued

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Question 8 continued

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13. On a roller coaster ride, passengers travel in carriages around a track.

On the ride, carriages complete multiple circuits of the track such that

- the maximum vertical height of a carriage above the ground is 60 m
- a carriage starts a circuit at a vertical height of 2 m above the ground
- the ground is horizontal

The vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is modelled by the equation

$$H = a - b(t - 20)^2$$

where a and b are positive constants.

(a) Find a complete equation for the model.

(3)

(b) Use the model to determine the height of the carriage above the ground when $t = 40$

(1)

In an alternative model, the vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is given by

$$H = 29\cos(9t + \alpha)^\circ + \beta \qquad 0 \leqslant \alpha < 360^\circ$$

where α and β are constants.

(c) Find a complete equation for the alternative model.

(2)

Given that the carriage moves continuously for 2 minutes,

(d) give a reason why the alternative model would be more appropriate.

(1)

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Question 13 continued

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Question 13 continued

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Question 13 continued

(Total for Question 13 is 7 marks)

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2. A sequence $u_1, u_2, u_3 \ldots$ is defined by

$$u_1 = 35$$

$$u_{n+1} = u_n + 7\cos\left(\frac{n\pi}{2}\right) - 5(-1)^n$$

(a) (i) Show that $u_2 = 40$

(ii) Find the value of u_3 and the value of u_4

(3)

Given that the sequence is periodic with order 4

(b) (i) write down the value of u_5

(ii) find the value of $\sum_{r=1}^{25} u_r$

(3)

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Question 2 continued

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4. Coffee is poured into a cup.

The temperature of the coffee, H °C, t minutes after being poured into the cup is modelled by the equation

$$H = A\mathrm{e}^{-Bt} + 30$$

where A and B are constants.

Initially, the temperature of the coffee was 85 °C.

(a) State the value of A.

(1)

Initially, the coffee was cooling at a rate of 7.5 °C per minute.

(b) Find a complete equation linking H and t, giving the value of B to 3 decimal places.

(3)

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Question 4 continued

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8. (a) Express $2\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants,

$R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The first three terms of an arithmetic sequence are

$$\cos x \qquad \cos x + \sin x \qquad \cos x + 2\sin x \qquad x \neq n\pi$$

Given that S_9 represents the sum of the first 9 terms of this sequence as x varies,

(b) (i) find the exact maximum value of S_9

(ii) deduce the smallest positive value of x at which this maximum value of S_9 occurs.

(3)

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Question 8 continued

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Question 8 continued

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15. A student attempts to answer the following question:

> Given that x is an obtuse angle, use algebra to prove by contradiction that
>
> $$\sin x - \cos x \geqslant 1$$

The student starts the proof with:

> Assume that $\sin x - \cos x < 1$ when x is an obtuse angle
>
> $$\Rightarrow (\sin x - \cos x)^2 < 1$$
>
> $$\Rightarrow \ldots$$

The start of the student's proof is reprinted below.

Complete the proof.

(3)

Assume that $\sin x - \cos x < 1$ when x is an obtuse angle

$$\Rightarrow (\sin x - \cos x)^2 < 1$$

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Question 15 continued

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Question 15 continued

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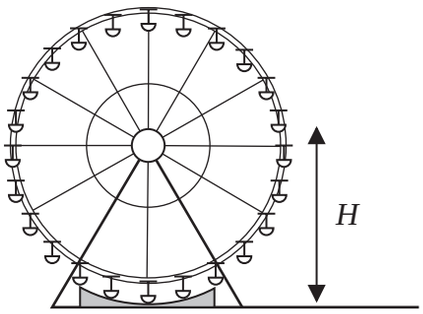


Figure 4

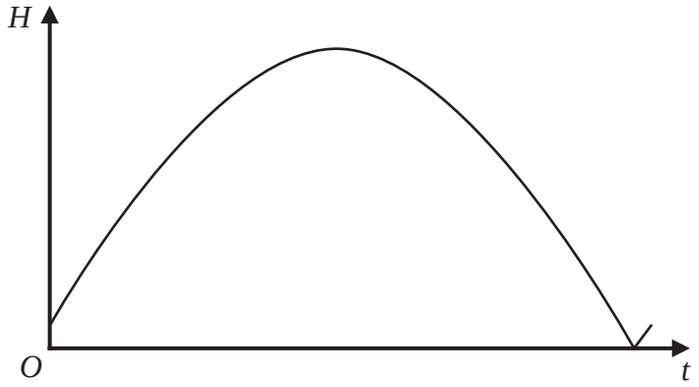


Figure 5

Figure 4 shows a sketch of a Ferris wheel.

The height above the ground, H m, of a passenger on the Ferris wheel, t seconds after the wheel starts turning, is modelled by the equation

$$H = \left|A\sin(bt + \alpha)^\circ\right|$$

where A, b and α are constants.

Figure 5 shows a sketch of the graph of H against t, for one revolution of the wheel.

Given that

- the maximum height of the passenger above the ground is 50 m
- the passenger is 1 m above the ground when the wheel starts turning
- the wheel takes 720 seconds to complete one revolution

(a) find a complete equation for the model, giving the exact value of A, the exact value of b and the value of α to 3 significant figures.

(4)

(b) Explain why an equation of the form

$$H = \left|A\sin(bt + \alpha)^\circ\right| + d$$

where d is a positive constant, would be a more appropriate model.

(1)

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Question 9 continued

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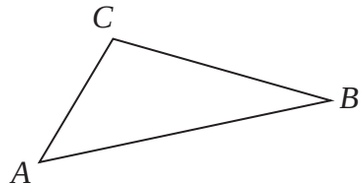


Figure 1

Figure 1 shows a sketch of triangle ABC.

Given that

- $\overrightarrow{AB} = -3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$
- $\overrightarrow{BC} = \mathbf{i} + \mathbf{j} + 4\mathbf{k}$

(a) find $\overrightarrow{AC}$

(2)

(b) show that $\cos ABC = \frac{9}{10}$

(3)

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Question 6 continued

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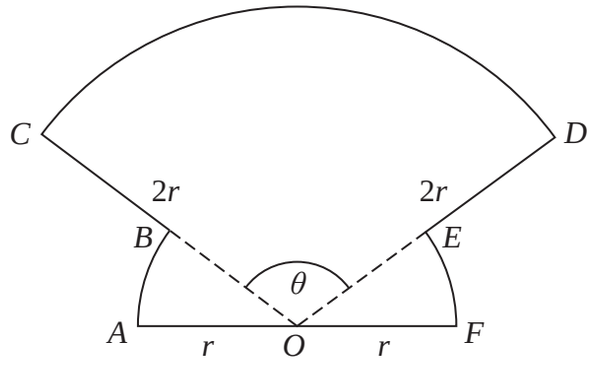


Figure 1

The shape *OABCDEFO* shown in Figure 1 is a design for a logo.

In the design

- *OAB* is a sector of a circle centre *O* and radius *r*
- sector *OFE* is congruent to sector *OAB*
- *ODC* is a sector of a circle centre *O* and radius 2*r*
- *AOF* is a straight line

Given that the size of angle *COD* is θ radians,

(a) write down, in terms of θ, the size of angle *AOB*

(1)

(b) Show that the area of the logo is

$$\frac{1}{2} r^2 (3\theta + \pi)$$

(2)

(c) Find the perimeter of the logo, giving your answer in simplest form in terms of r, θ and π.

(2)

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Question 6 continued

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9. Show that

$$\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^\circ = \frac{9}{28}$$

(3)

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15. (a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and the value of α in radians to 3 decimal places.

(3)

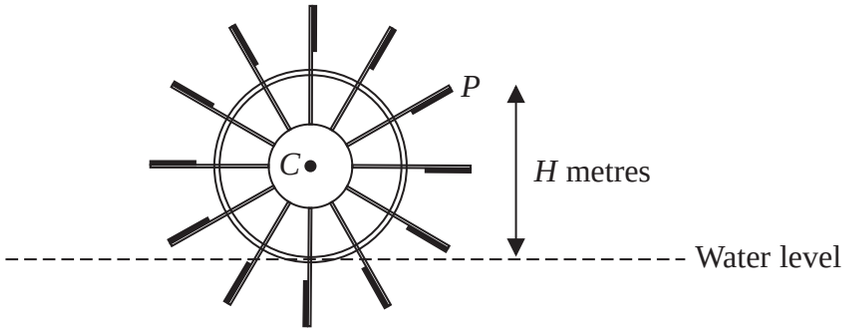


Figure 6

Figure 6 shows the cross-section of a water wheel.

The wheel is free to rotate about a fixed axis through the point C.

The point P is at the end of one of the paddles of the wheel, as shown in Figure 6.

The water level is assumed to be horizontal and of constant height.

The vertical height, H metres, of P above the water level is modelled by the equation

$$H = 3 + 4\cos(0.5t) - 2\sin(0.5t)$$

where t is the time in seconds after the wheel starts rotating.

Using the model, find

(b) (i) the maximum height of P above the water level,

(ii) the value of t when this maximum height first occurs, giving your answer to one decimal place.

(3)

In a single revolution of the wheel, P is below the water level for a total of T seconds.

According to the model,

(c) find the value of T giving your answer to 3 significant figures.

(*Solutions based entirely on calculator technology are not acceptable*.)

(4)

In reality, the water level may not be of constant height.

(d) Explain how the equation of the model should be refined to take this into account.

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Question 15 continued

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Question 15 continued

(Total for Question 15 is 11 marks)

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6. (a) Express $\sin x + 2\cos x$ in the form $R\sin(x + \alpha)$ where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The temperature, θ°C, inside a room on a given day is modelled by the equation

$$\theta = 5 + \sin\left(\frac{\pi t}{12} - 3\right) + 2\cos\left(\frac{\pi t}{12} - 3\right) \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

Using the equation of the model and your answer to part (a),

(b) deduce the maximum temperature of the room during this day,

(1)

(c) find the time of day when the maximum temperature occurs, giving your answer to the nearest minute.

(3)

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Question 6 continued

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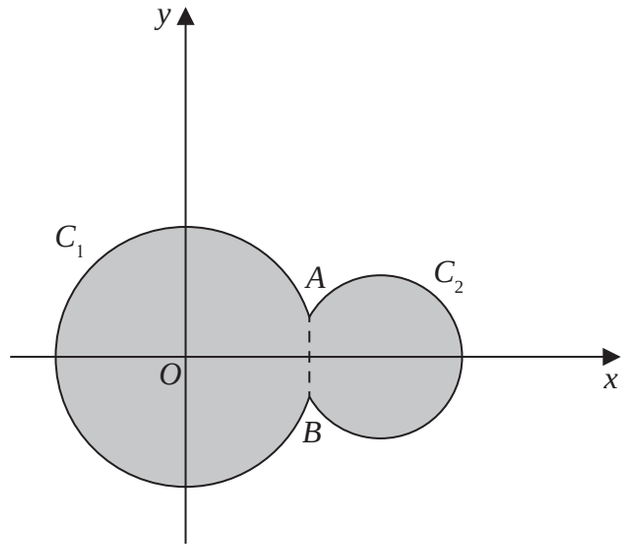


Figure 3

Circle C_1 has equation $x^2 + y^2 = 100$

Circle C_2 has equation $(x - 15)^2 + y^2 = 40$

The circles meet at points A and B as shown in Figure 3.

(a) Show that angle $AOB = 0.635$ radians to 3 significant figures, where O is the origin.

(4)

The region shown shaded in Figure 3 is bounded by C_1 and C_2

(b) Find the perimeter of the shaded region, giving your answer to one decimal place.

(4)

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Question 11 continued

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Question 11 continued

(Total for Question 11 is 8 marks)

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12. $f(x) = 10e^{-0.25x} \sin x, \quad x \geqslant 0$

(a) Show that the x coordinates of the turning points of the curve with equation $y = f(x)$ satisfy the equation $\tan x = 4$

(4)

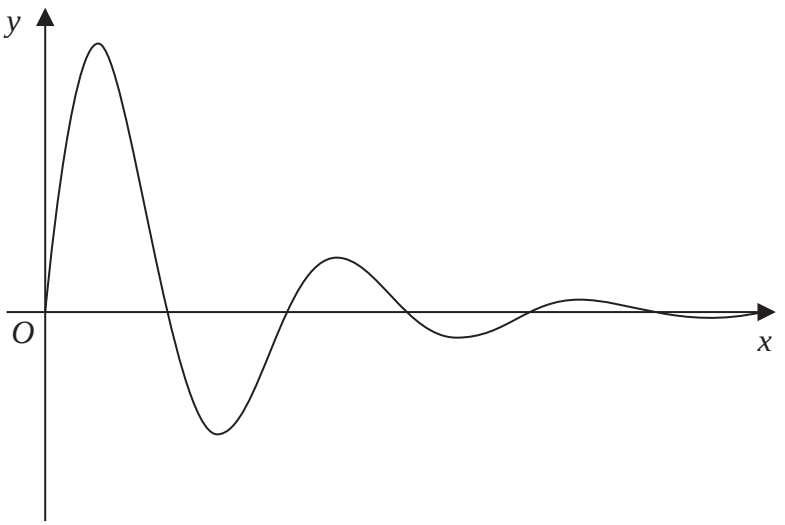


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$.

(b) Sketch the graph of H against t where

$$H(t) = \left|10e^{-0.25t} \sin t\right| \quad t \geqslant 0$$

showing the long-term behaviour of this curve.

(2)

The function $H(t)$ is used to model the height, in metres, of a ball above the ground t seconds after it has been kicked.

Using this model, find

(c) the maximum height of the ball above the ground between the first and second bounce.

(3)

(d) Explain why this model should not be used to predict the time of each bounce.

(1)

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Question 12 continued

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Question 12 continued

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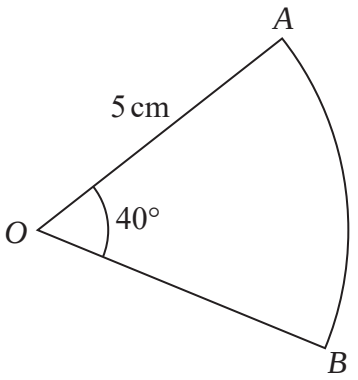


Figure 1

Figure 1 shows a sector AOB of a circle with centre O, radius 5 cm and angle $AOB = 40°$

The attempt of a student to find the area of the sector is shown below.

$$\text{Area of sector} = \frac{1}{2} r^2\theta$$

$$= \frac{1}{2} \times 5^2 \times 40$$

$$= 500\,\text{cm}^2$$

(a) Explain the error made by this student.

(1)

(b) Write out a correct solution.

(2)

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Question 3 continued

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12. (a) Prove

$$\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} \equiv 2\cot 2\theta \qquad \theta \neq (90n)^\circ,\ n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $90^\circ < \theta < 180^\circ$, the equation

$$\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} = 4$$

giving any solutions to one decimal place.

(3)

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Question 12 continued

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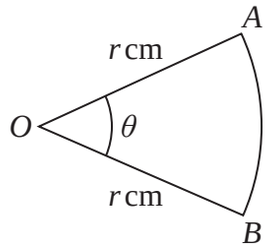


Figure 1

Figure 1 shows a sector AOB of a circle with centre O and radius r cm.

The angle AOB is θ radians.
The area of the sector AOB is $11\,\text{cm}^2$

Given that the perimeter of the sector is 4 times the length of the arc AB, find the exact value of r.

(4)

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Question 3 continued

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5. Given that

$$y = \frac{3\sin\theta}{2\sin\theta + 2\cos\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}\theta} = \frac{A}{1 + \sin 2\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

where A is a rational constant to be found.

(5)

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Question 5 continued

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8. The depth of water, D metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 2\sin(30t)° \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

A boat enters the harbour at 6:30 am and it takes 2 hours to load its cargo.
The boat requires the depth of water to be at least 3.8 metres before it can leave the harbour.

(a) Find the depth of the water in the harbour when the boat enters the harbour.

(1)

(b) Find, to the nearest minute, the earliest time the boat can leave the harbour.
(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(4)

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Question 8 continued

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6.

$$f(x) = -3x^3 + 8x^2 - 9x + 10, \quad x \in \mathbb{R}$$

(a) (i) Calculate $f(2)$

(ii) Write $f(x)$ as a product of two algebraic factors.

(3)

Using the answer to (a)(ii),

(b) prove that there are exactly two real solutions to the equation

$$-3y^6 + 8y^4 - 9y^2 + 10 = 0$$

(2)

(c) deduce the number of real solutions, for $7\pi \leqslant \theta < 10\pi$, to the equation

$$3\tan^3\theta - 8\tan^2\theta + 9\tan\theta - 10 = 0$$

(1)

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9. Given that θ is measured in radians, prove, from first principles, that

$$\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$$

You may assume the formula for $\cos(A \pm B)$ and that as $h \to 0$, $\dfrac{\sin h}{h} \to 1$ and $\dfrac{\cos h - 1}{h} \to 0$

(5)

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12. (a) Prove that

$$1 - \cos 2\theta \equiv \tan\theta \sin 2\theta, \quad \theta \neq \frac{(2n+1)\pi}{2}, \quad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, the equation

$$(\sec^2 x - 5)(1 - \cos 2x) = 3\tan^2 x \sin 2x$$

Give any non-exact answer to 3 decimal places where appropriate.

(6)

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