adamjee maths

A-LEVEL MATHS / EDEXCEL / FREE RESOURCES

Vectors

Mark scheme

10 questions

Exam questions and marking instructions remain credited to their board.

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Question	Scheme	Marks	AOs
10 (a)	Attempts to add $\overrightarrow{PQ} = 2\mathbf{i}+8\mathbf{j}-2\mathbf{k}$ and $\overrightarrow{QR} = 6\mathbf{i}+6\mathbf{k}$	M1	1.1b
	$\left(\overrightarrow{PR} =\right) 8\mathbf{i}+8\mathbf{j}+4\mathbf{k}$	A1	1.1b
		(2)	
(b)	Attempts to show the triangle is isosceles (**or** right-angled) e.g. Attempts $\left\|\overrightarrow{PQ}\right\| = \sqrt{2^2+8^2+(-2)^2}$ and $\left\|\overrightarrow{QR}\right\| = \sqrt{6^2+6^2}$	M1	3.1a
	Shows e.g. $\left\|\overrightarrow{PQ}\right\| = \left\|\overrightarrow{QR}\right\| = \sqrt{72}$ $(=6\sqrt{2})$	A1	1.1b
	Attempts to show the triangle is isosceles **and** right-angled e.g. attempts to find the lengths of all three sides AND e.g. attempts to compare lengths via use of "$a^2+b^2=c^2$"	M1	1.1b
	e.g. Shows that $\left\|\overrightarrow{PQ}\right\|^2+\left\|\overrightarrow{QR}\right\|^2=\left\|\overrightarrow{PR}\right\|^2$ as $72+72=144$ so PQR is a right-angled triangle	A1	2.1
		(4)	
			(6 marks)

Notes:

(a) If part (a) is not attempted and the correct $\overrightarrow{PR}$ is seen in part (b) then M1A1 can be awarded

M1: Attempts to add $\overrightarrow{PQ} = 2\mathbf{i}+8\mathbf{j}-2\mathbf{k}$ and $\overrightarrow{QR} = 6\mathbf{i}+6\mathbf{k}$ with at least one correct component of $8\mathbf{i}+8\mathbf{j}+4\mathbf{k}$. A typical misread of $\overrightarrow{QR}$ as $6\mathbf{i}+6\mathbf{j}$ can score for at least one correct component of $8\mathbf{i}+14\mathbf{j}-2\mathbf{k}$

A1: Correct vector. Allow $8\mathbf{i}+8\mathbf{j}+4\mathbf{k}$ or $\begin{pmatrix}8\\8\\4\end{pmatrix}$ but **not** $\begin{pmatrix}8\mathbf{i}\\8\mathbf{j}\\4\mathbf{k}\end{pmatrix}$ and **not** (8, 8, 4)

Condone $\begin{matrix}8\\8\\4\end{matrix}$ for $\begin{pmatrix}8\\8\\4\end{pmatrix}$ Do not apply isw here but award for e.g. $8\mathbf{i}+8\mathbf{j}+4\mathbf{k} = \begin{pmatrix}8\mathrm{i}\\8\mathrm{j}\\4\mathrm{k}\end{pmatrix}$

E.g. if they obtain $\overrightarrow{PR} = 8\mathbf{i}+8\mathbf{j}+4\mathbf{k}$ and then say $\overrightarrow{PR} = 2\mathbf{i}+2\mathbf{j}+\mathbf{k}$ then award A0

(b) **Note that M1A0M1A1 is not possible. If they have an incorrect vector in part (a) then the maximum score is M1A1M1A0. A misread of $\overrightarrow{QR}$ as $6\mathbf{i}+6\mathbf{j}$ in (b) can only score a maximum M1A1M1A0**

They may attempt to show the triangle is either isosceles or right-angled in either order. You will need to look through their solution and award the order which scores most marks. Usually it will be isosceles first. Condone slips to be recovered.

To show the triangle is isosceles they only need to show **two sides** (or **two angles**) are the same. They do not need to consider the other side to show it is isosceles.

A1: Fully shows that the triangle is isosceles (or right-angled). See table below. Allow slips in their method if recovered as long as they proceed to correct lengths or values. **A conclusion that the triangle is isosceles or right-angled is not required for this mark**.

Isosceles	**Requirement for M1 examples**	**Requirement for A1 examples**
Using lengths	Attempts to find the length or length^2 of PQ and QR: $\left(\left\|\overrightarrow{PQ}\right\| =\right) \sqrt{2^2+8^2+(-2)^2}$ $(=\sqrt{72}=6\sqrt{2})$ or seen as e.g. $2\sqrt{1^2+4^2+(-1)^2}$ $(=2\sqrt{18})$ $\left(\left\|\overrightarrow{QR}\right\| =\right) \sqrt{6^2+6^2}$ $(=\sqrt{72}=6\sqrt{2})$ or may be seen as e.g. $3\sqrt{2^2+2^2}$ $(=3\sqrt{8})$ May be implied by e.g. $6\sqrt{2}$ Condone missing brackets around $(-2)^2$ provided the intention is clear to square and add implied by e.g. $6\sqrt{2}$	States or shows that $\left\|\overrightarrow{PQ}\right\| = \left\|\overrightarrow{QR}\right\|$ $\left(=\sqrt{72}\,(=6\sqrt{2})\right)$, or equivalent. Accept e.g. $PQ^2 = QR^2$ or "both are 72" Condone poor notation and/or labelling of lengths provided they are not clearly referring to the longest length. e.g. achieves $6\sqrt{2}$ for both PQ and QR and states they are the same scores M1A1 **Only stating isosceles without a comparison of *PQ* and *QR* is A0**
		Uses the sine rule with the lengths and angles embedded in the correct places e.g. $\dfrac{\sin\angle QPR}{QR}=\dfrac{\sin\angle PRQ}{PQ}$ Deduces $\sin\angle QPR=\sin\angle PRQ$ so the angles are the same o.e.
Right-angled	**Requirement for M1 examples**	**Requirement for A1 examples**
Using lengths	Attempts to find all three lengths or lengths^2 $\left(\left\|\overrightarrow{PR}\right\| =\right) \sqrt{"8"^2+"8"^2+"4"^2}$ $(=12)$ or may be seen as e.g. $4\sqrt{"2"^2+"2"^2+"1"^2}$ or implied by their 12 **AND ATTEMPTS** $\left(PQ^2+QR^2=PR^2\Rightarrow\right)$ $"72"+"72"="144"$	States or shows that $\left\|PQ\right\|^2+\left\|QR\right\|^2=\left\|PR\right\|^2$, or equivalent Condone poor notation and/or labelling of lengths.
	Attempts to find all three lengths or lengths^2 (see above for guidance) **AND ATTEMPTS** the cosine rule correctly to find $\cos PQR=\dfrac{"72"+"72"-"144"}{2\times"\sqrt{72}"\times"\sqrt{72}"}$ o.e.	States or shows $\cos PQR=0$
Scalar dot product (Further Maths)	Attempts e.g. $\begin{pmatrix}2\\8\\-2\end{pmatrix}\bullet\begin{pmatrix}6\\0\\6\end{pmatrix}=(2\times6)+(8\times0)+(-2\times6)$ oe **Must see the calculation for M1**	States or shows that $(2\times6)+(8\times0)+(-2\times6)=0$ oe (they do not need to write anything more for this mark)
	$\left(\cos PQR=\right)\dfrac{(2\times6)+(8\times0)+(-2\times6)}{"\sqrt{72}"\times"\sqrt{72}"}$ oe	$\cos PQR=\dfrac{(2\times6)+(8\times0)+(-2\times6)}{\sqrt{72}\times\sqrt{72}}=0$

shown the triangle is isosceles and attempt to show that it is right-angled. **Here are some examples but there will be others**:

Right-angled	Method examples (required for second method mark)
Using lengths	Attempts to find all three lengths or lengths^2 **AND ATTEMPTS** $\left(PQ^2+QR^2=PR^2\Rightarrow\right)$ "72"+"72"="144"
	Attempts to find all three lengths or lengths^2 (see above for guidance) **AND ATTEMPTS** the cosine rule in an attempt to find $\cos PQR=\dfrac{"72"+"72"-"144"}{2\times"\sqrt{72}"\times"\sqrt{72}"}$
Scalar dot product	Attempts $\begin{pmatrix}2\\8\\-2\end{pmatrix}\bullet\begin{pmatrix}6\\0\\6\end{pmatrix}=(2\times6)+(8\times0)+(-2\times6)$
	$\left(\cos PQR=\right)\dfrac{(2\times6)+(8\times0)+(-2\times6)}{"\sqrt{72}"\times"\sqrt{72}"}$ oe

Alternatively, having already shown the triangle is right-angled they may:

- use trigonometry to show that the two base angles are both $45°$
- attempt to find the required lengths or lengths^2 of *PQ* and *PR* if not already found
(Note they may also find the length or length^2 of *QR* and may use the sine rule or cosine rule)

Note if they have shown either property then we will condone making an assumption of the other property to justify the size of angle *PRQ* and/or angle *QPR* and may use trigonometry to show either

$\left(\sin\theta=\dfrac{6\sqrt{2}}{12}\Rightarrow\right)\theta=\arcsin\left(\dfrac{6\sqrt{2}}{12}\right)=45°$ (we must see arcsin or $\sin^{-1}$)

$\sin\theta=\dfrac{6\sqrt{2}}{12}\Rightarrow\sin\theta=\dfrac{\sqrt{2}}{2}\left(\text{or}\dfrac{1}{\sqrt{2}}\right)\Rightarrow\theta=45°$ (since this is a known angle)

$\sin 45°=\dfrac{\sqrt{2}}{2}\left(\text{or}\dfrac{1}{\sqrt{2}}\right)=\dfrac{6\sqrt{2}}{12}$ (This requires all 3 equivalences (can be in any order))

Using trigonometry (SOHCAHTOA) to show the second property they will e.g.

- Use triangle *PQR* and assume right angled
- Split triangle *PQR* into two right angled congruent triangles using the isosceles property

If they use triangle *PQR* then it requires

- For M1: Calculations showing the required property to score M1
- For A1: To either draw a labelled right angled triangle or state the assumption that it is right angled and conclude

triangles which (provided they had two lengths the same already) will not be an assumption of a right angle. **Then it requires**

- For M1: Calculations to find either at least one of the base angles of $45°$ OR to find the right angle. It should be clear whether they
  - found a base angle of PQR and doubled it or
  - found half of the right angle and doubled it.
- For A1: Fully correct calculations and conclusion
  Note if they find a 45 degree angle and double it then it needs to be clear whether this is the right angle or if it is the sum of two base angles because there are four angles of size 45 degrees via this method.
  They would have to e.g. mention about the angle sum of a triangle oe or show a clearly labelled diagram and calculations with labels that match the diagram.

  Note that use of tan is unlikely to score because if they just use two equal lengths for the two shorter sides of their right angled triangle, then $\tan(A)=1$, so the angles will always be $45°$, $45°$ and $90°$ – it is the use of the length of 12 (or 6) which is going to lead to showing the triangle is right angled via these routes.

A1: Fully shows that the triangle is isosceles and right-angled **and concludes** that the triangle is **both isosceles and right-angled**. These conclusions may appear at separate stages of their solution. Condone poor notation and/or labelling of lengths provided the intention is clear. Condone slips if recovered. If they have a preamble then they must have a minimal conclusion e.g. proven, tick, QED

Note if they have an incorrect vector in part (a) then the maximum score is M1A1M1A0

Question	Scheme	Marks	AOs
14(a)	$\left(\overrightarrow{AD}=\overrightarrow{AB}-\overrightarrow{DB}=\right)\ 2\mathbf{a}+3\mathbf{b}-\left(-4\mathbf{a}+k\mathbf{b}\right)\ \left(=6\mathbf{a}+\left(3-k\right)\mathbf{b}\right)$	M1	1.1b
	$\frac{15}{6}=2.5\Rightarrow\frac{-5}{3-k}=2.5\rightarrow k=\ldots$	dM1	1.1b
	$k=5$ *	A1*	2.1
		(3)	

Notes

(a) **Note: Condone the use of column vectors throughout this question.**
There may be working on the diagram that can be awarded marks.

M1: Attempts either $\left(\overrightarrow{AD}=\overrightarrow{AB}-\overrightarrow{DB}=\right)\ 2\mathbf{a}+3\mathbf{b}-\left(-4\mathbf{a}+k\mathbf{b}\right)$ or $\left(\overrightarrow{DA}=\right)\ -4\mathbf{a}+k\mathbf{b}-\left(2\mathbf{a}+3\mathbf{b}\right)$ or $\left(\overrightarrow{DB}=\overrightarrow{DA}+\overrightarrow{AB}=\right)\ \alpha\left(15\mathbf{a}-5\mathbf{b}\right)+2\mathbf{a}+3\mathbf{b}$

Allow subtraction either way round and may be implied by one correct component or by e.g. $2\mathbf{a}+3\mathbf{b}=\overrightarrow{AD}-4\mathbf{a}+k\mathbf{b}$

For reference: $\overrightarrow{AD}=6\mathbf{a}+\left(3-k\right)\mathbf{b}$ and $\overrightarrow{DA}=-6\mathbf{a}+\left(k-3\right)\mathbf{b}$

Allow e.g. $\left(\overrightarrow{AD}=\right)\ \begin{pmatrix}6\mathbf{a}\\(3-k)\mathbf{b}\end{pmatrix}$ or $\left(\overrightarrow{AD}=\right)\ \begin{pmatrix}6\\3-k\end{pmatrix}$ including without the brackets $\begin{matrix}6\\3-k\end{matrix}$

Condone the use of gradients or ratios for $\overrightarrow{AD}$ e.g. $\frac{6}{3-k}$ or "6 : 3 – k" to imply this mark (either way round).

There are alternatives using e.g. $\overrightarrow{DC}$ but, in these cases, we require two expressions for the same vector, of which one expression must use $\overrightarrow{DB}$.

dM1: A full method to solve the problem. Some possible approaches:

- attempts to find a scale factor and uses it to find k
- sets up equivalent fractions e.g. $\frac{15}{6}=-\frac{5}{3-k}$ or e.g. $\frac{2--4}{3-k}=\frac{15}{-5}$ and solves for k
- sets up equivalent ratios e.g. 6 : 15 = 3 – k : –5 and solves for k
- sets up simultaneous equations and solves for k e.g. $2\mathbf{a}+3\mathbf{b}+4\mathbf{a}-k\mathbf{b}=\alpha\left(15\mathbf{a}-5\mathbf{b}\right)$ o.e. **or** $\left(\overrightarrow{DB}=\overrightarrow{DA}+\overrightarrow{AB}=\right)\ \alpha\left(-15\mathbf{a}+5\mathbf{b}\right)+2\mathbf{a}+3\mathbf{b}=-4\mathbf{a}+k\mathbf{b}$ leading to $6=15\alpha\ \left(\alpha=\frac{2}{5}\right)$ and $3-k=-5\alpha$ hence k = …

Note that their coefficient might be the reciprocal, from e.g. $\beta\left(2\mathbf{a}+3\mathbf{b}+4\mathbf{a}-k\mathbf{b}\right)=15\mathbf{a}-5\mathbf{b}$ $\left(\beta=2.5\right)$ and directions might be reversed in which case $\alpha=-0.4$ or $\beta=-2.5$ can be used.

A1*: Arrives at $k=5$ via a correct method. Usually this will be following:

- A correct expression for $\overrightarrow{AD}$ (e.g. $2\mathbf{a}+3\mathbf{b}-\left(-4\mathbf{a}+k\mathbf{b}\right)$) or $\overrightarrow{DA}$ or $\overrightarrow{BD}$ or $\overrightarrow{DB}$ which may be mislabelled.
- Correct scale factor stated (±2.5 or ±0.4) or implied (e.g., by $\frac{-5}{3-k}=\frac{15}{6}$ or $3\left(3-k\right)=-6$)
- A correct intermediate equation that leads to $k=5$

An example minimal response might look like:

e.g. $\alpha\left(15\mathbf{a}-5\mathbf{b}\right)=6\mathbf{a}+\left(3-k\right)\mathbf{b}\rightarrow6=15\alpha\rightarrow\alpha=\frac{2}{5}\rightarrow-5\left(\frac{2}{5}\right)=3-k\rightarrow k=5$

or e.g. $6\mathbf{a}+\left(3-k\right)\mathbf{b}\rightarrow\frac{15}{6}=-\frac{5}{3-k}\rightarrow k=5$

Condone missing/invisible brackets if recovered.

Alternative by verification:

M1: Sets $k = 5$, substitutes into $\overrightarrow{DB}$ and attempts $\left(\overrightarrow{AD} = \overrightarrow{AB} - \overrightarrow{DB} =\right)\ 2\mathbf{a} + 3\mathbf{b} - \left(-4\mathbf{a} + 5\mathbf{b}\right)$ o.e.

dM1: Attempts to compare $\overrightarrow{AD}$ o.e. with $\overrightarrow{BC}$ (usually $6\mathbf{a} - 2\mathbf{b} = \alpha\left(15\mathbf{a} - 5\mathbf{b}\right)$) and finds α

A1*: Requires:

- Correct $\overrightarrow{AD}$ o.e. e.g. $\overrightarrow{DA}$
- Correct scale factor $\alpha = \pm\frac{2}{5}$ or $\beta = \pm 2.5$ (sign dependent on their approach)
- Conclusion referencing the lines being parallel e.g. "hence $\overrightarrow{AD}$ and $\overrightarrow{BC}$ are parallel."

Question	Scheme	Marks	AOs
14(b)	e.g., $\left(\overrightarrow{BN}=\right)\ \frac{1}{5}\overrightarrow{BC}\ \left(=3\mathbf{a}-\mathbf{b}\right)$	B1	2.2a
	e.g., $\left(\overrightarrow{BX}=\lambda\overrightarrow{BD}=\right)\ \lambda\left(4\mathbf{a}-5\mathbf{b}\right)$	M1	2.1
	e.g., $\left(\overrightarrow{BX}=\lambda\overrightarrow{BD}=\right)\ \lambda\left(4\mathbf{a}-5\mathbf{b}\right)$ **and** e.g. $\left(\overrightarrow{BX}=\overrightarrow{BA}+\mu\overrightarrow{AN}=\right)\ \left(-2\mathbf{a}-3\mathbf{b}\right)+\mu\left(2\mathbf{a}+3\mathbf{b}+"3\mathbf{a}-\mathbf{b}"\right)$	dM1	3.1a
	$4\lambda=-2+5\mu$, $-5\lambda=-3+2\mu$ $\Rightarrow\lambda=\ldots\left(\frac{1}{3}\right)$ or $\mu=\ldots\left(\frac{2}{3}\right)$	ddM1	1.1b
	$1:2$	A1	2.2a
		(5)	
			(8 marks)

Notes

(b) **Note: Condone the use of column vectors throughout this question.**
There may be working on the diagram that can be awarded marks.

B1: Deduces a correct interpretation of the ratio $BN:NC=1:4$ that enables a start to a solution.
i.e., progresses to a correct statement that is at least as far as $\left(\overrightarrow{BN}=\right)\ \frac{1}{5}\overrightarrow{BC}$ (or e.g. $3\mathbf{a}-\mathbf{b}$) or $\left(\overrightarrow{CN}=\right)\ \frac{4}{5}\overrightarrow{CB}$ (or e.g. $4\mathbf{b}-12\mathbf{a}$). May be embedded in e.g. $\left(\overrightarrow{AN}=\right)\ 2\mathbf{a}+3\mathbf{b}+\frac{1}{5}\overrightarrow{BC}$
Allow e.g. $\frac{1}{5}\begin{pmatrix}15\\-5\end{pmatrix}$ or $\begin{pmatrix}3\\-1\end{pmatrix}$ or $\begin{pmatrix}3a\\-b\end{pmatrix}$ for this mark.

M1: For the key step in attempting a valid expression for $\overrightarrow{BX}$ (**or** $\overrightarrow{AX}$ **or** $\overrightarrow{DX}$ **or** $\overrightarrow{CX}$ **or** $\overrightarrow{NX}$) in terms of **a** and **b**. See diagram/notes on the next page for helpful vectors.
Condone slips provided their intention is clear.
Note that they might be using λ and μ the other way round or alternative variables.

Note: May be seen as a single expression such as $\overrightarrow{AB}=\overrightarrow{AX}+\overrightarrow{XB}\ \left(\text{i.e. }\overrightarrow{AB}=p\overrightarrow{AN}+q\overrightarrow{DB}\right)$ or $\overrightarrow{BN}=\overrightarrow{BX}+\overrightarrow{XN}\ \left(\text{i.e. }\overrightarrow{BN}=p\overrightarrow{BD}+q\overrightarrow{AN}\right)$ either of which scores M1dM1 simultaneously.

dM1: For the key step in attempting a **second** valid expression for their $\overrightarrow{BX}$ (**or** $\overrightarrow{AX}$ **or** $\overrightarrow{DX}$ **or** $\overrightarrow{CX}$ **or** $\overrightarrow{NX}$) in terms of **a** and **b** which enables the problem to be solved, i.e., it must not be parallel in approach to the first. See diagram/notes on the next page for helpful vectors.
One expression should involve $"\lambda"\left(4\mathbf{a}-5\mathbf{b}\right)$ and the other should involve $"\mu"\left("5\mathbf{a}+2\mathbf{b}"\right)$
Condone slips provided their intention is clear.
They must use different parameters in their approaches, e.g., λ and μ.
If using e.g. $\overrightarrow{DX}=-6\mathbf{a}+2\mathbf{b}+\mu\left(5\mathbf{a}+2\mathbf{b}\right)$ and $\overrightarrow{XB}=\lambda\left(-4\mathbf{a}+5\mathbf{b}\right)$ this mark is not scored until they set $\overrightarrow{DX}+\overrightarrow{XB}=\overrightarrow{DB}$ as $-6\mathbf{a}+2\mathbf{b}+\mu\left(5\mathbf{a}+2\mathbf{b}\right)+\lambda\left(-4\mathbf{a}+5\mathbf{b}\right)=-4\mathbf{a}+5\mathbf{b}$
Dependent on the previous method mark.

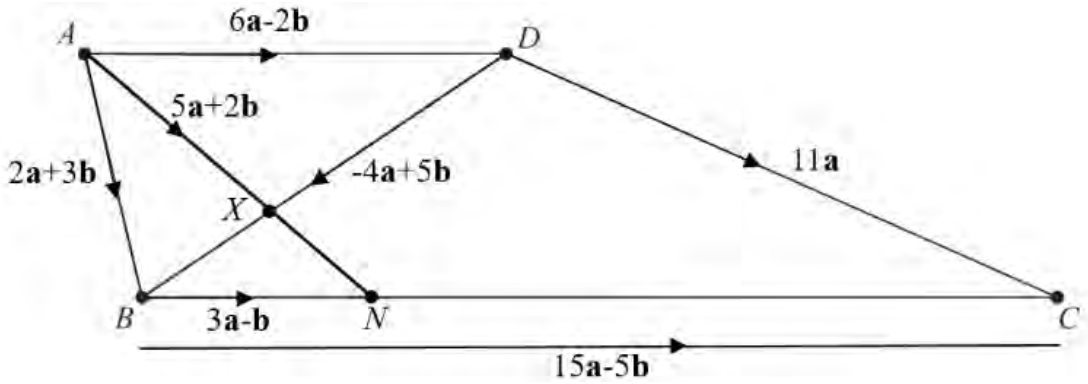
ddM1: Compares coefficients of **a** and **b** to create simultaneous equations in their parameters and attempts to solve (which may be by calculator) leading to a value for one of their parameters.
Condone slips provided the intention is clear.
This mark may be implied by a **correct** value for e.g. λ following their two **correct** expressions for e.g. $\overrightarrow{BX}$
Dependent on both previous method marks.

A1: $1:2$ o.e. Must follow a correct value for their parameter.
The correct ratio seen does **not** imply full marks. Candidates must show detailed reasoning.

Allow equivalent ratios e.g. $\frac{1}{3}:\frac{2}{3}$ and ISW (e.g. $1:3$) but they must be the correct way round.

There may be attempts using similar triangles. Send to review.

Helpful Diagram:



Note: Some examples of valid expressions for the M and dM marks for part (b) are:
In each expression they may use different parameters and e.g. $1-\lambda$ might just be e.g. ϕ.

- $\overrightarrow{BX}=\lambda\overrightarrow{BD}=\lambda\left(4\mathbf{a}-5\mathbf{b}\right)$
- $\overrightarrow{BX}=\overrightarrow{BA}+\mu\overrightarrow{AN}=\left(-2\mathbf{a}-3\mathbf{b}\right)+\mu\left(2\mathbf{a}+3\mathbf{b}+"3\mathbf{a}-\mathbf{b}"\right)$
- $\overrightarrow{BX}=\overrightarrow{BN}+\left(1-\mu\right)\overrightarrow{NA}=\left("3\mathbf{a}-\mathbf{b}"\right)+\left(1-\mu\right)\left(-2\mathbf{a}-3\mathbf{b}+"-3\mathbf{a}+\mathbf{b}"\right)$
- $\overrightarrow{DX}=\left(1-\lambda\right)\overrightarrow{DB}=\left(1-\lambda\right)\left(-4\mathbf{a}+5\mathbf{b}\right)$
- $\overrightarrow{DX}=\overrightarrow{DA}+\mu\overrightarrow{AN}=\left("-6\mathbf{a}+2\mathbf{b}"\right)+\mu\left(2\mathbf{a}+3\mathbf{b}+"3\mathbf{a}-\mathbf{b}"\right)$
- $\overrightarrow{DX}=\overrightarrow{DN}+\left(1-\mu\right)\overrightarrow{NA}=\left("-\mathbf{a}+4\mathbf{b}"\right)+\left(1-\mu\right)\left(-2\mathbf{a}-3\mathbf{b}+"-3\mathbf{a}+\mathbf{b}"\right)$
- $\overrightarrow{AX}=\mu\overrightarrow{AN}=\mu\left(2\mathbf{a}+3\mathbf{b}+"3\mathbf{a}-\mathbf{b}"\right)$
- $\overrightarrow{AX}=\overrightarrow{AB}+\lambda\overrightarrow{BD}=\left(2\mathbf{a}+3\mathbf{b}\right)+\lambda\left(4\mathbf{a}-5\mathbf{b}\right)$
- $\overrightarrow{AX}=\overrightarrow{AD}+\left(1-\lambda\right)\overrightarrow{DB}=\left("6\mathbf{a}-2\mathbf{b}"\right)+\left(1-\lambda\right)\left(-4\mathbf{a}+5\mathbf{b}\right)$
- $\overrightarrow{XN}=\mu\overrightarrow{AN}=\mu\left(2\mathbf{a}+3\mathbf{b}+"3\mathbf{a}-\mathbf{b}"\right)$
- $\overrightarrow{XN}=\lambda\overrightarrow{DB}+\overrightarrow{BN}=\lambda\left(-4\mathbf{a}+5\mathbf{b}\right)+\left("3\mathbf{a}-\mathbf{b}"\right)$
- $\overrightarrow{XN}=\left(1-\lambda\right)\overrightarrow{BD}+\overrightarrow{DN}=\left(1-\lambda\right)\left(4\mathbf{a}-5\mathbf{b}\right)+\left("-\mathbf{a}+4\mathbf{b}"\right)$

or alternatives using *C* or *N* as starting points, but these are unlikely.

Question	Scheme	Marks	AOs

7(a)	$\left(\overrightarrow{AB} =\right) 3\mathbf{i}+9\mathbf{j}+3\mathbf{k}$	B1	1.1b
		(1)	

Notes for (a)

B1: Correct vector. Allow $3\mathbf{i}+9\mathbf{j}+3\mathbf{k}$ or $\begin{pmatrix}3\\9\\3\end{pmatrix}$ but **not** $\begin{pmatrix}3\mathbf{i}\\9\mathbf{j}\\3\mathbf{k}\end{pmatrix}$ and **not** $(3, 9, 3)$

Condone $\begin{matrix}3\\9\\3\end{matrix}$ for $\begin{pmatrix}3\\9\\3\end{pmatrix}$

Do **not** apply isw here but award for e.g. $3\mathbf{i}+9\mathbf{j}+3\mathbf{k}=\begin{pmatrix}3\mathbf{i}\\9\mathbf{j}\\3\mathbf{k}\end{pmatrix}$

E.g. if they obtain $\overrightarrow{AB}=3\mathbf{i}+9\mathbf{j}+3\mathbf{k}$ and then say $\overrightarrow{AB}=\mathbf{i}+3\mathbf{j}+\mathbf{k}$ then award B0

If part (a) is **not attempted** and the correct $\overrightarrow{AB}$ is seen in part (b) the B1 can be awarded there.

General Guidance for part (b):

As with most vector questions we will see a variety of approaches (correct and incorrect).

In general, the marks are awarded as follows:

- **M1 for a correct complete strategy to find at least one position for *P* (May be implied by at least 2 correct components)**
- **A1 for one correct position for *P***
- **dM1 for a correct complete strategy to find both positions for *P* (May be implied by at least 2 correct components for both positions)**
- **A1 both correct positions for *P* and no others**

Various examples are shown below.
Other methods will be seen but the above marking principles should be applied.
You can condone slips in their algebra/processing as long as the intention is clear.
The examples given below give the detail to look for depending on the approach.
If you see a response and you are not sure if it deserves credit use Review.

Note that adding vectors when they should be subtracting will generally score M0 but use review if necessary.

(b)	Examples: $\overrightarrow{OP} = \overrightarrow{OA} + 2\overrightarrow{AB} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + 2(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = ...$ or $\overrightarrow{OP} = \overrightarrow{OB} + \overrightarrow{AB} = 5\mathbf{i} + 6\mathbf{j} + 8\mathbf{k} + (3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = ...$ or $\overrightarrow{OP} = \overrightarrow{OA} + \frac{2}{3}\overrightarrow{AB} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \frac{2}{3}(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = ...$ or $\overrightarrow{OP} = \overrightarrow{OB} + \frac{1}{3}\overrightarrow{BA} = 5\mathbf{i} + 6\mathbf{j} + 8\mathbf{k} - \frac{1}{3}(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = ...$	M1	3.1a
	$8\mathbf{i} + 15\mathbf{j} + 11\mathbf{k}$ or $4\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$	A1	1.1b
	Examples: $\overrightarrow{OP} = \overrightarrow{OA} + 2\overrightarrow{AB} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + 2(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = ...$ or $\overrightarrow{OP} = \overrightarrow{OB} + \overrightarrow{AB} = 5\mathbf{i} + 6\mathbf{j} + 8\mathbf{k} + (3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = ...$ **and** $\overrightarrow{OP} = \overrightarrow{OA} + \frac{2}{3}\overrightarrow{AB} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \frac{2}{3}(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = ...$ or $\overrightarrow{OP} = \overrightarrow{OB} + \frac{1}{3}\overrightarrow{BA} = 5\mathbf{i} + 6\mathbf{j} + 8\mathbf{k} - \frac{1}{3}(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = ...$	dM1	3.1a
	$8\mathbf{i} + 15\mathbf{j} + 11\mathbf{k}$ and $4\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$	A1	2.2a
		(4)	
			(5 marks)

Notes for (b)

Note that sight of at least one correct position for *P* implies M1A1

M1: Attempts at least one correct strategy for finding *P*

A1: One correct position vector or allow coordinates for this mark e.g. (8, 15, 11) or (4, 3, 7) or $x = \ldots, y = \ldots, z = \ldots$

If given as a vector, allow e.g. $8\mathbf{i} + 15\mathbf{j} + 11\mathbf{k}, \begin{pmatrix} 8 \\ 15 \\ 11 \end{pmatrix}$ but not $\begin{pmatrix} 8\mathbf{i} \\ 15\mathbf{j} \\ 11\mathbf{k} \end{pmatrix}$

dM1: Attempts two correct strategies for finding *P*

A1: Both correct position **vectors**

Must both be vectors so e.g. $8\mathbf{i} + 15\mathbf{j} + 11\mathbf{k}, \begin{pmatrix} 8 \\ 15 \\ 11 \end{pmatrix}$ and $4\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}, \begin{pmatrix} 4 \\ 3 \\ 7 \end{pmatrix}$ but not e.g. $\begin{pmatrix} 8\mathbf{i} \\ 15\mathbf{j} \\ 11\mathbf{k} \end{pmatrix}$

Condone e.g. $\begin{matrix} 8 \\ 15 \\ 11 \end{matrix}$ for $\begin{pmatrix} 8 \\ 15 \\ 11 \end{pmatrix}$

Alternative 1 using vector equation of *l*:

$$\mathbf{r} = \overrightarrow{OA} + \lambda\overrightarrow{AB} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 9 \\ 3 \end{pmatrix} \quad \left(\text{or e.g.} \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \right)$$

$$\left|\overrightarrow{AP}\right| = 2\left|\overrightarrow{BP}\right| \Rightarrow \left| \begin{pmatrix} 3\lambda \\ 9\lambda \\ 3\lambda \end{pmatrix} \right| = 2 \left| \begin{pmatrix} 3\lambda + 2 - 5 \\ 9\lambda - 3 - 6 \\ 3\lambda + 5 - 8 \end{pmatrix} \right| \Rightarrow 9\lambda^2 + 81\lambda^2 + 9\lambda^2 = 4\left[(3\lambda - 3)^2 + (9\lambda - 9)^2 + (3\lambda - 3)^2 \right]$$

$$\Rightarrow \lambda = 2, \frac{2}{3} \Rightarrow \mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + 2 \begin{pmatrix} 3 \\ 9 \\ 3 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + \frac{2}{3} \begin{pmatrix} 3 \\ 9 \\ 3 \end{pmatrix}$$

M1: Forms the vector equation of line *l* using their $\overrightarrow{AB}$ from part (a) or by starting again, forms the vectors $\overrightarrow{AP}$ and $\overrightarrow{BP}$ then uses $\left|\overrightarrow{AP}\right| = 2\left|\overrightarrow{BP}\right|$ and Pythagoras to produce a quadratic equation in "λ" which they then solve to find "λ" and use correctly to find at least one position for *P*.
Note if they use **i** + 3**j** + **k** for the direction, they should get $\lambda = 6, 2$
If all other work is correct, condone not squaring the "2" when applying Pythagoras

A1: See main scheme

dM1: As the first M and finds both positions for *P*

A1: See main scheme

Alternative 2 using *P* as $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and that $\overrightarrow{AP}$ and $\overrightarrow{BP}$ are parallel:

$$\overrightarrow{OP} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \overrightarrow{AP} = \begin{pmatrix} x - 2 \\ y + 3 \\ z - 5 \end{pmatrix}, \ \overrightarrow{BP} = \begin{pmatrix} x - 5 \\ y - 6 \\ z - 8 \end{pmatrix}$$

$$\left|\overrightarrow{AP}\right| = 2\left|\overrightarrow{BP}\right| \Rightarrow \left| \begin{pmatrix} x - 2 \\ y + 3 \\ z - 5 \end{pmatrix} \right| = 2 \left| \begin{pmatrix} x - 5 \\ y - 6 \\ z - 8 \end{pmatrix} \right| \Rightarrow \begin{matrix} (x-2)^2 = 4(x-5)^2 \\ (y+3)^2 = 4(y-6)^2 \\ (z-5)^2 = 4(z-8)^2 \end{matrix}$$

$$\begin{matrix} (x-2)^2 = 4(x-5)^2 \Rightarrow x = 4, 8 \\ (y+3)^2 = 4(y-6)^2 \Rightarrow y = 3, 15 \\ (z-5)^2 = 4(z-8)^2 \Rightarrow z = 7, 11 \end{matrix} \Rightarrow \begin{pmatrix} 4 \\ 3 \\ 7 \end{pmatrix}, \begin{pmatrix} 8 \\ 15 \\ 11 \end{pmatrix}$$

M1: Sets *P* as a general point, forms $\overrightarrow{AP}$ and $\overrightarrow{BP}$ (either way round) then uses $\left|\overrightarrow{AP}\right| = 2\left|\overrightarrow{BP}\right|$ then squares components and equates to produce quadratic equations in *x* and *y* and *z* which they then solve to find at least one position for *P*. It is not just for finding values which are not then used to form a point (or vector).
If all other work is correct, condone not squaring the "2" when squaring.

A1: See main scheme

dM1: As the first M and finds both positions for *P*.

A1: See main scheme

Note that if the modulus is not used, this method can lead to one correct position for P e.g.

$$\overrightarrow{AP} = 2\overrightarrow{BP} \Rightarrow \begin{pmatrix} x-2 \\ y+3 \\ z-5 \end{pmatrix} = 2\begin{pmatrix} x-5 \\ y-6 \\ z-8 \end{pmatrix} \Rightarrow \begin{matrix} x=8 \\ y=15 \\ z=11 \end{matrix} \text{ and scores M1 A1}$$

But it is possible to find the other position without squaring e.g.

$$\left|\overrightarrow{AP}\right| = 2\left|\overrightarrow{BP}\right| \Rightarrow \begin{pmatrix} x-2 \\ y+3 \\ z-5 \end{pmatrix} = 2\begin{pmatrix} 5-x \\ 6-y \\ 8-z \end{pmatrix} \Rightarrow \begin{matrix} x=4 \\ y=3 \\ z=7 \end{matrix} \text{ and scores dM1 then A1 as main scheme.}$$

This requires at least 2 correct equations for x, y or z for the dM1

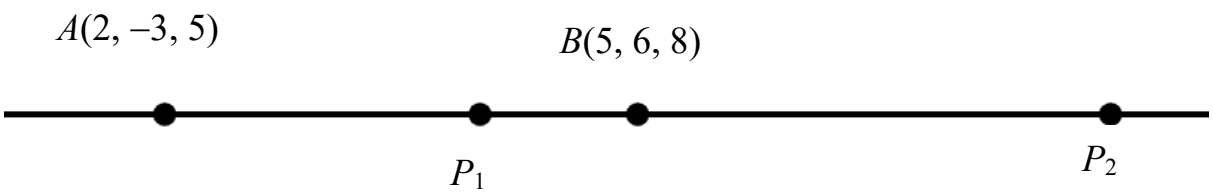
e.g. Alternative 3 using *P* as $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and eliminating 2 of the variables:

$$\overrightarrow{OP} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \overrightarrow{AP} = \begin{pmatrix} x-2 \\ y+3 \\ z-5 \end{pmatrix},\ \overrightarrow{BP} = \begin{pmatrix} x-5 \\ y-6 \\ z-8 \end{pmatrix}$$

$$\overrightarrow{AP} = \begin{pmatrix} x-2 \\ y+3 \\ z-5 \end{pmatrix} = \lambda\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \Rightarrow \overrightarrow{AP} = \begin{pmatrix} x-2 \\ 3x-6 \\ x-2 \end{pmatrix},\ \overrightarrow{BP} = \begin{pmatrix} x-5 \\ y-6 \\ z-8 \end{pmatrix} = \mu\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \Rightarrow \overrightarrow{BP} = \begin{pmatrix} x-5 \\ 3x-15 \\ x-5 \end{pmatrix}$$

$$\left|\overrightarrow{AP}\right| = 2\left|\overrightarrow{BP}\right| \Rightarrow (x-2)^2 + (3x-6)^2 + (x-2)^2 = 4\left[(x-5)^2 + (3x-15)^2 + (x-5)^2\right] \Rightarrow x = 4,\ 8$$

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}\left(\text{or } \overrightarrow{OB} + \overrightarrow{BP}\right) = \begin{pmatrix} x \\ 3x-9 \\ x+3 \end{pmatrix} \Rightarrow \begin{pmatrix} 4 \\ 3 \\ 7 \end{pmatrix} \text{ or } \begin{pmatrix} 8 \\ 15 \\ 11 \end{pmatrix}$$



Question	Scheme	Marks	AOs

3(a)	$\left(\lvert\overrightarrow{OA}\rvert =\right)\sqrt{5^2+3^2+2^2}\ =\sqrt{38}$ *	B1*	1.1b
		(1)	
(b)	$\lvert\overrightarrow{OB}\rvert=\sqrt{2^2+4^2+a^2}\ =\sqrt{20+a^2}$ so when $a=5$ $\lvert\overrightarrow{OB}\rvert=\sqrt{20+25}=\sqrt{45}$	M1	1.1b
	= 5	A1cso	2.3
		(2)	
			(3 marks)

Notes

(a)

B1*: Shows the magnitude of $\lvert\overrightarrow{OA}\rvert$ is $\sqrt{38}$. Must see $\sqrt{5^2+3^2+2^2}$ or e.g. $\sqrt{25+9+4}$. We need to see how the value 38 or $\sqrt{38}$ is formed using the three components. Withhold this mark for incorrect working such as $\lvert\overrightarrow{OA}\rvert=5^2+3^2+2^2=38\Rightarrow\lvert\overrightarrow{OA}\rvert=\sqrt{38}$ but do not penalise poor notation to denote vectors or the magnitude as long as the intention is clear as to what they are finding (*OA* instead of $\lvert\overrightarrow{OA}\rvert$ is fine). Do not penalise if their square root does not go fully over all three terms as long as the intention is clear. May find $\lvert\overrightarrow{AO}\rvert$ instead which is acceptable.

$\left(\lvert\overrightarrow{OA}\rvert^2=\right)25+9+4=38\Rightarrow\left(\lvert\overrightarrow{OA}\rvert=\right)\sqrt{38}$ scores B1 (we see how 38 is found)

$\left(\lvert\overrightarrow{OA}\rvert^2=\right)25+9+4\Rightarrow\sqrt{38}$ scores B1 (we see how 38 is found)

$25+9+4=\sqrt{38}$ scores B0 (they are not equal)

$\left(\lvert\overrightarrow{OA}\rvert^2=\right)38\Rightarrow\left(\lvert\overrightarrow{OA}\rvert=\right)\sqrt{38}$ scores B0 (no method seen to show how 38 is found)

(b)

M1: Attempts to find $\lvert\overrightarrow{OB}\rvert$ (or $\lvert\overrightarrow{OB}\rvert^2$) in terms of *a* and substitutes in a positive integer for *a* to find a value for $\lvert\overrightarrow{OB}\rvert$ (or $\lvert\overrightarrow{OB}\rvert^2$). e.g. $\lvert\overrightarrow{OB}\rvert=\sqrt{20+a^2}\Rightarrow$ when $a=2\Rightarrow\sqrt{24}$. Also accept e.g. $\lvert\overrightarrow{BO}\rvert$ (or $\lvert\overrightarrow{BO}\rvert^2$).

Alternatively sets up an equation or an inequality e.g. $\sqrt{20+a^2}>\sqrt{38}$ and proceeds to $a^2>...$ (or $a^2=...$). Condone sign slips in their rearrangement only.

Allow the use of =, < or > for this mark. $"20"+a^2...\ 38\Rightarrow a^2..."18"$

(may be implied by sight of $\sqrt{18}=4.24...$)

A1: 5 cso (answer on its own with no incorrect working seen scores M1A1). Withhold this mark if $\lvert\overrightarrow{OB}\rvert$ (or $\lvert\overrightarrow{OB}\rvert^2$) is incorrect (or $\lvert\overrightarrow{BO}\rvert$ or $\lvert\overrightarrow{BO}\rvert^2$). Do not be concerned with the notation as long as the intention is clear or implied as to what they are trying to calculate (the calculations must be correct)

Withhold this mark if at any point they set $\lvert\overrightarrow{OB}\rvert<\lvert\overrightarrow{OA}\rvert$ but accept an argument leading to an answer of 5 from either $\lvert\overrightarrow{OB}\rvert>\lvert\overrightarrow{OA}\rvert$ or $\lvert\overrightarrow{OB}\rvert=\lvert\overrightarrow{OA}\rvert$

Question	Scheme	Marks	AOs

6(a)	$\overrightarrow{AD} = 10\mathbf{i} + 24\mathbf{j}$ and $\overrightarrow{BC} = 50\mathbf{i} + 120\mathbf{j}$	**M1**	1.1b
	$\overrightarrow{AD} = \frac{1}{5}\overrightarrow{BC}$ therefore AD is parallel to BC *	**A1*cso**	2.2a
		(2)	
(b)	Attempt to find at least two lengths between AB, BC, CD and AD $\left\|\overrightarrow{BC}\right\| = \sqrt{50^2 + 120^2} = 130, \quad \left\|\overrightarrow{DA}\right\| = \sqrt{10^2 + 24^2} = 26$	**M1**	1.1b
	$\left\|\overrightarrow{AB}\right\| = \sqrt{12^2 + 16^2} = 20, \quad \left\|\overrightarrow{CD}\right\| = \sqrt{28^2 + 112^2} = 28\sqrt{17}$ (awrt 115 m)	**A1**	1.1b
	Average speed $= \dfrac{2\left(26+130+20+28\sqrt{17}\right)\big/_{1000}}{5\big/_{60}}$	**dM1**	3.1b
	awrt $= 6.99$ (km/h)	**A1**	3.2a
		(4)	
			(6 marks)

Notes

(a)

M1: Attempts to find both $\overrightarrow{AD} = \pm(10\mathbf{i} + 24\mathbf{j})$ and $\overrightarrow{BC} = \pm(50\mathbf{i} + 120\mathbf{j})$.

May be seen as column vectors.

Condone poor notation with column vectors e.g. $\begin{pmatrix} 10\mathbf{i} \\ 24\mathbf{j} \end{pmatrix}$ or $\begin{matrix} 10 \\ 24 \end{matrix}$ for $\begin{pmatrix} 10 \\ 24 \end{pmatrix}$

This mark can be scored for at least one correct component for each vector if no method is shown.

May be implied if they go straight for ratios (gradients) e.g. $\pm\frac{24-0}{22-12}$, $\pm\frac{16-136}{0-50}$, $\pm\frac{0-50}{16-136}$

Some candidates use distances in an attempt to prove part (a) e.g. finding $10^2 + 24^2$ and $50^2 + 120^2$ in which case the M1 can be implied. Adding vectors scores M0

A1*cso: This mark requires

- correct work showing AD is parallel to BC by showing that e.g. $\overrightarrow{AD} = \pm\frac{1}{5}\overrightarrow{BC}$ or equivalent e.g. $\overrightarrow{BC} = \pm 5\overrightarrow{AD}$ or e.g. $\overrightarrow{AD} = 2(5\mathbf{i} + 12\mathbf{j})$ $\overrightarrow{BC} = 10(5\mathbf{i} + 12\mathbf{j})$ or e.g. $BC = \pm 5AD$ (i.e. the vector arrows are not required) or by showing the ratio/gradient of the lines through AD and BC are equal e.g. $\frac{24}{10} = \frac{120}{50}$. Condone e.g. $\frac{50\mathbf{i} + 120\mathbf{j}}{10\mathbf{i} + 24\mathbf{j}} = 5$
- a (minimal) conclusion e.g. ✓, hence shown, etc. which may be in a preamble e.g. if they are parallel then $AD = kBC$…etc. If using ratios/gradients they need to say that they are parallel.
- vectors correctly calculated but allow e.g. $\overrightarrow{AD} = -10\mathbf{i} - 24\mathbf{j}$ and allow poor column vector notation as above

Using reciprocal gradients for both is acceptable for A1 even if they call them "gradients".

Do not credit work in part (b) in part (a) unless used in part (a)

(b)

M1: Attempts to use Pythagoras to find **at least two** of the lengths of the quadrilateral.

May be implied by at least 2 correct lengths.

For reference $\pm\overrightarrow{AB} = \pm(-12\mathbf{i} + 16\mathbf{j})$, $\pm\overrightarrow{BC} = \pm(50\mathbf{i} + 120\mathbf{j})$, $\pm\overrightarrow{CD} = \pm(-28\mathbf{i} - 112\mathbf{j})$, $\pm\overrightarrow{DA} = \pm(10\mathbf{i} + 24\mathbf{j})$

Allow ft if using their vectors from part (a) provided subtraction was used. Do not be concerned about the signs of individual components but must be using subtraction (but condone $\pm\overrightarrow{AB} = \pm(12\mathbf{i} + 16\mathbf{j})$) to find the vectors and then squaring and adding components and then taking the square root.

A1: At least 2 lengths correct: If units are given they must be correct.

$\left|\overrightarrow{AB}\right| = \sqrt{12^2+16^2} = 20,$ $\left|\overrightarrow{CD}\right| = \sqrt{28^2+112^2} = 28\sqrt{17}$ (allow awrt 115 m)

$\left|\overrightarrow{BC}\right| = 10\sqrt{5^2+12^2} = 130,$ $\left|\overrightarrow{DA}\right| = 2\sqrt{5^2+12^2} = 26$

Allow if they are working in km e.g. $\left|\overrightarrow{AB}\right| = 0.02$ etc.

M1A1 is implied by a total distance of awrt 291 (m) or possibly a multiple of this if they are doubling (awrt 583) or e.g. multiplying by 24 (awrt 6990) etc.

dM1: For an attempt at an average speed ignoring any attempt to get the correct units.

They must have attempted all 4 lengths for this mark.

There must be some indication that they have divided by 5 but this may be implied.

Allow this mark if they calculate the average speed for 2 laps or 1 lap e.g.

$\frac{"291"\times 2}{5}$, $\frac{"291"}{5}$, $"291"\times 12$, $"291"\times 2\times 12$ or e.g. if they divide by 2.5 or multiply by 24.

A1: awrt 6.99 (km/h). or anything which truncates to 6.99 e.g. 6.995. Units are **not** required but if they are given they must be correct. Isw once a correct answer is seen.

An exact answer is acceptable for the final A1 in (b): $4.224+0.672\sqrt{17}$

Special Case:

Some candidates are misinterpreting/misreading the position vector for *B* as 16**i** rather than 16**j**
This is usually implied by their vectors/ratios e.g.

$$\overrightarrow{AD} = \pm(10\mathbf{i}+24\mathbf{j}) \quad \text{and} \quad \overrightarrow{BC} = \pm(34\mathbf{i}+136\mathbf{j})$$

or e.g.

$$\pm\frac{24-0}{22-12},\ \pm\frac{50-16}{136-0}$$

For part (a), the maximum possible score is **M1A0** with the conditions for the M mark as described in the main scheme.

For part (b) the maximum possible score is **M1A1M1A0** as follows:

M1: Attempts to use Pythagoras to find **at least two** of the lengths of the quadrilateral as defined in the main scheme.

For reference $\pm\overrightarrow{AB} = \pm 4\mathbf{i}$, $\pm\overrightarrow{BC} = \pm(34\mathbf{i}+136\mathbf{j})$, $\pm\overrightarrow{CD} = \pm(-28\mathbf{i}-112\mathbf{j})$, $\pm\overrightarrow{DA} = \pm(10\mathbf{i}+24\mathbf{j})$

A1: Correct lengths for *AD* and *CD*. If units are given they must be correct.
This may **not** be scored for correct ft lengths for *AB* or *BC*
So requires both:

$\left|\overrightarrow{CD}\right| = \sqrt{28^2+112^2} = 28\sqrt{17}$ (allow awrt 115 m)

$\left|\overrightarrow{DA}\right| = \sqrt{10^2+24^2} = 26$

dM1: As above for an attempt at an average speed ignoring any attempt to get the correct units.

A0: Not available

If the position vector for *B* is not misinterpreted/misread in part (b) then full marks are available.

Question	Scheme	Marks	AOs

9(a)	Attempts both $\left\|\overrightarrow{PQ}\right\| = \sqrt{2^2+3^2+(-4)^2}$ and $\left\|\overrightarrow{QR}\right\| = \sqrt{5^2+(-2)^2}$	M1	3.1a
	States that $\left\|\overrightarrow{PQ}\right\| = \left\|\overrightarrow{QR}\right\| = \sqrt{29}$ so $PQRS$ is a rhombus	A1	2.4
		(2)	
(b)	Attempts BOTH $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = 7\mathbf{i}+3\mathbf{j}-6\mathbf{k}$ AND $\overrightarrow{QS} = -\overrightarrow{PQ} + \overrightarrow{PS} = 3\mathbf{i}-3\mathbf{j}+2\mathbf{k}$	M1	3.1a
	Correct $\overrightarrow{PR} = 7\mathbf{i}+3\mathbf{j}-6\mathbf{k}$ and $\overrightarrow{QS} = 3\mathbf{i}-3\mathbf{j}+2\mathbf{k}$	A1	1.1b
	Correct method for area $PQRS$. E.g. $\frac{1}{2}\times\left\|\overrightarrow{PR}\right\|\times\left\|\overrightarrow{QS}\right\|$	dM1	2.1
	$= \sqrt{517}$	A1	1.1b
		(4)	
			(6 marks)

Alt (b) Example using the cosine rule	Attempts $\left\|\overrightarrow{QS}\right\| = \sqrt{3^2+(-3)^2+2^2} = \sqrt{22}$ and so $22 = 29+29-2\sqrt{29}\sqrt{29}\cos SPQ$	M1	3.1a
	$\cos PQR = -\frac{18}{29}$ or $\cos SPQ = \frac{18}{29}$ Condone angles in degrees 51.6, 128.4 (1dp) or radians 2.24, 0.901 (3sf) here	A1	1.1b
	Correct method for area $PQRS$. E.g. $PQ\times QR\sin PQR = \sqrt{2^2+3^2+(-4)^2}\times\sqrt{5^2+(-2)^2}\times\frac{\sqrt{517}}{29}$	dM1	2.1
	$= \sqrt{517}$	A1	1.1b
		(4)	

FYI

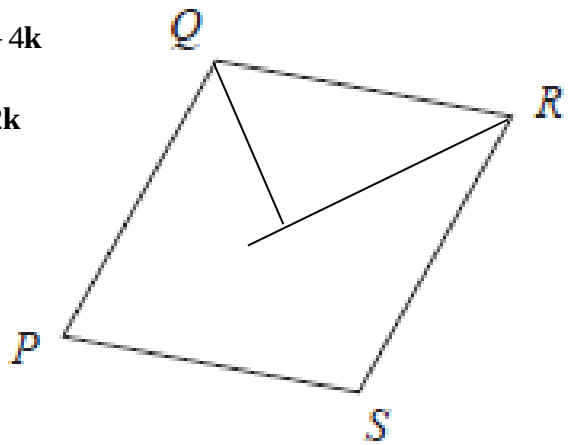
$\overrightarrow{QR} = 5\mathbf{i}+0\mathbf{j}-2\mathbf{k}$

$\overrightarrow{PQ} = 2\mathbf{i}+3\mathbf{j}-4\mathbf{k}$

$\overrightarrow{SQ} = -3\mathbf{i}+3\mathbf{j}-2\mathbf{k}$

$\overrightarrow{MQ} = -1.5\mathbf{i}+1.5\mathbf{j}-1\mathbf{k}$

$\overrightarrow{PR} = 7\mathbf{i}+3\mathbf{j}-6\mathbf{k}$



M

$\overrightarrow{PM} = 3.5\mathbf{i}+1.5\mathbf{j}-3\mathbf{k}$

(a) Do not award marks in part (a) from work in part (b).

M1: Attempts both $\left\|\overrightarrow{PQ}\right\| = \sqrt{2^2+3^2+(\pm4)^2}$ and $\left\|\overrightarrow{QR}\right\| = \sqrt{5^2+(\pm2)^2}$ or PQ^2 and QR^2. For this mark only, condone just the correct answers $\left\|\overrightarrow{PQ}\right\| = \sqrt{29}$ and $\left\|\overrightarrow{QR}\right\| = \sqrt{29}$. Alternatively attempts $\overrightarrow{PR}\bullet\overrightarrow{QS}$ or PM^2, MQ^2 and PQ^2 where M is the mid point of PR

A1: Shows that $\left\|\overrightarrow{PQ}\right\| = \left\|\overrightarrow{QR}\right\| = \sqrt{29}$ (with calculations) and states $PQRS$ is a rhombus

Requires both a reason and a conclusion. The reason may be given at the start of their solution.

In the alternatives $\overrightarrow{PR} \bullet \overrightarrow{QS} = (7\mathbf{i}+3\mathbf{j}-6\mathbf{k}) \bullet (3\mathbf{i}-3\mathbf{j}+2\mathbf{k}) = 21-9-12 = 0$ so diagonals cross at $90°$ so $PQRS$ is a rhombus or $PM^2 + MQ^2 = PQ^2 = 23.5+5.5 = 29 \Rightarrow \angle PMQ = 90° \Rightarrow$ Rhombus

(b) **Candidates can transfer answers from (a) to use in part (b) to find the area**

Look through their complete solution first. The first two marks are for finding the elements that are required to calculate the area. The second set of two marks is for combining these elements correctly. If the method is NOT shown on how to find vector it can be implied by two correct components. Allow as column vectors.

M1: For a key step in solving the problem. It is scored for attempting to find both key vectors.

Attempts both $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = (7\mathbf{i}+3\mathbf{j}-6\mathbf{k})$ **AND** $\overrightarrow{QS} = -\overrightarrow{PQ} + \overrightarrow{PS} = (3\mathbf{i}-3\mathbf{j}+2\mathbf{k})$

You may see $\overrightarrow{PM} = \frac{1}{2}\overrightarrow{PQ} + \frac{1}{2}\overrightarrow{QR} = \left(\frac{7}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}-3\mathbf{k}\right)$ **AND** $\overrightarrow{QM} = -\frac{1}{2}\overrightarrow{PQ} + \frac{1}{2}\overrightarrow{PS} = \left(\frac{3}{2}\mathbf{i}-\frac{3}{2}\mathbf{j}+1\mathbf{k}\right)$

A1: Accurately finds both key vectors whose lengths are required to solve the problem.

Score for both $\overrightarrow{PR} = 7\mathbf{i}+3\mathbf{j}-6\mathbf{k}$ and $\overrightarrow{QS} = 3\mathbf{i}-3\mathbf{j}+2\mathbf{k}$ (Allow either way around.)

or both $\overrightarrow{PM} = \frac{7}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}-3\mathbf{k}$ and $\overrightarrow{QM} = \frac{3}{2}\mathbf{i}-\frac{3}{2}\mathbf{j}+1\mathbf{k}$ (Allow either way around.)

dM1: Constructs a rigorous method leading to the area $PQRS$. Dependent upon previous M.

E.g. See scheme. Alt: the sum of the area of four right angled triangles e.g. $4 \times \frac{1}{2} \times \left|\overrightarrow{PM}\right| \times \left|\overrightarrow{QM}\right|$,

A1: $\sqrt{517}$

Alternatives for (b). Two such ways are set out below

Alt 1-Examples via cosine rule but you may see use of scalar product via a Further Maths method.

M1: For a key step in solving the problem. In this case it for an attempt at cos PQR or cos SPQ.

Don't be too concerned with the labelling of the angle which may appear as θ.

Attempts $\pm\begin{pmatrix}2\\3\\-4\end{pmatrix} \bullet \pm\begin{pmatrix}5\\0\\-2\end{pmatrix} = \sqrt{2^2+3^2+(-4)^2} \times \sqrt{5^2+(-2)^2} \cos PQR$

A1: Finds the cosine of one of the angles in the Figure.

Look for $\cos ... = -\frac{18}{29}$ or $\cos ... = \frac{18}{29}$ which may have been achieved via the cosine rule.

Accept rounded answers and the angles in degrees 51.6, 128.4 (1dp) or radians 2.24, 0.901 (3sf) here.

dM1: Constructs a rigorous method leading to the area $PQRS$. Implied by awrt 22.7

E.g. $PQ \times QR \sin PQR = \sqrt{2^2+3^2+(-4)^2} \times \sqrt{5^2+(-2)^2} \times \frac{\sqrt{517}}{29}$

A1: $\sqrt{517}$

Alt 2-Example via vector product via a Further Maths method.

M1: For a key step in solving the problem. In this case it for an attempt at $\pm\overrightarrow{PQ} \times \overrightarrow{QR}$

E.g. $\overrightarrow{PQ} \times \overrightarrow{QR} = \begin{pmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2 & 3 & -4\\ 5 & 0 & -2\end{pmatrix} = (3\times-2-0\times-4)\mathbf{i} - (2\times-2-5\times-4)\mathbf{j} + (2\times0-3\times5)\mathbf{k}$

A1: E.g. $\overrightarrow{PQ} \times \overrightarrow{QR} = -6\mathbf{i}-16\mathbf{j}-15\mathbf{k}$

dM1: Constructs a rigorous method leading to the area $PQRS$. In this case $\left|\overrightarrow{PQ} \times \overrightarrow{QR}\right|$

A1: $= \sqrt{(-6)^2+(-16)^2+(-15)^2} = \sqrt{517}$

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
13(a)	Attempts two of the relevant vectors $\pm\overrightarrow{AB} = \pm(-4\mathbf{i}+7\mathbf{j}+\mathbf{k})$ $\pm\overrightarrow{AC} = \pm(-20\mathbf{i}+(p+3)\mathbf{j}+5\mathbf{k})$ $\pm\overrightarrow{BC} = \pm(-16\mathbf{i}+(p-4)\mathbf{j}+4\mathbf{k})$	M1	3.1a
	Uses two of the three vectors in such a way as to find the value of p. E.g. $p+3=5\times 7$	dM1	2.1
	$p=32$	A1	1.1b
		(3)	
	(a) Alternative:		
	$r_{AB} = 4\mathbf{i}-3\mathbf{j}+5\mathbf{k}+\lambda(-4\mathbf{i}+7\mathbf{j}+\mathbf{k})$	M1	3.1a
	$4\mathbf{i}-3\mathbf{j}+5\mathbf{k}+\lambda(-4\mathbf{i}+7\mathbf{j}+\mathbf{k}) = -16\mathbf{i}+p\mathbf{j}+10\mathbf{k} \Rightarrow \lambda = 5$ $4\mathbf{i}-3\mathbf{j}+5\mathbf{k}+\lambda(-4\mathbf{i}+7\mathbf{j}+\mathbf{k}) = -16\mathbf{i}+p\mathbf{j}+10\mathbf{k} \Rightarrow p = 35-3$	dM1	2.1
	$p=32$	A1	1.1b
(b)	Deduces that $\overrightarrow{OD} = \lambda\overrightarrow{OB} = 4\lambda\mathbf{j}+6\lambda\mathbf{k}$ and attempts $\overrightarrow{CD} = 16\mathbf{i}+(4\lambda-"32")\mathbf{j}+(6\lambda-10)\mathbf{k}$	M1	3.1a
	Correct attempt at λ using the fact that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$ $\overrightarrow{CD} = 16\mathbf{i}+(4\lambda-"32")\mathbf{j}+(6\lambda-10)\mathbf{k}$ $\overrightarrow{OA} = 4\mathbf{i}-3\mathbf{j}+5\mathbf{k}$ $4\lambda-32=-12\Rightarrow\lambda=\ldots$ OR $6\lambda-10=20\Rightarrow\lambda=\ldots$	dM1	1.1b
	$\left\lvert\overrightarrow{OD}\right\rvert = 5\times\sqrt{4^2+6^2} = 10\sqrt{13}$	A1	1.1b
		(3)	
	(b) Alternative:		
	Deduces that $\overrightarrow{OD} = \lambda\overrightarrow{OB} = 4\lambda\mathbf{j}+6\lambda\mathbf{k}$ and attempts $\overrightarrow{OD} = \overrightarrow{OC}+\mu\overrightarrow{OA} = -16\mathbf{i}+32\mathbf{j}+10\mathbf{k}+\mu(4\mathbf{i}-3\mathbf{j}+5\mathbf{k})$	M1	3.1a
	Correct attempt at λ **or** μ using the fact that $\lambda\overrightarrow{OB} = \overrightarrow{OC}+\mu\overrightarrow{OA}$ E.g. $-16+4\mu=0\Rightarrow\mu=4$	dM1	1.1b
	$\left\lvert\overrightarrow{OD}\right\rvert = 5\times\sqrt{4^2+6^2} = 10\sqrt{13}$	A1	1.1b
		(3)	
			(6 marks)

Notes:

(a)

M1: Attempts two of the three relevant vectors by **subtracting** either way around. See scheme.

Allow equivalent work e.g. $\pm\overrightarrow{AB} = \pm(\overrightarrow{OB}+\overrightarrow{AO})$

E.g. $p+3=5\times 7,\ \ p-4=\frac{4}{5}(p+3),\ p-4=4\times 7$

A1: $p=32$ (Condone 32**j**)

For reference, $\overrightarrow{BC}=4\overrightarrow{AB},\ \overrightarrow{AC}=5\overrightarrow{AB},\ \overrightarrow{BC}=\frac{4}{5}\overrightarrow{AC},\ \overrightarrow{AC}=\frac{5}{4}\overrightarrow{BC}$

Note that candidates generally only need to use 2 components to find *p* and if the 3rd component has errors but is not used, full marks can be awarded.

Alternative:

M1: Forms the vector equation using *A* or *B* as position and $\pm\overrightarrow{AB}$ as the direction

dM1: For the key step in using the fact that *C* lies on the line to find *p*

A1: $p=32$ (Condone 32**j**)

For reference, $\overrightarrow{BC}=4\overrightarrow{AB},\ \overrightarrow{AC}=5\overrightarrow{AB},\ \overrightarrow{BC}=\frac{4}{5}\overrightarrow{AC},\ \overrightarrow{AC}=\frac{5}{4}\overrightarrow{BC}$

Note that candidates generally only need to use 2 components to find *p* and if the 3rd component has errors but is not used, full marks can be awarded.

There will be other approaches e.g. using "gradients" and "ratios" and the method marks can be implied – if you are unsure if such attempts deserve credit use Review

(b) Vector approach

M1: Deduces that $\overrightarrow{OD}=\lambda\overrightarrow{OB}=4\lambda\mathbf{j}+6\lambda\mathbf{k}$ and attempts $\overrightarrow{CD}=16\mathbf{i}+(4\lambda-"32")\mathbf{j}+(6\lambda-10)\mathbf{k}$

dM1: Correct attempt at finding λ using the fact that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$

E.g. $16\mathbf{i}+(4\lambda-"32")\mathbf{j}+(6\lambda-10)\mathbf{k}=4\alpha\mathbf{i}-3\alpha\mathbf{j}+5\alpha\mathbf{k}\Rightarrow\alpha=4\Rightarrow 4\lambda-"32"=-3\times"4"\Rightarrow\lambda=...$

A1: $\left|\overrightarrow{OD}\right|=10\sqrt{13}$

Alternative:

M1: Deduces that $\overrightarrow{OD}=\lambda\overrightarrow{OB}=4\lambda\mathbf{j}+6\lambda\mathbf{k}$ and attempts

$\overrightarrow{OD}=\overrightarrow{OC}+\mu\overrightarrow{OA}=-16\mathbf{i}+32\mathbf{j}+10\mathbf{k}+\mu(4\mathbf{i}-3\mathbf{j}+5\mathbf{k})$

dM1: Correct attempt at finding λ or μ using the fact that $\lambda\overrightarrow{OB}=\overrightarrow{OC}+\mu\overrightarrow{OA}$

E.g. $(-16+4\mu)\mathbf{i}+("32"-3\mu)\mathbf{j}+(10+5\mu)\mathbf{k}=4\lambda\mathbf{j}+6\lambda\mathbf{k}\Rightarrow -16+4\mu=0\Rightarrow\mu=...$

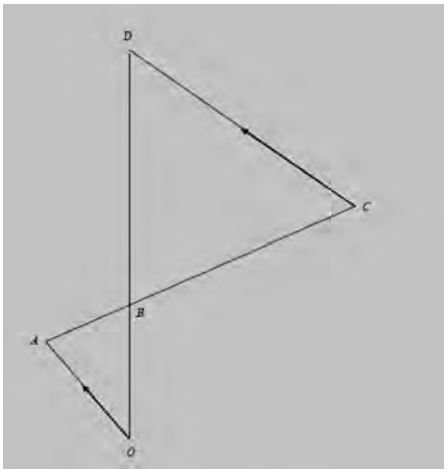
May also solve simultaneously using *y* and *z* components to find λ or μ

A1: $\left|\overrightarrow{OD}\right|=10\sqrt{13}$

Note that the correct vector is 20**j** + 30**k**

PTO for similar triangle approach

(b) Similar triangle approach



M1: For the key step in recognising that triangle *BCD* and triangle *BAO* are similar with a ratio of lengths of 4:1

dM1: States or uses the fact that $\left|\overrightarrow{OD}\right| = 5 \times \left|\overrightarrow{OB}\right|$

Stating this will score M1 dM1 provided there is no evidence of incorrect work

Note that they may establish this result using the work from (a) but must be used here to score.

A1: $\left|\overrightarrow{OD}\right| = 10\sqrt{13}$

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
3 (a)	$\overrightarrow{AB} = (3\mathbf{i}-3\mathbf{j}-4\mathbf{k})-(2\mathbf{i}+5\mathbf{j}-6\mathbf{k})$	M1	1.1b
	$= \mathbf{i}-8\mathbf{j}+2\mathbf{k}$	A1	1.1b
		(2)	
(b)	States $\overrightarrow{OC} = 2\times\overrightarrow{AB}$	M1	1.1b
	Explains that as *OC* is parallel to *AB*, so *OABC* is a trapezium.	A1	2.4
		(2)	
			(4 marks)
Notes:			

(a)

M1: Attempts to subtract either way around. If no method is seen it is implied by two of $\pm 1\mathbf{i} \pm 8\mathbf{j} \pm 2\mathbf{k}$.

A1: $\mathbf{i}-8\mathbf{j}+2\mathbf{k}$ or $\begin{pmatrix}1\\-8\\2\end{pmatrix}$ but not $(1,-8,2)$

(b)

M1: Compares their $\mathbf{i}-8\mathbf{j}+2\mathbf{k}$ with $2\mathbf{i}-16\mathbf{j}+4\mathbf{k}$ by stating **any one of**

- $\overrightarrow{OC} = 2\times\overrightarrow{AB}$
- $\begin{pmatrix}2\\-16\\4\end{pmatrix} = 2\times\begin{pmatrix}1\\-8\\2\end{pmatrix}$
- $\overrightarrow{OC} = \lambda\times\overrightarrow{AB}$ or vice versa

This may be awarded if *AB* was subtracted "the wrong way around" or if there was one numerical slip

A1: A full explanation as to why *OABC* is a trapezium.

Requires fully correct calculations, so part (a) must be $\overrightarrow{AB} = (\mathbf{i}-8\mathbf{j}+2\mathbf{k})$

It requires a reason and minimal conclusion.

Example 1:

$\overrightarrow{OC} = 2\times\overrightarrow{AB}$, therefore *OC* is parallel to *AB* so *OABC* is a trapezium

Example 2:

A trapezium has one pair of parallel sides. As $\overrightarrow{OC} = 2\times\overrightarrow{AB}$, they are parallel, so ✓.

Example 3

As $\begin{pmatrix}2\\-16\\4\end{pmatrix} = 2\times\begin{pmatrix}1\\-8\\2\end{pmatrix}$, *OC* and *AB* are parallel, so proven

Example 4

Accept as $\overrightarrow{OC} = \lambda\times\overrightarrow{AB}$, they are parallel so true

Note: There are two definitions for a trapezium. One stating that it is a shape with one pair of parallel sides, the other with **only one** pair of parallel sides. Any calculations to do with sides *OA* and *CB* in this question may be ignored, even if incorrect.

Question	Scheme	Marks	AOs

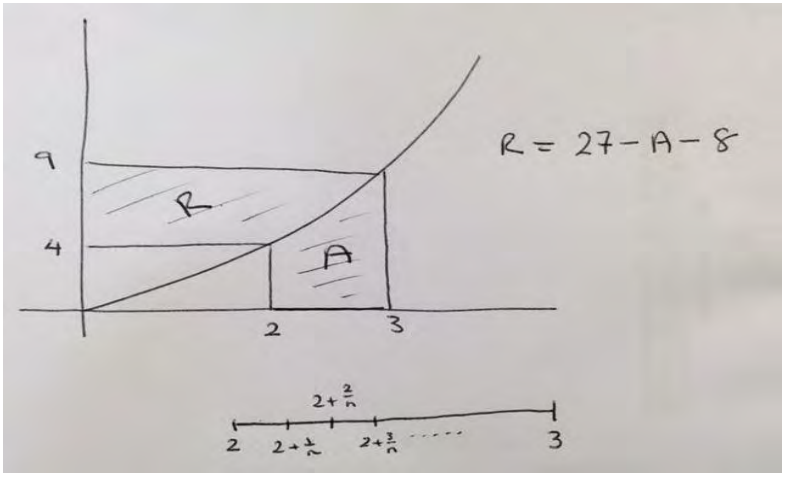
5	States $\left\{\lim_{\delta x\to 0}\sum_{x=4}^{9}\sqrt{x}\,\delta x \text{ is}\right\}\ \int_4^9 \sqrt{x}\,\mathrm{d}x$	B1	1.2
	$= \left[\frac{2}{3}x^{\frac{3}{2}}\right]_4^9$	M1	1.1b
	$= \frac{2}{3}\times 9^{\frac{3}{2}} - \frac{2}{3}\times 4^{\frac{3}{2}} = \frac{54}{3} - \frac{16}{3}$		
	$= \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7	A1	1.1b
		(3)	

(3 marks)

Notes for Question 5

B1:	States $\int_4^9 \sqrt{x}\,\mathrm{d}x$ with or without the '$\mathrm{d}x$'
M1:	Integrates $\sqrt{x}$ to give $\lambda x^{\frac{3}{2}};\ \lambda \neq 0$
A1:	See scheme
Note:	You can imply B1 for $\left[\lambda x^{\frac{3}{2}}\right]_4^9$ or for $\lambda\times 9^{\frac{3}{2}} - \lambda\times 4^{\frac{3}{2}}$
Note:	Give B0 for $\int_1^9 \sqrt{x}\,\mathrm{d}x - \int_1^3 \sqrt{x}\,\mathrm{d}x$ or for $\int_3^9 \sqrt{x}\,\mathrm{d}x$ without reference to a correct $\int_4^9 \sqrt{x}\,\mathrm{d}x$
Note:	Give B1 M1 A1 for no working leading to a correct $\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for $\int_4^9 \sqrt{x}\,\mathrm{d}x = \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for $\left[\frac{2}{3}x^{\frac{3}{2}} + c\right]_4^9 = \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for no working followed by an answer $\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give M0 A0 for use of a trapezium rule method to give an answer of awrt 12.7, but allow B1 if $\int_4^9 \sqrt{x}\,\mathrm{d}x$ is seen in a trapezium rule method
Note:	Otherwise, give B0 M0 A0 for using the trapezium rule to give an answer of awrt 12.7

Notes for Question 5 Continued	
Alt	The following method is correct:



$$\text{Area } (A) = \lim_{n\to\infty}\sum_{i=1}^{n}(x_i - x_{i-1})\mathrm{f}(x_i) = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\left(2+\frac{i}{n}\right)^2$$

$$= \lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n}4 + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{4i}{n}\right) + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{i^2}{n^2}\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n}4 + \frac{4}{n^2}\sum_{i=1}^{n}i + \frac{1}{n^3}\sum_{i=1}^{n}i^2\right]$$

$$= \lim_{n\to\infty}\left[\frac{4n}{n} + \frac{4}{n^2}\left(\frac{1}{2}n(n+1)\right) + \frac{1}{n^3}\left(\frac{1}{6}n(n+1)(2n+1)\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{4}{n} + \frac{4n^2+4n}{2n^2} + \frac{2n^3+3n^2+n}{6n^3}\right]$$

$$= \lim_{n\to\infty}\left[4 + 2 + \frac{2}{n} + \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}\right]$$

$$= 4 + 2 + \frac{1}{3} = \frac{19}{3}$$

$$\text{So, } \lim_{\delta x\to 0}\sum_{x=4}^{9}\sqrt{x}\,\delta x = \text{Area}(R) = (3\times 9) - (2\times 4) - \frac{19}{3}$$

$$= \frac{38}{3} \quad \text{or} \quad 12\frac{2}{3} \quad \text{or awrt } 12.7$$

Question	Scheme	Marks	AOs

2	$\overrightarrow{OA} = 2\mathbf{i}+3\mathbf{j}-4\mathbf{k},\ \overrightarrow{OB} = 4\mathbf{i}-2\mathbf{j}+3\mathbf{k},\ \overrightarrow{OC} = a\mathbf{i}+5\mathbf{j}-2\mathbf{k},\ a<0$ $\overrightarrow{AB} = \overrightarrow{BD},\ \left\lvert\overrightarrow{AB}\right\rvert = 4$		
(a)	E.g. $\overrightarrow{OD} = \overrightarrow{OB}+\overrightarrow{BD} = \overrightarrow{OB}+\overrightarrow{AB}$ **or** $\overrightarrow{OD} = \overrightarrow{OB}+\overrightarrow{BD} = \overrightarrow{OB}+\overrightarrow{AB} = \overrightarrow{OB}+\overrightarrow{OB}-\overrightarrow{OA} = 2\overrightarrow{OB}-\overrightarrow{OA}$ **or** $\overrightarrow{OD} = \overrightarrow{OB}+\overrightarrow{BD} = \overrightarrow{OB}+\overrightarrow{AB} = \overrightarrow{OA}+\overrightarrow{AB}+\overrightarrow{AB} = \overrightarrow{OA}+2\overrightarrow{AB}$		
	$= \begin{pmatrix}4\\-2\\3\end{pmatrix}+\begin{pmatrix}4\\-2\\3\end{pmatrix}-\begin{pmatrix}2\\3\\-4\end{pmatrix}\ \left\{= \begin{pmatrix}4\\-2\\3\end{pmatrix}+\begin{pmatrix}2\\-5\\7\end{pmatrix}\right\}$ **or** $= \begin{pmatrix}2\\3\\-4\end{pmatrix}+2\left(\begin{pmatrix}4\\-2\\3\end{pmatrix}-\begin{pmatrix}2\\3\\-4\end{pmatrix}\right)\ \left\{= \begin{pmatrix}2\\3\\-4\end{pmatrix}+2\begin{pmatrix}2\\-5\\7\end{pmatrix}\right\}$	M1	3.1a
	$= \begin{pmatrix}6\\-7\\10\end{pmatrix}$ or $6\mathbf{i}-7\mathbf{j}+10\mathbf{k}$	A1	1.1b
		(2)	
(b)	$(a-2)^2+(5-3)^2+(-2--4)^2$	M1	1.1b
	$\left\{\left\lvert\overrightarrow{AC}\right\rvert = 4 \Rightarrow\right\} (a-2)^2+(5-3)^2+(-2--4)^2 = (4)^2$ $\Rightarrow (a-2)^2 = 8 \Rightarrow a = \ldots$ **or** $\Rightarrow a^2-4a-4=0 \Rightarrow a = \ldots$	dM1	2.1
	(as $a<0 \Rightarrow$) $a = 2-2\sqrt{2}$ (or $a = 2-\sqrt{8}$)	A1	1.1b
		(3)	
			(5 marks)

Notes for Question 2

(a)	
M1:	Complete ***applied*** strategy to find a vector expression for $\overrightarrow{OD}$
A1:	See scheme
Note:	Give M0 for subtracting the wrong way wrong to give e.g. $(4\mathbf{i}-2\mathbf{j}+3\mathbf{k})+(2\mathbf{i}+3\mathbf{j}-4\mathbf{k})\ -(4\mathbf{i}-2\mathbf{j}+3\mathbf{k}) = (4\mathbf{i}-2\mathbf{j}+3\mathbf{k})+(-2\mathbf{i}+5\mathbf{j}-7\mathbf{k}) = (2\mathbf{i}+3\mathbf{j}-4\mathbf{k})$
Note:	Writing e.g. $\overrightarrow{OD} = \overrightarrow{OB}+\overrightarrow{AB}$ or $\overrightarrow{OD} = 2\overrightarrow{OB}-\overrightarrow{OA}$ with no other work is M0
Note:	Finding ***coordinates***, i.e. $(6,-7,10)$ without reference to the correct position vectors is A0
Note:	Allow M1A1 for writing down $6\mathbf{i}-7\mathbf{j}+10\mathbf{k}$ with no working
Note:	M1 can be implied for at least two correct components in their position vector of D
(b)	
M1:	Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OC}$, then squares and adds each of the 3 components **Note:** Ignore labelling
dM1:	Complete method of ***correctly*** applying Pythagoras' Theorem on $\left\lvert\overrightarrow{AC}\right\rvert = 4$ and using a correct method of solving their resulting quadratic equation to find at least one of $a = \ldots$
Note:	Condone at least one of either awrt 4.8 or awrt -0.83 for the dM mark
A1:	Obtains **only one** exact value, $a = 2-2\sqrt{2}$
Note:	Writing $a = 2\pm 2\sqrt{2}$, without evidence of rejecting $a = 2+2\sqrt{2}$ is A0
Note:	Allow exact alternatives such as $2-\sqrt{8}$ or $\frac{4-\sqrt{32}}{2}$ for A1, and isw can be applied
Note:	Writing $a = -0.828\ldots$, without reference to a correct exact value is A0

Question	Scheme	Marks	AOs
	Statement: "If m and n are irrational numbers, where $m \neq n$,		